Transport through a Strongly Interacting Electron System: Theory of Periodic Conductance Oscillations

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The conductance through a quantum dot is calculated via an Anderson model of a site weakly coupled to ideal leads with an on-site Coulomb interaction. As the chemical potential is varied, peaks occur periodically in the conductance whenever an electron is added to the site. The participation of multiple electronic levels in each conductance peak explains the anomalous temperature dependence of peak heights observed in recent narrow-channel experiments.

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As electronic confinement in quantum-dot structures approaches atomic dimensions, behavior characteristic of atomic impurities has emerged in capacitance[1] and IR (Ref. 2) spectroscopies. The two characteristic features of an impurity, the quantization of the electronic spectrum and the quantization of the electronic charge, manifested through the electron-electron interactions, play a major role in the physics of the nanostructures. It is the purpose of this Letter to demonstrate how the interplay of these two quantization effects produces the hitherto unexplained features of the narrow-channel experiments of Scott-Thomas *et al.*[3] and Meirav, Kastner, and Wind.[4] In fact, we will show that the phenomenology of these experiments can be readily understood in terms of an Anderson-type Hamiltonian for a single magnetic impurity.[5]

The conductance of a narrow channel between two-dimensional electron gases has been found to display a number of remarkable features.[3,4] Most striking are the periodic oscillations as a function of gate voltage or, equivalently, chemical potential. Near threshold, the peaks in conductance are well separated with a line shape closely matching the derivative of the Fermi function, and a width proportional to temperature. The heights of the peaks, however, have an irregular and even nonmonotonic dependence on temperature, in contrast to the expected $1/T$ dependence of the Fermi function. The periodicity of the conductance oscillations has been explained[6,7] in terms of single-electron charging of a segment of the channel isolated by impurities or by lithographic tunnel barriers (Fig. 1, inset). To our knowledge, no explanation has been presented for the unusual temperature dependence of the peaks.

In this Letter, we address the experimental phenomenology of periodic conductance oscillations in terms of a microscopic model, the Wolff model,[8] similar to the Anderson model for a single magnetic impurity. The conductance is determined by a Landauer formula involving the interaction density of states (DOS) on the impurity site. Since the temperatures of interest are high compared to the Kondo temperature, the density of states can be obtained by an equations-of-motion technique that properly treats on-site correlations but neglects correlations in the leads. We find peaks in the conductance spaced by the Coulomb-interaction energy, each peak corresponding to the addition of an electron to the site. The observed anomalous temperature dependence of the conductance peaks is due to the presence of two very different energy scales: (1) the Coulomb-interaction energy between electrons U, which is approximately equal to the capacitive-charging energy of the quantum dot $e^2/C \sim 0.5$ meV, and (2) the bare-energy-level spacing $\Delta\epsilon \sim 0.05$ meV for a 100-nm$\times$1-μm quantum dot in GaAs. For $k_BT \simeq \Delta\epsilon$, transport occurs simultaneously through multiple levels, but the conductance peaks remain well resolved up to a much higher temperature $k_BT \simeq U$. This leads to anomalous temperature dependence in the range $\Delta\epsilon < k_BT < U$. This is to be contrasted with the noninteracting case, where a temperature larger than the level spacing would wash out the resonance structure altogether.

The model we treat is the simplest one that takes account of the two dominant quantizations: charge and energy. It consists of two ideal leads coupled to a single site on which there is a Coulomb interaction of energy U. (The isolated 1-μm segment of the channel can be treated as a site, or "quantum dot," since the wave functions are coherent over lengths $\gg 1$ μm. For a more complete discussion of the physical approximations involved see Ref. 7.) Initially, we will consider only two levels on the site, labeled by a spin index α, with energies ϵ_α which may be degenerate. The Hamiltonian is

$$H = \sum_{\alpha;k \in L,R} \epsilon_{k\alpha} \mathbf{c}^\dagger_{k\alpha} \mathbf{c}_{k\alpha} + \sum_\alpha \epsilon_\alpha \mathbf{c}^\dagger_\alpha \mathbf{c}_\alpha + U n_\uparrow n_\downarrow + \sum_{\alpha;k \in L,R} (V_{k\alpha} \mathbf{c}^\dagger_{k\alpha} \mathbf{c}_\alpha + \text{H.c.}) . \quad (1)$$

In (1), the states in the leads, labeled L and R, have energies $\epsilon_{k\alpha}$ and are connected to the site by hopping matrix elements $V_{k\alpha}$.

To make contact with experiment, the conductance is calculated using a Landauer-type formula[9] generalized to interacting systems,

$$\sigma = \frac{e^2}{h} \sum_\alpha \int_{-\infty}^{\infty} d\epsilon f'_{\text{FD}}(\epsilon) \frac{2\Gamma_{L\alpha}(\epsilon)\Gamma_{R\alpha}(\epsilon)}{\Gamma_{L\alpha}(\epsilon)+\Gamma_{R\alpha}(\epsilon)} \text{Im}[G_\alpha(\epsilon)] . \quad (2)$$

Equation (2) expresses the linear-response conductance

as the sum of the elastic-transmission probabilities in each spin channel weighted by the derivative of the Fermi function. The elastic-transmission probabilities are constructed as a product of the elastic coupling to the leads and the interacting DOS on the site, $-\mathrm{Im}[G_\alpha(\omega)]/\pi$.[10] Here $G_\alpha(\omega)$ is the Fourier transform of the retarded Green's function,

$$G_\alpha(t) = -i\Theta(t)\langle\{\mathbf{c}_\alpha(t),\mathbf{c}_\alpha^\dagger(0)\}\rangle , \tag{3}$$

where the curly brackets denote the anticommutator. We expect (2) to apply even in the presence of phonons or other inelastic-scattering mechanisms provided that the inelastic broadening is smaller than k_BT.

In the noninteracting case ($U=0$), one finds $G_\alpha(\omega) = (\omega-\epsilon_\alpha-\Sigma_{\alpha 0})^{-1}$, where the self-energy due to tunneling into the leads is

$$\Sigma_{\alpha 0}(\omega) = \sum_{k\in L,R} |V_{k\alpha}|^2 \frac{1}{\omega-\epsilon_{k\alpha}} . \tag{4}$$

The full width of the resonance, which is given by $-2\,\mathrm{Im}[\Sigma_{\alpha 0}(\omega+i\delta)]$, is just the sum of the elastic couplings to the two leads, $\Gamma_\alpha(\omega) = \Gamma_{L\alpha}(\omega)+\Gamma_{R\alpha}(\omega)$, where

$$\Gamma_{L(R)\alpha}(\omega) = 2\pi \sum_{k\in L(R)} |V_{k\alpha}|^2 \delta(\omega-\epsilon_{k\alpha}) . \tag{5}$$

For simplicity, we assume symmetric barriers, so $\Gamma_{L\alpha}(\omega) = \Gamma_{R\alpha}(\omega) = \Gamma_\alpha(\omega)/2$.

In the interacting case ($U\neq 0$), the elastic-coupling factor in the conductance is unchanged, but the DOS now includes the interactions. Since, experimentally, the Coulomb-interaction energy U is dominant, it is essential to properly treat correlations on the site. As indicated in the inset of Fig. 1, when one level on the site is occupied, the energy of the unoccupied level is raised by U. Thus, double occupancy of the site is energetically costly. To calculate $G_\alpha(\omega)$, we employ an equations-of-motion method[11,12] that accounts for these on-site correlations correctly.

Briefly, the equations-of-motion method consists of differentiating the Green's function $G_\alpha(t)$ with respect to time, thereby generating higher-order Green's functions which must be approximated to close the equation for $G_\alpha(t)$. Since the time derivative of a Heisenberg operator is determined by its commutator with the Hamiltonian, one finds

$$i\hbar\frac{\partial}{\partial t}G_\alpha(t) = \hbar\delta(t)+\epsilon_\alpha G_\alpha(t) + \sum_{\alpha;k\in L,R} V_{k\alpha}G_{k\alpha}(t)+UG_{\bar\alpha\alpha}(t) . \tag{6}$$

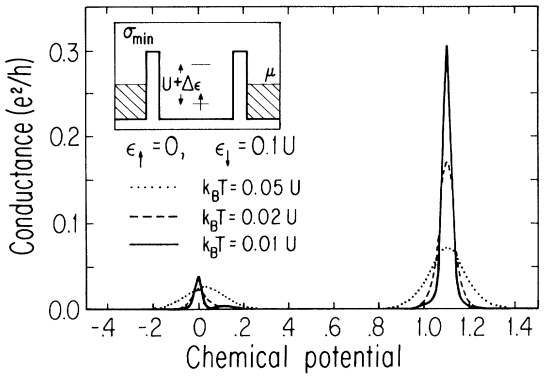

FIG. 1. Conductance, in units of e^2/h, vs chemical potential, in units of U, for three temperatures, calculated from Eq. (2). The full self-energies in $G_\alpha(\omega)$, Eq. (8), are evaluated assuming spin-dependent elastic couplings, $\Gamma_\uparrow = 0.001U$ and $\Gamma_\downarrow = 0.01U = 10\Gamma_\uparrow$. Though the bare splitting of the levels is $\epsilon_\downarrow - \epsilon_\uparrow = 0.1U$, the Coulomb interaction separates the conductance peaks by $U+\epsilon_\downarrow-\epsilon_\uparrow = 1.1U$. As k_BT approaches the bare-level splitting, the broad upper level at $\epsilon_\downarrow$ begins to contribute to the first conductance peak, causing it to rise with temperature. Inset: Schematic band-edge diagram corresponding to the conductance valley at $\mu \simeq (U+\epsilon_\downarrow)/2$.

There are two new Green's functions on the right-hand side of Eq. (6): The first, $G_{k\alpha}(t)$, is generated by the hopping term in the Hamiltonian and is given by (3) with the on-site lowering operator $\mathbf{c}_\alpha(t)$ replaced by $\mathbf{c}_{k\alpha}(t)$. The equation of motion for $G_{k\alpha}(t)$ closes since the only Green's functions generated are $G_{k\alpha}(t)$ and $G_\alpha(t)$. The second new term in (6) is generated by the Coulomb interaction $Un_\uparrow n_\downarrow$ and involves a two-particle Green's function

$$G_{\bar\alpha\alpha}(t) = -i\Theta(t)\langle\{\mathbf{c}_{\bar\alpha}^\dagger(t)\mathbf{c}_{\bar\alpha}(t)\mathbf{c}_\alpha(t),\mathbf{c}_\alpha^\dagger(0)\}\rangle , \tag{7}$$

where α and $\bar\alpha$ are opposite spin directions. In the absence of hopping, $G_{\bar\alpha\alpha}(t)$ can be obtained exactly via its equation of motion and the identity $n_\alpha n_\alpha = n_\alpha$. For finite hopping, an approximate solution, valid for temperatures higher than the Kondo temperature,[12] $k_BT_K \simeq (U\Gamma)^{1/2} \times \exp(-\pi|\mu-\epsilon_\alpha|/\Gamma)$, is obtained by neglecting terms in the equation of motion for $G_{\bar\alpha\alpha}(t)$ which involve correlations in the leads.[13]

After truncation of the higher-order equations of motion, the equation for $G_\alpha(t)$ closes. We find that the Fourier transform $G_\alpha(\omega)$ has two resonances—one at ϵ_α weighted by the probability, $1-\langle n_{\bar\alpha}\rangle$, that the other level is vacant, and one at $\epsilon_\alpha+U$, weighted by the probability, $\langle n_{\bar\alpha}\rangle$, that the other level is occupied,[14]

$$G_\alpha(\omega) = \frac{1-\langle n_{\bar\alpha}\rangle}{\omega-\epsilon_\alpha-[\Sigma_{\alpha 0}-U\Sigma_{\alpha 1}(\omega-\epsilon_\alpha-U-\Sigma_{\alpha 0}-\Sigma_{\alpha 3})^{-1}]} + \frac{\langle n_{\bar\alpha}\rangle}{\omega-\epsilon_\alpha-U-[\Sigma_{\alpha 0}+U\Sigma_{\alpha 2}(\omega-\epsilon_\alpha-\Sigma_{\alpha 0}-\Sigma_{\alpha 3})^{-1}]} . \tag{8}$$

Expression (8) for $G_\alpha(\omega)$ is exact both in the noninteracting limit ($U=0$) and in the isolated-site limit ($V_{k\alpha}=0$). Generally, and in contrast to the noninteracting limit, $G_\alpha(\omega)$ depends on the temperature and chemical potential through $\langle n_{\bar\alpha}\rangle$ and through the self-energies $\Sigma_{\alpha 1}$ and $\Sigma_{\alpha 2}$. In Eq. (8), $\Sigma_{\alpha 0}(\omega)$ is the ordinary self-energy due to tunneling of the α

electron (4), while the other self-energies are due to tunneling of the $\bar{\alpha}$ electron,

$$\Sigma_{ai}(\omega)=\sum_{k\in L,R}A_{k\bar{\alpha}}^{(i)}|V_{k\bar{\alpha}}|^2\left[\frac{1}{\omega-\epsilon_a+\epsilon_{\bar{\alpha}}-\epsilon_{k\bar{\alpha}}}+\frac{1}{\omega-\epsilon_a-\epsilon_{\bar{\alpha}}-U+\epsilon_{k\bar{\alpha}}}\right],\quad i=1,2,3\,, \tag{9}$$

where $A_{k\bar{\alpha}}^{(1)}=f_{\rm FD}(\epsilon_{\bar{\alpha}})$, $A_{k\bar{\alpha}}^{(2)}=1-f_{\rm FD}(\epsilon_{\bar{\alpha}})$, and $A_{k\bar{\alpha}}^{(3)}=1$. It is interesting to note that for $T<T_K$, the equations-of-motion solution has a Kondo resonance at the Fermi surface, as expected for an Anderson Hamiltonian. An analysis of the conductance in this limit has been given by Glazman and Raikh,[15] and by Ng and Lee.[16]

The equations-of-motion solution (8) for $G_a(\omega)$ can be employed to calculate the conductance via the Landauer formula (2). Because the elastic couplings increase rapidly near the top of the barriers, it is instructive to consider two levels, spin split by $\epsilon_\downarrow-\epsilon_\uparrow=0.1U$, with very different couplings to the leads, which are taken to have broad, flat densities of states. For this case the elastic couplings (5), from which the full self-energies (9) can be determined, are constant and are chosen to be spin dependent, $\Gamma_\uparrow=0.001U$ and $\Gamma_\downarrow=0.01U=10\Gamma_\uparrow$. The conductance is plotted, in Fig. 1, as a function of chemical potential for three temperatures. Several features are noteworthy: (1) Although the bare-level spacing is $\Delta\epsilon=0.1U$, so that in principle four peaks occur in the interacting DOS, there are only two peaks in the conductance, split by $U+\Delta\epsilon=1.1U$. This suppression of peaks follows from the dependence of the DOS of each level on the occupancy of the other level. As μ passes $\epsilon_\uparrow$, the up-spin level fills and all the weight of the density of states of the down-spin level is pushed up to $\epsilon_\downarrow+U$, giving rise to the conductance peak at $\mu=1.1U$. (2) The temperature dependence of the second conductance peak is similar to that of a noninteracting, single-particle resonance—i.e., width $\sim T$, height $\sim 1/T$—but the height of the first peak actually *increases* with temperature between the two higher-temperature traces. At temperatures comparable to the level spacing, the down-spin channel begins to contribute to the first conductance peak. Since the down-spin level is more strongly coupled to the leads than the up-spin level, the conductance at the first peak increases with temperature. At even higher temperatures, both conductance peaks fall because of the $1/T$ dependence of the amplitude of $f'_{\rm FD}(\epsilon)$. In the opposite limit of temperatures much less than the resonance widths, the peak conductances approach e^2/h, the value for a single ideal channel.

To compare directly to the experiments, it is necessary to generalize our results to multiple levels on the site. In the relevant high-temperature regime ($k_BT\gg\Gamma$), the integral in (2) can be carried out exactly, since in this limit the interacting density of states for a level n can be approximated by a sum of delta functions $\sum_m P_n(m)\delta(\epsilon_n+mU-\epsilon)$. The weight $P_n(m)$, obtained exactly from the equations of motion for multiple levels, turns out to be the probability that m levels, other than level n, will be occupied for the *isolated* site, and is therefore given by the corresponding Boltzmann weight. From Eq. (2) for the conductance, we find the multilevel conductance formula[17]

$$\sigma=-\frac{\pi e^2}{2h}\sum_n\sum_m\Gamma_nP_n(m)f'_{\rm FD}(\epsilon_n+mU)\,. \tag{10}$$

The strengths of coupling to the leads, Γ_n, are still essential in (10) in setting the magnitude of the conductance even though their effect of broadening the DOS is negligible for $k_BT\gg\Gamma_n$.

In Fig. 2(a), we have plotted experimental conductance data for a narrow-channel GaAs structure with lithographic barriers.[18] For comparison, in Fig. 2(b), we have plotted the theoretical conductance (10) as a function of chemical potential for a system of 10 nondegenerate levels. The coupling strengths Γ_n of successive levels increase by a factor of 1.5 to reflect the increase in the tunneling matrix elements near the top of the tunnel

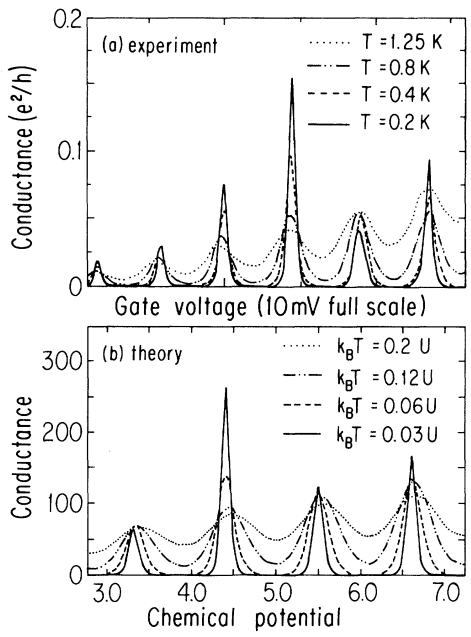

FIG. 2. (a) Experimental conductance of a narrow GaAs channel with two lithographically defined barriers plotted against gate voltage for four temperatures (Ref. 18). (b) Theoretical conductance, in units of $(\Gamma_1/U)(e^2/h)$, for ten levels, spaced by $\Delta\epsilon=0.1U$, vs chemical potential, in units of U, at four temperatures, calculated from (10). The elastic couplings of the levels increase geometrically, $\Gamma_n=1.5^n\Gamma_1$ (to simulate disorder Γ_4 is increased by an additional factor of 4). While for $k_BT\ll\Delta\epsilon$ only one bare level contributes to each conductance peak, for $k_BT\simeq\Delta\epsilon$ many levels contribute, permitting the conductance to rise with temperature.

barriers. The constant, bare-level spacing is taken to be $\Delta\epsilon = 0.1U$. (These parameters are obtained assuming parabolic barriers with the lithographic width of 100 nm and a height of 0.75 meV, estimated from the product of the level spacing, 0.05 meV, and the fifteen oscillations observed before the top of the barrier is reached. A similar rapid increase of couplings is also expected near the parabolic maxima of the screened-impurity-potential barriers in the Si structures.[3]) At low temperatures ($k_BT \ll \Delta\epsilon$), each conductance resonance corresponds to transport through a single level. As μ increases and each level is filled, the peaks in the density of states of the remaining levels are pushed up by U, leading to conductance peaks spaced by $U+\Delta\epsilon = 1.1U$. For temperatures comparable to $\Delta\epsilon$, multiple levels participate in transport, and the stronger coupling of higher levels leads to an increase of conductance with temperature. The effects of random fluctuations in coupling strength, represented by a factor-of-4 enhancement of Γ_4, are smoothed out for $k_BT > \Delta\epsilon$ by the averaging over multiple levels. Similarly, random fluctuations in the bare-level spacing lead to fluctuations in the conductance-peak spacing. However, these fluctuations would no longer be noticeable for $k_BT > \Delta\epsilon$. The phenomenology of the experimental conductance is well reproduced by the theoretical results at a realistic ratio of the bare-level spacing to the Coulomb-interaction energy.

In conclusion, we have analyzed the narrow-channel experiments of Scott-Thomas *et al.*[3] and Meirav, Kastner, and Wind[4] in terms of an Anderson model for a site coupled to ideal leads. The observed periodic conductance oscillations are explained by the periodic vanishing of the energy gap to addition of an electron to the site. Although the separation between conductance peaks is set by the large Coulomb-interaction energy, it is the much smaller bare-energy-level spacing which determines the temperature dependence of the conductance peaks. The observed temperature dependence is explained by simultaneous transport through multiple levels whose coupling to the leads increases rapidly with energy near the top of the tunnel barriers.

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Note added.— Following the submission of this paper, the high-temperature conductance formula (10) has been quantitatively verified on a GaAs structure in the quantum Hall regime.[19]

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$$\langle\{\mathbf{c}^\dagger_{k\bar{a}}(t)\mathbf{c}_{k'\bar{a}}(t)\mathbf{c}_a(t),\mathbf{c}^\dagger_{\bar{a}}(0)\}\rangle = \delta_{k,k'}f_{\mathrm{FD}}(\epsilon_{k\bar{a}})\langle\{\mathbf{c}_a(t),\mathbf{c}^\dagger_{\bar{a}}(0)\}\rangle$$

in the second iteration of the equation of motion for $G_{\bar{a}a}(t)$.

[14]The occupation probability $\langle n_a\rangle$ is given by an integral of the interacting DOS weighted by the Fermi factor, $\langle n_a\rangle = \int d\epsilon f_{\mathrm{FD}}(\epsilon)\{-\mathrm{Im}[G_a(\epsilon)]/\pi\}$.

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