Measuring T_c of the Quark-Gluon Plasma with e^+e^- Pairs

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The e^+e^- spectrum resulting from ultrarelativistic heavy-ion collisions is shown to have a sharp resonance at an invariant mass $M \approx 0.2T_c$ (≈ 40 MeV for typical $T_c = 200$ MeV). Observation of this resonance would signal the existence of the quark-gluon plasma, confirm the first-order nature of the transition, and determine the value of T_c.

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Modern cosmology rests on the expectation that when the Universe was very hot, quarks and gluons were not confined inside nuclei. Lattice studies of quantum chromodynamics predict a first-order phase transition[1] at a critical temperature $T_c \sim 175$–275 MeV. To recreate the high-temperature phase, accelerators will collide heavy ions at energies of 200 GeV per nucleon.[2,3]

The production of lepton pairs (e^+e^- or $\mu^+\mu^-$) will be an important signal because they escape the collision region without reinteracting and therefore can convey information about the hot interior.[4] High-energy lepton pairs are produced by a virtual γ, whose energy q^0 and momentum $\mathbf{q}$ are the observed total energy and momentum of the lepton pair. Their invariant mass if $M^2 = (q^0)^2 - \mathbf{q}^2$. For M large (500 MeV–5 GeV) the multiplicity of dileptons per unit space-time volume is[5]

$$\frac{dN}{d^4x\,d^4q} = \frac{\alpha}{12\pi^4}\frac{\Gamma^\mu_\mu}{M^2}, \tag{1}$$

where Γ^μ_μ is the polarization-averaged "rate" for producing a virtual γ. Γ^μ_μ can be computed by squaring the amplitude for each contributing process (e.g., $Q\bar{Q} \rightarrow \gamma$, $\pi^+\pi^- \rightarrow \gamma$) and integrating over the thermal phase space of all the particles except the γ. The strong interactions produce resonances in Γ^μ_μ at each of the vector mesons ($\rho, \omega, \phi, J/\psi$). Since each point in the plasma may have four-velocity u_μ, the Bose-Einstein and Fermi-Dirac factors in the thermal phase space are functions of $p \cdot u$ for a particle of momentum p. This is the starting point for most dilepton calculations.[5–13] One can also relate Γ^μ_μ to the photon proper self-energy at finite T,

$$\Gamma^\mu_\mu = -\mathrm{Im}\Pi^\mu_\mu/(e^{\beta q\cdot u} - 1) > 0. \tag{2}$$

Consequently (1) can be calculated from the photon self-energy.[10–13]

The formula (1) is not completely general, for it assumes that the probability amplitude for the virtual γ to propagate through the plasma is the free photon propagator, $1/M^2$. In ordinary plasmas (or even in dielectrics) the propagation of electromagnetic waves is strongly dependent on frequency. For a particular wave vector there is usually one frequency that enjoys resonance propagation and most other frequencies are so severely damped that they are neglected. When the effect of propagation is included, the multiplicity of lepton pairs per space-time volume becomes[14]

$$\frac{dN}{d^4x\,d^4q} = \frac{\alpha}{12\pi^4}\frac{2\Re_T + \Re_L}{e^{\beta q\cdot u} - 1}, \tag{3a}$$

$$\Re_j = \frac{-M^2\mathrm{Im}\Pi_j}{(M^2 - \mathrm{Re}\Pi_j)^2 + (\mathrm{Im}\Pi_j)^2} > 0, \tag{3b}$$

where $j = T$ or L. Π_T and Π_L are the transverse and longitudinal self-energy functions of the γ. If the plasma has four-velocity u^μ, the two functions depend on temperature, M, and the energy $q \cdot u$. The real functions $\Re_T$ and $\Re_L$ represent the probability that the virtual γ will propagate through the plasma. If one ignores the denominators in (3) then the result agrees with (1) since $\Pi^\mu_\mu = 2\Pi_T + \Pi_L$. The validity of (3) depends on the wavelength being much smaller than the system size. The mean free path can be much larger than the system size. The larger the mean free path ($\approx k/|\mathrm{Im}\Pi|$) becomes, the more nearly $\Re$ approaches $\delta(M^2 - \mathrm{Re}\Pi)$. This is the case in geometrical optics, where $q^0 = |\mathbf{q}|/n$ is the only frequency that can propagate in a media with index of refraction n.

The hadrons $\rho, \omega, \phi, J/\psi$ still occur as strong-interaction resonances in the numerator of (3b). The new feature is the possibility of a plasma resonance when the denominator of (3b) is a minimum: i.e., at $M^2 = \mathrm{Re}\Pi$. Since the one-quark-loop contribution to the self-energy is $\mathrm{Re}\Pi \approx e^2T^2$, for reasonable T a resonance would occur at M less than 100 MeV. This can only appear in the e^+e^- channel, since $\mu^+\mu^-$ production requires $M > 210$ MeV. It will not be possible to detect an e^+e^- resonance if the energy $q \cdot u$ is small because the backgrounds will dominate. However, at larger values of $q \cdot u$ the backgrounds fall rapidly. We are thus led to consider the kinematic region

$$q \cdot u \gg T \gg M. \tag{4}$$

In this region the longitudinal contribution $\Re_L$ is negligible because $M^2 = \mathrm{Re}\Pi_L$ is satisfied very near the light cone[15] and the factor $M^2 \approx 0$ in the numerator of (3b) will suppress $\Re_L$. In the same region (4) the transverse self-energy is[15]

$$\mathrm{Re}\Pi_T = \tfrac{5}{3} e^2T^2/6 , \quad (5)$$

where the factor $\frac{5}{3}$ results from the squared charges of the light quarks, u and d, summed over color. This gives a resonance at $M^2 = \mathrm{Re}\Pi_T$, or $M = 0.16T$. There will be QCD corrections to (5) of order $e^2g^2T^2$ which can raise this value. So the resonance is expected at $M \approx 0.2T$.

The width of the resonance is determined by the imaginary part of Π. The one-loop contribution resulting from $Q\bar{Q}$ annihilating into a virtual γ is $|\mathrm{Im}\Pi| \sim \alpha M^2$ provided the virtual photon mass is above threshold: $M > 2m_Q$. For large M the denominators in (3) are irrelevant.[4-10] Then $\Re = |\mathrm{Im}\Pi_T|/M^2 \sim \alpha$ is small.

Near the resonance at $M \approx 0.2T$ the threshold condition for $Q\bar{Q}$ annihilation probably cannot be satisfied for several reasons: (a) The confinement transition is close to, but not identical with, the chiral-symmetry-breaking transition. Consequently, the residual quark masses may be large enough that $M \not> 2m_Q$. (b) Even if chiral symmetry is exact, the quarks acquire effective thermal masses[16] $m_Q = gT/\sqrt{6}$ that would be too heavy to annihilate into one photon of mass M.

Moreover, the QCD corrections to $\mathrm{Im}\Pi$ are larger than the one-loop result because of phase space. The phase space for $Q\bar{Q}$ annihilation is $\sim M^2$, which is small near resonance. The phase space for scattering contributions ($ab \to c\gamma$) to $\mathrm{Im}\Pi$ is $\sim T^2$, which is much larger. There are three QCD processes: gluon emission ($Q\bar{Q} \to G+\gamma$), Compton scattering from a quark ($QG \to Q+\gamma$), and Compton scattering from an antiquark ($\bar{Q}G \to \bar{Q}+\gamma$). All are two-loop contributions to $\mathrm{Im}\Pi$. These are free of infrared and mass singularities.[11,12] Braaten, Pisarski, and Yuan[13] have calculated the value of $\mathrm{Im}\Pi$ when $M < T$, but for a γ that is not moving with respect to the plasma (i.e., $q\cdot u = M$). In the kinematic region (4) the γ has a large momentum with respect to the plasma and a new calculation is necessary. By squaring each amplitude and integrating over the three unobserved particles, the calculation can be reduced to a four-dimensional numerical integration. In the kinematic region (4) the result is

$$|\mathrm{Im}\Pi_T| \approx \tfrac{10}{9}\,[e^2g^2/(2\pi)^4]aT^2 , \quad (6)$$

where a is determined numerically. Each of the three processes contributes about equally. When the quarks are massive, $a \approx 25$. Consequently, $|\mathrm{Im}\Pi_T| \approx e^2g^2T^2/56$. For typical $g^2 \approx 1.2$ (Ref. 1) and $T \sim 200$–400 MeV the width is 1–2 MeV. The corresponding lifetime of 100–200 fm/c is intermediate between those of the ω (24 fm/c) and ϕ (45 fm/c) and that of the J/ψ (2940 fm/c). All of these are longer than the expected lifetime of the fireball, which is why they are able to convey information about the hot interior. (By contrast the ρ lifetime is so short, 1.3 fm/c, that it can only convey information about the surface of the fireball.)

The experimentally observable multiplicity dN/d^4q is obtained by integrating (3) over the space-time volume of the plasma in the standard manner.[2,4,6-9] The colliding-beam axis is $\hat{\mathbf{z}}$ and the collision occurs at $t=0$. Because the initial energy density is so high, quark and gluon interactions will rapidly lead to thermalization in the central region.[17] Perfect fluid hydrodynamics[3] shows that if the expansion is only along the z axis, then the thermodynamic functions only depend on the proper time $\tau = (t^2 - z^2)^{1/2}$. An element of the plasma at (t,x,y,z) expands along the beam axis $\hat{\mathbf{z}}$ with velocity $\mathbf{v} = \hat{\mathbf{z}}z/t$. Its four-velocity is $u^\mu = (\cosh\eta, 0, 0, \sinh\eta)$, where η is the coordinate-space rapidity. The integration element is $d^4x = d^2x_\perp \tau d\tau d\eta$, but because the expansion is one dimensional, $\int d^2x_\perp = \pi R_A^2$, with $R_A \approx 8$ fm for uranium. The rate to be integrated (3) depends on the proper time τ through the temperature and depends on the rapidity η through $q\cdot u$. The four components of the dilepton momentum in the laboratory frame are conventionally chosen as

$$q^\mu = (M_\perp \cosh y, q_{\perp x}, q_{\perp y}, M_\perp \sinh y) , \quad (7)$$

where $M_\perp = (M^2 + q_\perp^2)^{1/2}$. Because $q\cdot u = M_\perp \cosh(\eta - y)$, the kinematic region (4) is guaranteed by choosing $q_\perp \gg T$. The integral over η can be performed analytically[7,8] (whenever $\Re$ is independent of η) and gives

$$\frac{dN}{d^4q} = \frac{\alpha R_A^2}{6\pi^3} \int \tau d\tau f(\tau) \Re K_0(M_\perp/T) , \quad (8)$$

where $\Re = 2\Re_T + \Re_L$ and K_0 is the modified Bessel function.

The essential parameters in the thermal history are the initial time τ_0 of thermalization and three temperatures: the initial temperature T_0, the critical temperature T_c, and the final temperature T_f. The values used here are $\tau_0 = 1.0$ fm/c, $T_0 = 400$ MeV, $T_c = 200$ MeV, $T_f = 100$ MeV. The ratio of color degrees of freedom (two light quarks plus gluons) to pion degrees of freedom is $r = \frac{37}{3}$. There are three stages in the cooling process:[3-9] (i) For $\tau_0 < \tau < \tau_1$ the system is in a pure quark-gluon phase ($f=1$) with $T_c < T < T_0$. Proper time and temperature are related by $\tau = \tau_0(T_0/T)^3$ and this determines the transition time τ_1. (ii) For $\tau_1 < \tau < \tau_2 = r\tau_1$ the system is in a mixed phase undergoing a first-order phase transition at fixed temperature T_c. The quark-gluon fraction of the system is $f = (\tau_2/\tau - 1)/(r-1)$, which decreases from 1 to 0 during the mixed phase. (iii) For $\tau_2 < \tau < \tau_f$ the system is in the hadron phase ($f=0$), i.e., a pion plasma, with $T_f < T < T_c$. Proper time and temperature are related by $\tau = \tau_2(T_c/T)^3$ and this determines the final time τ_f at which the decoupling temperature T_f is reached.

(1) Plasma resonance.—When the integration in (8) is performed over the pure quark-gluon phase ($\tau_0 < \tau < \tau_1$), the resonance at $M \approx 0.2T$ will produce a broad

shoulder for $0.2T_c < M < 0.2T_0$. However, during the phase transition, the temperature remains fixed at T_c and the resonance in $\Re_T$ is not smeared at all. Integrating (8) over $\tau_1 < \tau < \tau_2$ gives

$$\frac{dN}{d^4q} = \frac{\alpha R_A^2}{6\pi^3}(r-1)\tau_1^2 \Re_T K_0(M_\perp/T_c). \quad (9)$$

The peak in $\Re_T$ at $M \approx 0.2T_c$ is a direct signal of the first-order phase transition. The results are shown in Fig. 1 for two choices of $q_\perp$. The backgrounds to this signal will be considered next.

(2) Pion bremsstrahlung.— From the mixed phase and the hadronic phase the major background is pion bremsstrahlung ($\pi\pi \to \pi\pi e^+e^-$). There is no resonance effect and so $\Re = |\mathrm{Im}\Pi_\mu^\mu|/M^2$ in (8). To estimate this one can use a quartic interaction $\mathcal{L}_I = \lambda(\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2/4$, with $\lambda \approx 1.4$ from $\pi\pi$ scattering lengths.[18] Because the pions can

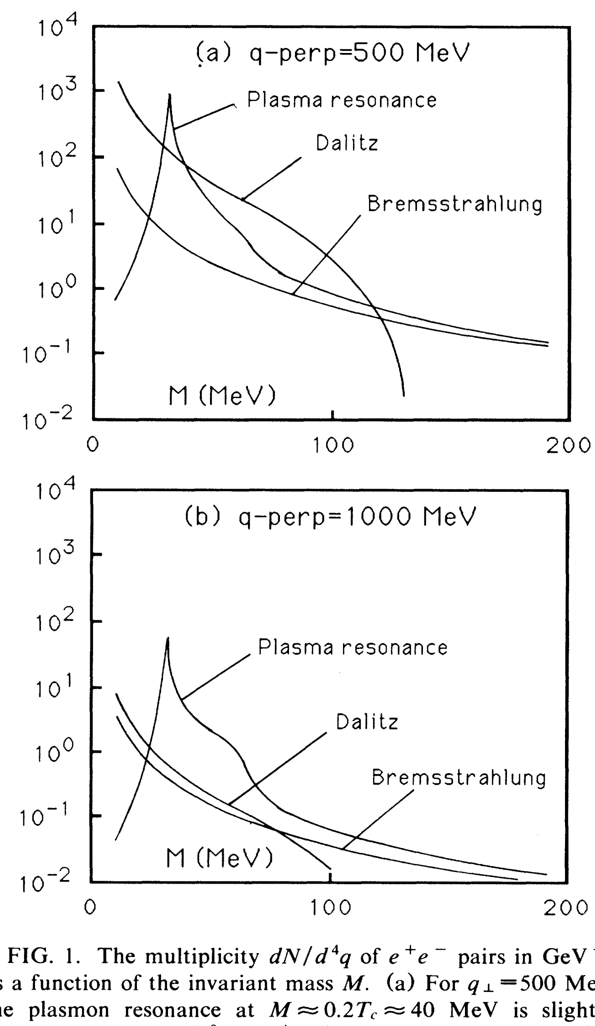

FIG. 1. The multiplicity dN/d^4q of e^+e^- pairs in GeV^{-4} as a function of the invariant mass M. (a) For $q_\perp = 500$ MeV the plasmon resonance at $M \approx 0.2T_c \approx 40$ MeV is slightly above the Dalitz ($\pi^0 \to \gamma e^+e^-$) and bremsstrahlung ($\pi\pi \to \pi\pi e^+e^-$) backgrounds. (b) For $q_\perp = 1000$ MeV the plasmon resonance is 70 times the background.

have various charges, altogether there are twenty Feynman diagrams. A crude bound on the overall rate is

$$|\mathrm{Im}\Pi_\mu^\mu| < \frac{\alpha\lambda^2}{4\pi^4}(q\cdot u)Te^{-(2m_\pi + q\cdot u)/T}. \quad (10)$$

When the bremsstrahlung bound is substituted into (8), the contribution from the pure hadron phase predominates (by a factor of 2 or 3) over that of the mixed phase. Their sum is plotted in Fig. 1.

(3) $\pi^0 \to \gamma e^+e^-$.— The major background is the Dalitz three-body decay of the π^0. The lifetime of the π^0 is so long that very few will decay during the mixed or hadronic phases. Eventually the reaction rates become so infrequent that the pions decouple. Integrating the Bose-Einstein function for massive pions at $T_f = 100$ MeV gives $dN_\pi/dy = 4400$. After decoupling, the pions cease interacting and their momenta remains constant. The free-streaming π^0's decay in flight and produce the e^+e^- background. From PCAC (partial conservation of axial-vector current), the differential branching ratio for $\pi^0 \to \gamma e^+e^-$ with a dilepton momentum q^μ is

$$\frac{dB}{d^4q} = \frac{4\alpha}{\pi^3}\frac{(m_\pi^2 - M^2)^2}{M^2 m_\pi^4}\delta(m_\pi^2 + M^2 - 2p_\pi\cdot q). \quad (11)$$

Integrating this over all q (with $2m_e < M < m_\pi$) gives the known branching ratio 1.2%. At the decoupling time τ_f all the pions on the hypersurface $d\sigma_\mu = d^4x\delta(x\cdot u - \tau_f)u_\mu$ have a momentum distribution characterized by temperature T_f. The number of dilepton pairs they eventually produce is

$$\frac{dN}{d^4q} = \int d\sigma_\mu \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu/E}{\exp(p\cdot u/T_f) - 1}\frac{dB}{d^4q}. \quad (12)$$

For one-dimensional expansion, $d\sigma_\mu = \pi R_A^2 \tau_f \int d\eta\, u_\mu$. At large $q_\perp$ this gives

$$\frac{dN}{d^4q} = \frac{\alpha\tau_f R_A^2 T_f}{\pi^4}\frac{(m_\pi^2 - M^2)^2}{M^2 m_\pi^4}K_0(m_{\pi\perp}/T_f), \quad (13)$$

with $m_{\pi\perp} = (m_\pi^2 + q_\perp^2)^{1/2}$. Figure 1 shows that at $q_\perp = 500$ MeV the transverse plasmon resonance is about 7 times the Dalitz background; at $q_\perp = 1000$ MeV it is 70 times the background. This should be a spectacular signal.

It is perhaps useful to display the dependence of the results on the four parameters τ_0, T_0, T_c, and T_f. The ratio of the resonance peak (9) to the Dalitz background (13) can be approximated for $q_\perp \gg m_\pi \gg M$ as

$$0.17\frac{\tau_0 T_0^3}{T_c^2}\left(\frac{T_f}{T_c}\right)^{3/2}\frac{\exp(q_\perp/T_f)}{\exp(q_\perp/T_c)}. \quad (14)$$

For example, if T_c is reduced to 150 MeV with the other parameters fixed, the resonance is still 37 times larger than the Dalitz background for $q_\perp = 1000$ MeV. Generally, no matter what the values of τ_0, T_0, T_c, and T_f, one can always choose a $q_\perp$ large enough that the resonance stands out above the Dalitz background.

The most important effect that has not been included here is the transverse expansion of the plasma, perpendicular to the beam axis.[8,9] Because there is little transverse motion before T_c, the resonance will not be affected much. In the hadron phase, the velocity u_μ will develop transverse components whose value depends on r and τ. This will increase the backgrounds but will be partially compensated by a much smaller decoupling time τ_f.

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