Spontaneous Poloidal Spin-Up of Tokamaks and the Transition to H Mode

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(Received 9 October 1990)

The radial transport of toroidal angular momentum and circulation in a tokamak resulting from diffusion which is poloidally asymmetric is shown to produce an instability of the poloidal rotation. This instability, due to Stringer, sets in where the local particle-confinement time is smaller than the damping time of poloidal flow and leads to poloidal velocity shear. The nonlinear interplay between the poloidal spin-up and turbulence-driven anomalous transport is shown to lead to bifurcated equilibria of the type observed in the L-to-H-mode transition in tokamaks.

PACS numbers: 52.25.Fi, 52.55.−s

The success of future tokamak operation, especially insofar as the attainment of energy multiplication and eventual ignition is concerned, is crucially dependent on the quality of energy confinement. In recent years, the characterization of tokamak energy confinement has increasingly been made in terms of the so-called L and H modes of confinement.[1,2] The L mode is characterized by a decrease of confinement time with increasing input power. It has become evident, however, that beyond a critical power threshold, an enhanced-confinement regime (H mode) is achievable. Confinement time in the H mode can be up to a factor of 3 higher than in the L mode, and indications are that the L and H modes constitute a bifurcation in tokamak transport equilibria. In particular, the H-mode equilibria are characterized by the appearance of steep density and temperature profiles at the plasma edge and an almost complete disappearance, over the L mode, of edge microturbulence (see, e.g., Ref. 3).

Recently, there have come the discoveries, from the Continuous Current Tokamak[4] (CCT) and the DIII-D tokamak,[5] that the H-mode-like enhanced confinement is accompanied by and, perhaps, a result of a large increase in the poloidal rotation of the edge plasma. In the CCT device, improved confinement is achieved by applying an external torque to rotate the edge plasma; in DIII-D, the rotation is seen to set in spontaneously, with improvement in confinement appearing in a concomitant manner.

In this Letter, we present a theory to show that tokamaks can spontaneously develop large poloidal velocity shears. We show that the generation of such spontaneous rotation is favored at tokamak edges and that the rotation may extend inward over macroscopic scale sizes. This spontaneous spin-up is a direct consequence of *particle transport,* a feature which makes the theory highly appealing because of the well accepted experimental fact that the H mode is intimately related to edge particle confinement.[3] What is required for the spin-up to occur is that the particle transport be sufficiently poloidally asymmetric and that the rate of transport exceed the damping rate of poloidal rotation. Experimentally, a poloidally asymmetric particle transport is indicated in most tokamaks, and, for the present-day, hotter devices, the local rate of particle transport, especially at the edge, can exceed the local rate of poloidal damping.

We show further that there is an intimate relation among the spin-up, a consequence of particle loss, the fact that poloidal velocity shear can quell instability and microturbulence,[6,7] thus reducing particle loss, and the transition to the H mode. We demonstrate the existence of an L-H type bifurcation of the density profile, arising as a dynamic consequence of the interplay between the tendency of particle fluxes to initiate spin-up and the tendency of spin-up to quell the particle flux.

Several theories have been proposed to account for the L-H transition. Preferential loss of ions at the edges of the tokamak is well recognized as a mechanism whereby a radial electric field may be built up with concomitant rotation. This phenomenon, when considered in conjunction with attendant electron losses,[8] or the flow-dependent drag from parallel viscosity,[9] has led to models of the L-H transition that exhibit a bifurcation. The appearance of poloidal flow associated with the H-mode onset[4,5] has led to the realization that poloidal flow shear can quell microturbulence, thus causing a reduction in anomalous heat losses and improved confinement.[7] Finally, it has been suggested that the bifurcation can be explained by diamagnetic flow effects alone without recourse to a radial electric field.[10] The theory presented in this Letter proposes a mechanism for poloidal spin-up that does not rely on preferential, nonambipolar particle loss at the edge; rather, the theory suggests that spin-up is favored at the edges but may penetrate inward over macroscopic scale sizes, depending on the parameters of the experiment (for present day experiments, a penetration of order a few cm is indicated). The theory also has the feature that it does not depend on the tokamak collisionality parameter ν_{*i} *per se*; the poloidal spin-up may occur over a wide range in ν_{*i}.

The fact that tokamaks can spontaneously spin up poloidally was discovered by Stringer who showed that an initial poloidal rotation in the presence of Pfirsch-Schlüter particle diffusion was unstable.[11] It was pointed out, however, that poloidal rotation is strongly damped by magnetic pumping.[12,13] The Pfirsch-Schlüter transport rate being generally much smaller than the magnetic pumping rate, the Stringer spin-up was considered unimportant.

For tokamak experiments, however, the particle loss rate is much larger than that predicted by neoclassical theory, so spin-up is possible. A closer examination of the theory of poloidal spin-up reveals that the key ingredient necessary for instability is that particle transport be poloidally asymmetric. By incorporating a general poloidally dependent particle transport in the theory, we find that spin-up occurs on those surfaces where the poloidal asymmetry, measured by δ, the fractional difference between the particle transport inside and outside, is sufficiently large, and where the particle transport rate is larger than the damping due to magnetic pumping, roughly according to the condition [see also Eqs. (14) and (15)]

$$(\delta/\epsilon)D/L_n^2 > \gamma_{\mathrm{MP}}\,. \tag{1}$$

In Eq. (1), ϵ is the inverse aspect ratio, D is the particle diffusivity, L_n is the local scale length of variation in the density profile, and γ_{MP} is the magnetic-pumping damping rate.[13,14] For DIII-D-type L-mode parameters[3] ($D=10^4$ cm^2/s, $n=10^{13}$ cm^{-3}, $T=100$ eV, with γ_{MP} taken to be ν_{ii}), condition (1) becomes $L_n < (\delta/\epsilon)^{1/2}$ cm. For DIII-D, L_n is of order 1-5 cm at the edge.

The equations governing tokamak particle and momentum transport can be written, in "magnetic differential" form, as[13]

$$\mathbf{B}\cdot\nabla(nv_{\parallel}/B) = -\nabla\cdot n\mathbf{v}_{\perp} - \partial n/\partial t\,, \tag{2}$$

$$T\mathbf{B}\cdot\nabla n = -nM\mathbf{B}\mathbf{v}:\nabla\mathbf{v} - \mathbf{B}\nabla:\mathbf{\Pi} - nM\partial(\mathbf{B}\cdot\mathbf{v})/\partial t\,, \tag{3}$$

$$\mathbf{B}\cdot\nabla\phi = 0\,, \tag{4}$$

$$\mathbf{B}\cdot\nabla(j_{\parallel}/B) = -\nabla\cdot\mathbf{j}_{\perp}\,, \tag{5}$$

with the auxiliary expressions for $\mathbf{v}_{\perp}$ and $\mathbf{j}_{\perp}$ given by

$$\mathbf{v}_{\perp} = \frac{\mathbf{B}\times\nabla\phi}{B^2} + \frac{\mathbf{R}_{\perp}\times\mathbf{B}}{B^2}\,, \tag{6}$$

$$\mathbf{j}_{\perp} = \frac{\mathbf{B}}{B^2}\times\left[T\nabla n + \nabla\cdot\mathbf{\Pi} + nM\frac{d\mathbf{v}}{dt}\right]. \tag{7}$$

Standard notation is employed with ϕ being the electrostatic potential and $\mathbf{\Pi}$ the parallel viscous stress. A generalized electron-ion momentum transfer term $R_{\perp}$ has been assumed. $\mathbf{R}_{\perp}$ is taken to be perpendicular to $\mathbf{B}$ for simplicity. The parallel viscous stress is what provides magnetic pumping; its exact form depends on tokamak collisionality. Temperature is assumed to be constant on each flux surface. The magnetic field is assumed to be given. For the purposes of this paper, we will assume for simplicity that the flux surfaces are concentric circles. In that case, in the usual (r,θ,ϕ) coordinate system, $\mathbf{B}=(0,\Theta(r),1)B_0(r)/h$, where $h\equiv 1+(r/R)\cos\theta$, $\Theta(r)\equiv r/qR\equiv\epsilon/q$, with $q=q(r)$ being the tokamak "safety factor." These equations are valid for flows which are sub-Alfvénic and superdiamagnetic.

The standard procedure to solve these equations takes advantage of the separation of time scales between the fast, "ideal" time scale (the time scale for sound waves and the rotation period) and the slow, transport time scale (particle-confinement and magnetic-pumping time scale).[13] Accordingly, to lowest order, all terms proportional to $\mathbf{\Pi}$, $R_{\perp}$, and $\partial/\partial t$ are neglected and an equilibrium with flow is obtained. If, in addition, a subsidiary expansion in $|\mathbf{v}|/c_s \ll \epsilon$ is made [$c_s \equiv (T/M)^{1/2}$ being the sound speed], the equilibrium is characterized by three "flux functions:" $n(r)$ and two flux functions associated with the flow. For the latter functions, we use the average poloidal flow, defined as $V_p(r)\equiv\langle v_\theta h\rangle$, and the average toroidal flow, defined as $V_t(r)\equiv\langle v_\phi h\rangle$. Here flux-surface averages, $\langle f\rangle$, are defined by $\langle f\rangle \equiv \oint (d\theta/2\pi)hf$. In terms of V_t and V_p, it can be readily shown from Eqs. (2), (4), and (6), to lowest order in the dissipation and first order in ϵ, that v_θ and v_ϕ are given by the following expressions:

$$v_\theta = V_p/h\,, \tag{8}$$

$$v_\phi \approx V_t - 2qV_p\cos\theta + \epsilon[V_t\cos\theta + 2qV_p(1+\tfrac{1}{4}\cos 2\theta)]\,. \tag{9}$$

By demanding periodicity in θ, $n(r)$, $V_p(r)$, and $V_t(r)$ must satisfy consistency conditions obtained by applying the $\langle\cdots\rangle$ operator to Eqs. (2)-(5). This results in the flux-surface-averaged toroidal transport equations of mass $\langle n\rangle = n$, toroidal angular momentum $\langle nRv_\phi\rangle \approx nR_0V_t$, and circulation $\langle v_\parallel B/B_0\rangle \approx V_t + \Theta(1+2q^2)V_p$. In the absence of sources, mass and toroidal angular momentum are conserved; circulation, however, is only convected. The resulting equations expressing the two conservation laws and the convection of circulation, expanded to lowest order in ϵ, are

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rn\bar{v}_r) = 0\,, \tag{10}$$

$$\frac{\partial}{\partial t}[nV_t] + \frac{1}{r}\frac{\partial}{\partial r}rn[V_t\bar{v}_r - qV_p\tilde{v}_r] = 0\,, \tag{11}$$

$$\frac{\partial}{\partial t}[V_t + \Theta(1+2q^2)V_p] + \bar{v}_r\frac{\partial V_t}{\partial r} - \tilde{v}_r\frac{\partial}{\partial r}[qV_p] + \text{magnetic pumping} = 0\,, \tag{12}$$

where in (12) damping terms from magnetic pumping will be included later. In (10)-(12), the quantities $\bar{v}_r$ and $\tilde{v}_r$ represent radial diffusive velocities. Diffusive velocities arise as a consequence of electron-ion momentum transfer. For simplicity, we have assumed that $\mathbf{R}_{\perp}$ is parallel to $\mathbf{B}\times\nabla n$. With this form, the radial diffusive flux is $nv_r = R_{\perp}/eB$; $\bar{v}_r$ and $\tilde{v}_r$ are defined according to $\bar{v}_r \equiv \langle v_r\rangle$ and $\tilde{v}_r \equiv \langle 2\cos\theta\, v_r\rangle$. Note that within the context of the present model, both the density and the angular momenta are transported radially by the same diffusive velocity $v_r \propto R_{\perp}$. In general, the instability causing the anomalous losses might affect particle and momentum transport differently—this would have to be accounted for in a more sophisticated model.

We now show that poloidal spin-up arises provided the radial flux is poloidally asymmetric, i.e., $\tilde{v}_r \neq 0$. Equations (10)-(12) can be reworked to obtain an equation for $V_p(r,t)$; we eliminate $\partial n/\partial t$ from (11) using (10) and use the resulting equation to eliminate $\partial V_t/\partial t$ from (12) to obtain

$$\Theta(1+2q^2)\left[\frac{\partial V_p}{\partial t}+\gamma_{\rm MP}V_p\right]+qV_p\frac{1}{\partial r}(nr\tilde{v}_r)=0\,, \qquad (13)$$

where we have now introduced the damping due to magnetic pumping as $\gamma_{\rm MP}V_p$. From (13), we see that poloidal flow may be unstable if $\gamma_{\rm MP}$ is small enough. In particular, since the $\tilde{v}_r$ term is a particle transport term, we obtain condition (1) if we assume $\tilde{v}_r \sim \delta D/L_n$. Note that even if (1) is satisfied, the rotation is unstable only if the particle transport term in (13) is negative, i.e., a necessary condition for instability is

$$(\partial/\partial r)(nr\tilde{v}_r)<0\,. \qquad (14)$$

The physical mechanism underlying this spin-up can be traced to the so-called "Pfirsch-Schluter" flows of a poloidally rotating tokamak (in direct analogy to the Pfirsch-Schluter current). These flows are harmonic toroidal flows that are necessarily associated with poloidal rotation, i.e., there cannot be a purely poloidally rotating tokamak plasma. As a tokamak plasma rotates poloidally, the flux tubes alternately compress and decompress thus driving parallel flow to keep $\mathbf{\nabla}\cdot\mathbf{v}\approx 0$ [Eq. (2) to lowest order]. The latter equation and the fact that the electric potential is a flux function led to the expressions for v_ϕ and v_θ given in Eqs. (8) and (9). These clearly show that if $v_\theta \neq 0$, v_ϕ has a harmonic component, independent of ϵ, even if there is no average toroidal flow. This situation is depicted in Fig. 1. Given the nature of the flow pattern, it is clear that poloidally asymmetric transport ($\tilde{v}_r \neq 0$) can affect the transport of toroidal angular momentum in a nontrivial manner since the harmonic parts of v_ϕ and v_r can couple. This coupling results in the "cross terms" in (11) and (12) (proportional to qV_p and $\tilde{v}_r$) which are of the same order as the "direct" terms (proportional to V_t and $\bar{v}_r$) and are responsible for the spin-up. As the plasma diffuses radially, the energy released from adiabatic expansion ends up in a buildup of poloidal rotation.

From Eq. (13) and the fact that shear in the poloidal rotation can quell microturbulence,[7] it can be shown that a bifurcation in transport equilibria results. To show this, we first assume $v_r=-D(1+\delta\cos\theta)(n'/n)-v_0(r/a)$. Thus, the radial velocity is made up of a poloidally asymmetric diffusive part and an azimuthally symmetric pinch, in which case $\bar{v}_r=-D(n'/n)-v_0(r/a)$ and $\tilde{v}_r=-\delta D(n'/n)$. For this choice of v_r, the complete condition for instability becomes

$$\delta D(n''/n+n'/nr)>\epsilon\gamma_{\rm MP}(1+2q^2)/q^2\,. \qquad (15)$$

[We emphasize that (15) is based on the specific model for v_r chosen above; the more general condition is obtainable from (13). For our specific model, the necessary condition (14) becomes $(\partial/\partial r)(\delta Dn'r)>0$; for δD constant, this implies that only density profiles which are sufficiently concave inwards ($n''r>-n'$) are susceptible to spin-up.] We next assume, in accordance with the quelling of microturbulence that would result from poloidal flow shear, that D is a decreasing function of $|\partial V_p/\partial r|$, i.e., $D(V_p')=D_1+(D_0-D_1)\exp(-\alpha V_p'^2)$, with $D_1<D_0$. With these modifications, Eqs. (10) and (13) were solved numerically for $n(r,t)$ and $V_p(r,t)$. A nonlinear damping term proportional to $-V_p^3$ was added to the right-hand side of (13) to provide saturation at large V_p; a small amount of perpendicular viscosity was also added. The numerical solution yields bifurcated equilibria for n, indicated by the solid curves A and B in Fig. 2. Identical parameters were used to obtain A and B; the only difference was the initial condition $n(r,0)$, indicated by the dashed curves. (The parameters are specified in the caption for Fig. 2.) In Fig. 2, we also show the profile of the poloidal rotation speed $V_p(r)$ (dotted curve) for state B.

The bifurcation results as follows. If we start with a discharge where (15) is not satisfied anywhere, then there is no rotation and the transport is large for all sur-

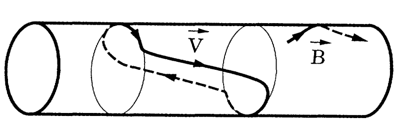

FIG. 1. The "Pfirsch-Schluter" flow pattern expected in a tokamak of large aspect ratio: The size of the harmonic toroidal flows is comparable to the poloidal flow.

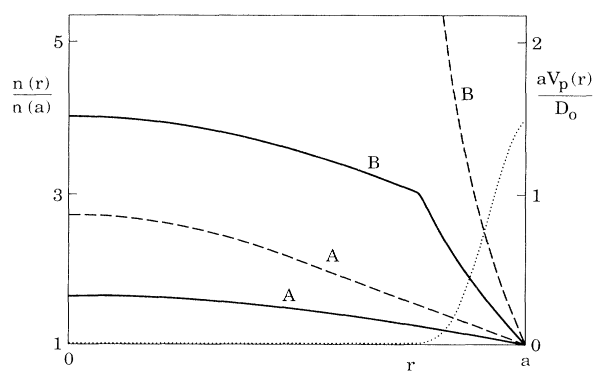

FIG. 2. Radial profiles of n showing bifurcated states A and B (solid lines) and radial profile of V_p for case B (dotted line). Dashed lines are initial conditions for n. Identical parameters were used for A and B: $D_1/D_0=0.2$, $\alpha=10$, $av_0/D_0=1$, $a^2\gamma_{\rm MP}/D_0=2$; we let $q(r)=1$ and $\delta(r)=3\epsilon$.

faces (profile A). Suppose now we locally modify the profile such that n'' is positive and large enough so that (15) is locally satisfied. Rotation then commences, thus depressing D, in which case the n profile must steepen to maintain particle flux, thus accentuating the magnitude of n''. The system then locks into the profile B. In an alternative scenario, we would begin in state A and assume that ambient plasma conditions change such that $\gamma_{\rm MP}$ decreases. If the latter drops sufficiently so that condition (1) is attained, the system will make a transition to an enhanced-confinement state such as B. The time scale of the transition will be governed by the time scale obtained from a near cancellation of the magnetic-pumping and particle-flux terms in (13).

To summarize, the feature of prime importance that emerges from the present study is that the coupling of radial transport and poloidal and toroidal flows in a tokamak can potentially induce a buildup of poloidal flow shear, favored on the outermost surfaces but not necessarily confined to those surfaces. The helical geometry of magnetic flux tubes and the incompressible nature of the flows conspire to produce, in the presence of a radial electric field, the remarkable flow pattern depicted in Fig. 1. (Such a flow pattern has been experimentally verified.[15]) If this flow pattern is transported outward by a diffusive process which is in-out asymmetric, the thermal energy released from the radial expansion is converted to poloidal kinetic energy with the **B** field geometry again mediating the process. This phenomenon appears to be quite general, occurring for any asymmetric transport of particles and angular momentum, classical or anomalous. The transport rate of momentum must exceed the magnetic pumping rate for the spin-up to occur. This condition is more likely to be satisfied for the hotter, present-day tokamaks. The tendency of tokamak plasmas to spin up spontaneously may be desirable in view of the fact that poloidal flow shears can quell microturbulence. In this sense, hotter tokamaks, being more inviscid, would tend to self-regulate transport losses. Note, however, that Eq. (14) places requirements on the radial dependences of the density profile and the particle flux for this spin-up to occur.

The present theory has several features that make it appealing when it is tested against experiment. First, as mentioned earlier, the fact that particle transport plays a central role in the theory is in accordance with the experimental observations (see, e.g., Ref. 3). Second, experimental evidence indicates that both poloidal and toroidal flows undergo significant change at the L-H transition;[3,5] the theory predicts concomitant changes in both. Third, the observed time scale of the transition is of order of or less than a millisecond; theoretically, the transition time scale would be governed by the difference between the spin-up and damping terms, as in Eq. (15): For DIII-D, $\gamma_{\rm MP}$ can be expected to be of order $\nu_{ii} \sim 10^4\ {\rm s}^{-1}$, which is not inconsistent with observations.[3] Fourth, the length scales over which sharp variations in n are observed[3] are of the order 2–5 cm, consistent with the estimate obtained from condition (1) and the predicted macroscopic inward extent of the rotation.

Finally, the question of the parametric dependence of the spin-up threshold, condition (1), or more specifically, condition (15), must be addressed. Since the parametric dependences of D and the pinch speed v_0 are not well known, this question cannot be resolved unequivocally. Roughly speaking, however, the magnetic pumping $\gamma_{\rm MP}$ must be overcome. For $\nu_{*i} \lesssim \epsilon^{-3/2}$, $\gamma_{\rm MP} \sim v_{ii}$. Since $\nu_{ii} \propto T_i^{-3/2} M^{-1/2}$, where M is the ion mass, lower density and higher temperatures at the edge are favorable to spin-up as are higher-mass isotopes (at least for tokamaks with $\nu_{*i} \sim 1$). The latter dependences are consistent with the experimental observations that there is an input-power threshold for the H mode and that this threshold increases with n (e.g., Ref. 3). Wall conditions are known to play an important role in achieving the H mode—in particular, a reduction in inward flow of recycling particles, effected by a divertor or wall conditioning, facilitates obtaining the H mode. This fact is consistent with a lowered edge n favoring poloidal spin-up. Finally, the isotope dependence is also in the right direction.

We acknowledge beneficial discussions with K. Burrell, A. M. Dimits, P. N. Guzdar, R. M. Kulsrud, and Y. T. Lau. This work was supported by the U.S. Department of Energy.

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