Comment on "Collapse and Revival of the State Vector in the Jaynes-Cummings Model: An Example of State Preparation by a Quantum Apparatus"

The Jaynes-Cummings model (JCM) of quantum optical resonance is an important fundamental theoretical model of the interaction between two dissimilar quantum systems. The model consists of a single quantized field mode interacting with a single two-level atom.[1] In a recent Letter,[2] Gea-Banacloche has studied the evolution of the atomic and field state vectors in the JCM. He showed that the atom and field (initially prepared in pure states) most closely return to pure states during the so-called collapse region. It is pointed out in this Comment that the von Neumann entropy, rather than the square of the density operator used by Gea-Banacloche, is a more *sensitive* measure of the purity of a quantum state, in general. Furthermore, we give an exact form for the entangled atom-field state as it evolves under the Jaynes-Cummings Hamiltonian. This entangled state allows one to determine, without any of the approximations used by Gea-Banacloche,[2] the form of the pure states to which the atom and field most closely evolve in the course of the interaction. We also seek to clarify the apparent claim that the field state evolves on a different time scale from that of the atomic state.

The crucial parameter which determines the purity of a quantum state is the von Neumann entropy,[3] $S = -\mathrm{Tr}\rho\ln\rho$. For pure states this entropy is zero, implying that the information we can retrieve from such a state is the maximum that quantum mechanics will allow. The purity of a quantum state is mathematically determined by the exact requirement that $\rho^n = \rho$, where n is an integer. Thus, in general, all moments $\mathrm{Tr}\rho^n$ must be determined to *fully* assess the purity of the state, although for two-state systems $\mathrm{Tr}\rho^2$ is, in fact, sufficient to give a complete measure. This procedure is automatically performed in a calculation of the entropy. The entropy of the atom in the one-photon JCM has been calculated, in some detail, by Phoenix and Knight,[4] who show that, for the parameters considered, the approach to a pure state is only approximate (about 85%) and occurs towards the end of the collapse region.[4]

The most surprising result of the work of Phoenix and Knight is that the field in the JCM can, *at all times*, be described by *just two quantum states.* The explicit expressions for these states have been given.[4] This remarkable fact follows from an inequality derived by Araki and Lieb[5] which states that for two quantum systems, labeled by a and b, their entropies are linked to the total system entropy S by the relation

$$|S_a - S_b| \le S \le S_a + S_b . \tag{1}$$

This inequality shows us that if we prepare a two-component quantum system in a pure state then the entropies of the component systems are equal throughout their subsequent evolution. The inequality (1) shows that the *purity* of the component systems cannot evolve on different time scales as this would be reflected in the entropy. This also tells us that both the atom and field in the JCM approach a pure state *at the same time.* The entangled atom-field state can be determined using the methods developed earlier[4] and the form of this state is given by

$$|\psi_{af}(t)\rangle = (\pi^{(+)})^{1/2}|\psi_f^{(+)}\rangle \otimes |\psi_a^{(+)}\rangle + (\pi^{(-)})^{1/2}|\psi_f^{(-)}\rangle \otimes |\psi_a^{(-)}\rangle , \tag{2}$$

where the states $|\psi_{a(f)}^{(\pm)}\rangle$ are the eigenstates of the atom (field) density operator and $\pi^{(\pm)}$ are the eigenvalues. We note here that the pure state to which the system most closely evolves during the collapse region is given by $|\psi_f^{(+)}\rangle \otimes |\psi_a^{(+)}\rangle$, and the atomic state at this time can be shown to reduce to Eq. (6) of the paper by Gea-Banacloche.[2]

The use of the entropy as a dynamical parameter is necessary to make a precise and quantitative statement about the evolution of correlations between the atom and field in the JCM.[6] We see that the atom and field decorrelate during the collapse region with the correlations being reestablished during the first revival. It is the establishment of "entanglement" between the field and atomic states which governs the evolution of the correlation and the degree of entanglement is determined by the eigenvalues $\pi^{(\pm)}$.

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[1]For a review of the JCM, see the article by S. M. Barnett, P. Filipowicz, J. Javanainen, P. L. Knight, and P. Meystre, in *Frontiers in Quantum Optics,* edited by E. R. Pike and S. Sarkar (Adam Hilger, Bristol, 1986).

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