Vortex Variable-Range-Hopping Resistivity in Superconducting Films

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We study both classical and quantum vortex creep in disordered thin-film superconductors in an applied magnetic field. Quantum tunneling of vortices leads to a variable-range-hopping resistivity with a non-Arrhenius temperature dependence at low T. Mott's $\frac{1}{3}$ law is modified by long-range vortex interactions, and a numerical analysis enables us to estimate a temperature exponent between roughly $\frac{2}{3}$ and $\frac{4}{5}$. At higher T, a classical hopping regime is expected with a current-density scale for nonlinearities varying (roughly) as T^3.

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Since the discovery of the high-temperature oxide superconductors, there has been a resurgence of interest in vortex creep.[1-3] A question of interest in bulk materials has been the possible existence of a truly superconducting vortex-glass phase at low temperatures, in the presence of a penetrating magnetic field.[4-6] In very thin superconducting films, where the vortices become pointlike, long-range vortex-glass order is expected only *at* zero temperature[6] and this will be confirmed by our numerics below. Nevertheless, at low temperatures transport is greatly modified by the growth of vortex-glass correlations, as recently emphasized.[6] Moreover, at sufficiently low temperatures, quantum tunneling of vortices should dominate the transport.[7] In this Letter, we study a simple model for the low-temperature properties of disordered superconducting films in a field and explore its consequences for both classical and quantum vortex creep. Our main prediction is the existence of a novel quantum regime, where vortices undergo variable-range hopping (VRH), in direct analogy to variable-range hopping in dirty electron insulators.[8] Taking explicitly into account multivortex-hopping processes leads to a nonactivated form for the *resistivity,* with a temperature exponent which depends upon a critical exponent of the $T=0$ vortex-glass fixed point. Combining our numerical estimate for the exponent with a physical argument about tunneling rates enables us to estimate a temperature exponent roughly in the range $\frac{2}{3}$ to $\frac{4}{5}$, in contrast to Mott's $\frac{1}{3}$ law. At higher temperatures we obtain a classical VRH regime with a current-density scale for nonlinearities varying roughly as T^3, in contrast to the Anderson-Kim theory which predicts the first power of T.

Consider then a thin disordered superconducting film in an applied magnetic field. The long-range crystalline correlations in the Abrikosov vortex lattice will be destroyed by disorder.[9] For simplicity, we assume throughout "strong disorder," so that the lattice translational correlation length is not significantly greater than the intervortex spacing. At nonzero temperatures the vortices will be mobile, leading to a vortex-creep resistance and a loss of superconductivity. As $T\rightarrow 0$ the motion of vortices will cease, and the film should freeze into a superconducting vortex-glass phase—a phase which lacks conventional off-diagonal long-range order, but has Edwards-Anderson spin-glass-like order.[6]

A crucial dimensional parameter characterizing the 2D vortex-glass phase at $T=0$ is the ratio of the vortex localization length,[10] denoted a_v, and the distance between vortices, $l=(\phi_0/B)^{1/2}$, with $\phi_0=h/2e$ and B the applied magnetic field. As argued below, the classical regime is when $a_v \ll l$, but near the superconductor-insulator transition[10] for larger B, where a_v diverges, vortex quantum effects will dominate. The most important energy scale is set by the vortex core energy, $K=\hbar^2\rho_s/m$, where ρ_s is the areal superfluid number density and m the Cooper-pair mass. Both the characteristic pinning energy (per vortex) and the vortex interaction strength $[\sim K\ln(r)]$ will be set by K.

At finite temperatures vortex motion destroys the 2D vortex-glass order. For low T one expects[6] that the vortex-glass correlation length ξ_T, which sets the scale for exponential decay of the correlation function $[|\langle\psi^*(r)\psi(0)\rangle|^2]_{\rm av}$, where ψ is the Cooper-pair field and the square brackets denote an ensemble average, will diverge as a power law in T, with exponent ν_T. For temperatures comparable to K, vortices should be completely depinned and mobile, and ξ_T will be given roughly by the vortex separation l. Thus we expect ($k_B=1$)

$$\xi_T \approx l(K/T)^{\nu_T}. \tag{1}$$

Since the vortices are localized by disorder in the 2D vortex-glass phase, many properties can be deduced by ignoring quantum fluctuations and treating the vortices (and the phase of the pair field) classically. This is directly analogous to the classical treatment of electrons in the Coulomb-glass models of localized insulators by Shklovskii and Efros.[8] The simplest model imaginable, in terms of the phase ϕ of the pair field, $\psi=|\psi|e^{i\phi}$, is the so-called gauge glass,[11]

$$H=-K\sum_{r,\delta}\cos(\phi_r-\phi_{r+\delta}-A_r^{\delta}), \tag{2}$$

where r denotes sites of a square lattice, $\delta=\hat{x},\hat{y}$ are near-neighbor vectors, and A_r^δ is a random quenched gauge field taken as uniformly distributed on the interval $[0,2\pi]$. If the potential $\exp[\beta K\cos(\phi)]$ is replaced by a Villain potential,

$$\exp[\beta V_K(\phi)]\equiv\sum_n \exp[-K(\phi-2\pi n)^2/2T],$$

in the partition function for H in (2), standard duality transformations[12] can be used to expose the vortex degrees of freedom, with a dual Hamiltonian:

$$\tilde{H}_0=\frac{K}{2}\sum_{R,R'}(2\pi N_R-B_R)G(R-R')(2\pi N_{R'}-B_{R'}), \quad (3)$$

where N_R, an integer, is the number of vortices on sites of the dual lattice (plaquettes of the original lattice), B_R is the magnetic field penetrating each plaquette obtained from a directed sum of A_r^δ around the plaquette, and $G(R)$ is the 2D lattice Green's function (inverse lattice Laplacian) which varies as $\ln(R)$ for large R. There is also a "neutrality" constraint on (3) that $\sum_R(2\pi N_R-B_R)=0$. Since the model (2) is invariant under $B_R\rightarrow B_R+2\pi n_R$, for integers n_R, all the B's entering (3) can be shifted into the interval $[0,2\pi]$ and act as a compensating "negative charge background" for the vortices. With this shift, on average, half of the (dual) sites will be occupied by positive ($N=+1$) vortices and half of the sites will be empty ($N=0$). Because of this, the typical distance between vortices, l, can be equated with the lattice spacing in (2) or (3).

In order to characterize the $T=0$ vortex-glass phase described by (2), it is useful to take a square L by L lattice (in units of lattice spacing l) with periodic boundary conditions, and consider the magnitude of the typical (e.g., rms) change in ground-state energy U_L upon changing the boundary conditions in one direction to antiperiodic, which is equivalent to the replacement $A_r^x\rightarrow A_r^x+\pi/L$. This energy is expected to vary as a power law in L:

$$U_L\sim KL^\theta, \quad (4)$$

where θ is the renormalization-group eigenvalue of temperature.[13] Since θ is negative (see below), there are excitations of arbitrarily low energy at long length scales, which can be thermally excited and should destroy the glass order at any finite temperature. More specifically, at temperature T excitations on scale L will be active provided $K/L^{|\theta|}\lesssim T$. The smallest such excitations, of size $l(K/T)^{1/|\theta|}$, will set the range of correlations, that is, ξ_T in (1). Thus we can identify $\nu_T\equiv 1/|\theta|$.

The above twist in boundary conditions can be absorbed either by a smooth spin-wave variation or by moving a single vortex a distance $L/2$ (or moving many vortices, see below). This expectation can be quantified by redoing the duality transformation for model (2) on a finite lattice with periodic boundary conditions. Besides changing $G(R)$ in (3) to be an inverse Laplacian periodic in L, an additional term in the Hamiltonian is generated due to the nontrivial topology of a torus. The dual vortex Hamiltonian takes the form $\tilde{H}=\tilde{H}_0+\tilde{H}_1$, with $\tilde{H}_0$ given in (3) and

$$\tilde{H}_1=V_K\left[\sum_x(E^y_{x,y=1}-A^x_{x,y=1})\right]+V_K\left[\sum_y(E^x_{x=1,y}-A^y_{x=1,y})\right], \quad (5)$$

where E_R is an "electric field" which lives on the links of the dual lattice and satisfies $\nabla\cdot\mathbf{E}_R=2\pi N_R-B_R$ and $\nabla\times\mathbf{E}=0$. The values of the Villain functions in (5) do not depend on the choice $y=1$ in the first term (or $x=1$ in the second) since shifting y changes the argument by 2π times an integer.

At $T=0$ the 2π-periodic Villain functions reduce to $V_K(\phi)=\frac{1}{2}K\phi^2$ in the interval $[-\pi,\pi]$. $\tilde{H}_1$ thus accounts for the spin-wave energy, since upon changing the boundary conditions by π via $A_r^x\rightarrow A_r^x+\pi/L$, without rearranging vortices, the only change in the energy is due to the first term in $\tilde{H}_1$, whose argument changes by π. This gives an energy change of order K, independent of L. Alternatively, if a single vortex is moved a distance $L/2$ in the y direction, or a number of vortices are moved with an equivalent cumulative change, then the changes due to the new electric field cancel exactly the changes from the boundary conditions, so $\tilde{H}_1$ is unchanged. $\tilde{H}_0$, on the other hand, changes because the values of the N_R have been altered. Below, we find that θ in (4) is clearly negative, so that for large L, moving vortices is energetically the most favorable.

Moving a single vortex a distance L without any relaxation of other vortices costs a typical energy of $\ln(L)$. Since the minimal-energy vortex excitations discussed above cost substantially less energy ($\sim L^{-|\theta|}$), they must involve rearrangements of *many* vortices. We argue below that such multivortex hops will dominate over single hops in the quantum VRH regime.

It is useful to generalize the notion of such minimal-energy (multi)vortex excitations to an infinite film. Specifically, an excitation on scale L is defined as the minimal-energy vortex rearrangement in a given L by L region, which has a net (center-of-mass) displacement equivalent to moving one vortex a distance $L/2$. We expect their *typical* (e.g., rms) energy on scale L to vary with the *same* exponent θ as the excitations in an L by L system with periodic boundary conditions.

Creation of such optimal vortex excitations will typically involve surmounting barriers V_L with energies much higher than U_L itself, due to intermediate nonoptimal vortex configurations. The usual assumption[13] is that these barriers vary as $V_L\sim KL^\psi$, with $\psi\geq\theta$. One also expects $\psi\geq 0$, since each of the intermediate moves (i.e., a vortex moving one lattice spacing) will cost an energy of order K. In Ref. 6 it was suggested that ψ might in fact be zero.

Consider now the effects of vortex creep on the I-V curves, ignoring initially quantum tunneling. At temperature T, motion of vortices will proceed by activation

over barriers V_L into locally optimal vortex excitations (defined above) with energies of order k_BT. Creation of such excitations involve moving vortices on the scale of the coherence length ξ_T, so that the relevant barrier is $V_{L=\xi_T}$. Since ξ_T varies with temperature, this is a *classical* variable-range-hopping picture. This process leads to a linear resistance, $R_L \sim \exp(-V_{\xi_T}/T)$ or

$$R_L \sim \exp[-(K/T)^{1+\psi\nu_T}], \tag{6}$$

which is the simple Arrhenius form if[6] $\psi=0$. In the presence of an areal current density J, the Magnus force contributes an energy $\phi_0 J\xi_T$ in creating these vortex excitations. At a current density $J_{nl}\equiv T/\phi_0\xi_T$ this energy becomes comparable to the intrinsic thermal energy scale and should result in nonlinearities in the I-V curve.[14] Because of the temperature dependence of ξ_T, J_{nl} vanishes at $T^{1+|1/\theta|}$, faster than the linear T dependence predicted in the Anderson-Kim theory. Below, we show that $\theta \simeq -0.5$, and so we predict (roughly) a T^3 dependence.

Equation (6) is modified by quantum tunneling of vortices. As in Mott variable-range-hopping theory,[8] there will be a competition between finding states of energy within k_BT of the ground state and states which are close enough to tunnel into. In generalizing Mott's theory to charged systems with a $1/r^\sigma$ interaction between particles, Shklovskii and Efros[8] argue that hopping of a single particle (vortex) a distance r, without additional relaxation, will cost a typical energy $\sim 1/r^\sigma$. Multiplying by the rate of tunneling, $\exp(-r/a_v)$, and optimizing with respect to r yields a linear resistance due to vortex tunneling of the form

$$R_L \sim \exp[-(T_0/T)^p], \tag{7}$$

with $p=(1+\sigma)^{-1}$ and $T_0 \simeq K(l/a_v)^\sigma$. Logarithmically interacting particles (vortices) might then be expected to correspond to the $\sigma=0$ limit of this formula, i.e., an Arrhenius form.

Note, however, that a logarithmic interaction differs qualitatively from an inverse power law, since it grows without bound with particle separation. This fact requires modifying the Shklovskii-Efros argument[8] since single-particle "ionization" energies can no longer be defined. In this case the typical minimal energy to move a single particle a distance r will *grow* with increasing r [as $\ln(r)$] due to direct attraction to the "hole" left by the particle, rather than decreasing (as $1/r^\sigma$). This implies a nonlinear current-voltage characteristic (i.e., a superconductor), as in a 2D superconductor in zero field. Our results yield an energy scale for vortex motion which decreases with distance ($1/r^{|\theta|}$), so that multivortex hopping dominates the above single-particle effects. As we argue below, quantum tunneling for such multivortex hops then produces a linear resistivity which dominates the classical expression [Eq. (6)] at low enough temperatures.

Consider then the minimal-energy vortex excitations introduced after Eq. (5), which involve many vortices hopping and cost an energy $U_r \sim r^{-|\theta|}$ on scale r. These multivortex hops, although energetically more accessible, will be harder to tunnel into than the single hops considered in Ref. 8. A lower bound for this multihop tunneling rate can be estimated as follows: In the worst-case scenario, creation of an excitation on scale r will involve tunneling of *all* of the $(r/l)^2$ vortices in the r by r region a distance comparable to the intervortex spacing l. The rate for this should be proportional to the single-vortex rate, $\exp(-l/a_v)$, raised to the power of the number of vortices: $\exp[-(r/l)^2(l/a_v)]$. A natural ansatz for the actual tunneling rate would be to replace the power of 2 in this expression by an exponent ψ_Q, with $1 \le \psi_Q \le 2$. Multiplying this rate by $\exp(-U_r/T)$ and optimizing with respect to r gives a resistance with precisely the same form as in (7) but with $p=(1+|\theta|/\psi_Q)^{-1}$ and $T_0 \simeq K(l/a_v)^{|\theta|/\psi_Q}$, rather than the $\sigma \to 0$ limit of the Shklovskii-Efros results after Eq. (7). With θ near $-\frac{1}{2}$ and $1 \le \psi_Q \le 2$, the hopping exponent p falls (roughly) in the range $\frac{2}{3}$ to $\frac{4}{5}$. We note that such multiparticle-hopping processes as considered above might dominate over the single-particle-hopping processes considered by Shklovskii and Efros even for an inverse power-law interaction ($\sigma > 0$).

Upon cooling, the classical Arrhenius form in (6) for the resistivity should cross over into the quantum VRH form in (7), at some temperature T^*, which can be deduced by equating the arguments of the two exponentials. For simplicity, assuming $\psi=0$ in (6), this gives

$$T^* \simeq (a_v/l)K. \tag{8}$$

There are several possible regimes. For vortices localized on scales short compared to their separation, $a_v \ll l$, T^* is much smaller than K and the classical Arrhenius form in (6) will hold down to inaccessibly low tempera-

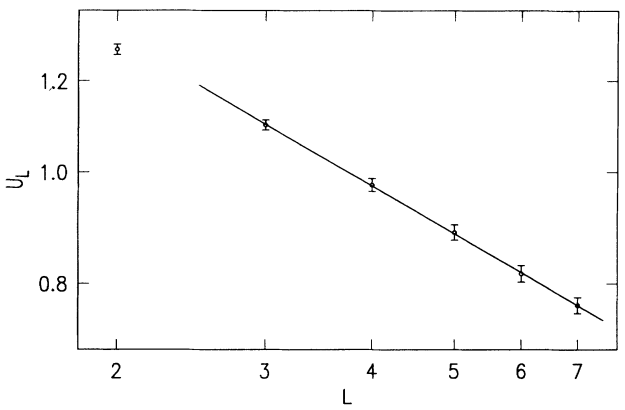

FIG. 1. Ensemble-averaged modulus of the defect energy, U_L in (4), vs system size for the gauge-glass model (2) on square L by L lattices, using the spin-quench method (see text). Error bars represent 2 standard deviations, and the straight line is a fit to the data, excluding the point for $L=2$.

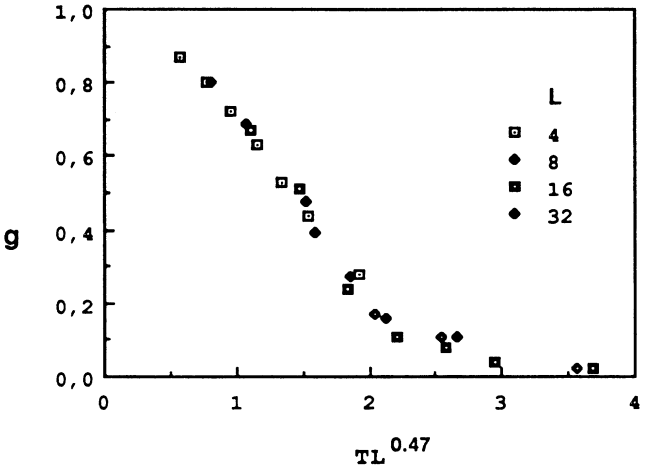

FIG. 2. Scaling plot of the ratio of moments g [Eq. (9)] computed with Monte Carlo for the same model as in Fig. 1.

tures. This regime will occur for small applied magnetic fields. However, a_v increases[10] with increasing field, and for $a_v > l$, quantum VRH dominates the resistivity and a nonactivated form as in (7) with multivortex-hopping exponents is expected. Upon approaching the field-tuned superconductor-insulator transition[10] at some critical field B_c, a_v diverges, T_0 in (7) vanishes, and the form of the resistivity becomes dominated by quantum critical fluctuations.[10] Thus the nonactivated form in (7) should be most accessible at intermediate fields, say, $B \sim B_c/2$.

The above discussion involves a critical exponent θ describing the $T=0$ vortex-glass phase. We now describe three different calculations of θ for the gauge-glass model of Eq. (3), which all give a value of θ close to $-\frac{1}{2}$. The most direct method involves searching for the ground state of H (or $\tilde{H}$), with both periodic and antiperiodic boundary conditions and ensemble averaging the modulus of the difference to get U_L. For very small system sizes (L by L with $L=2,3,4$), we computed U_L for the Villain model in the vortex representation, (3) and (5), by exhaustive searches for the ground state, allowing each site to either be vacant or have one vortex. Slightly larger sizes were possible using a spin-quench method,[15] which can also be applied to the cosine model, Eq. (2). In this approach, a random initial configuration of the $\{\phi_r\}$ is relaxed to a stationary (possibly metastable) state, which is then repeated for a large number of initial configurations (typically 200 to 1000) until further attempts cease to find states of lower energy. Where they overlapped, the two methods agreed within the statistical uncertainties. Spin-quench results for the cosine model are shown in Fig. 1, for systems up to $L=7$, on a double-logarithmic plot. The error bars represent 2 standard deviations and are limited by the number of samples in the ensemble average. Ignoring possible systematic errors due to small system sizes, the slope gives an estimate of $|\theta| = 0.43 \pm 0.03$. The value of θ for the Villain model was consistent with this.

An estimate for θ was also obtained from Monte Carlo simulations on the cosine model. Both the spin-glass susceptibility, $\chi_{SG} = N[\langle |q|^2 \rangle]_{av}$, and a ratio of moments,

$$g = 2 - [\langle |q|^4 \rangle]_{av} / [\langle |q|^2 \rangle]^2_{av}, \quad (9)$$

were calculated.[16,17] Here $q = N^{-1} \sum_r \exp[i(\phi_r^{(1)} - \phi_r^{(2)})]$ is the overlap between the configurations of two identical independent copies of the system. The quantity g is expected to have the finite-size scaling form $g = \tilde{g}(L^{1/\nu_T} T)$ from which the estimate $1/\nu_T = |\theta| = 0.47$ is found, as shown in Fig. 2. It is gratifying that quite different calculations give consistent results, namely, $\theta \simeq -0.5$. It would be very interesting to do experiments on thin superconducting films to test this prediction.

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