"Low-Temperature" Behavior of a Phase-Slip Center

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The differential resistance of the voltage-current characteristic generated by an isolated phase-slip center in a superconducting tin whisker has been measured down to 50 mK below its critical temperature. As a function of the temperature the differential resistance shows a resonancelike behavior with a maximum at the temperature where charge-imbalance waves excited by the phase-slip center have their maximum decay length, suggesting that the observed maximum of the differential resistance gives experimental evidence for charge-imbalance waves arising from a phase-slip center.

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For temperatures some millikelvins below the critical temperature T_{c0}, the voltage-current (V-I) characteristics at fixed temperature T of superconducting tin whiskers show a steplike structure[1,2] generated by localized phase-slip centers.[3-5] At the core of a phase-slip center the order parameter performs a relaxation oscillation at the Josephson frequency with a periodic production of nonequilibrium quasiparticle excitations. The diffusion and relaxation of these nonequilibrium quasiparticles govern the so-called "normal-like length" L_{Anl} which is proportional to the differential resistance of the first step in a voltage-current characteristic related to a single, i.e., isolated, phase-slip center. A recent review on this topic of nonequilibrium superconductivity is given in Ref. 6.

Most experiments on whiskers were carried out in a small temperature range (down to about 15 mK) below their critical temperature T_{c0}. In this paper we report measurements on tin whiskers for temperatures down to 50 mK below T_{c0}. The tin whiskers used were grown by a squeeze technique.[1,2] The measurements were performed in a ^{4}He bath cryostat.[6]

In Fig. 1 we show a sketch of the first voltage step. It is characterized by the critical current I_c, the height V_1 of the voltage jump at I_c, the differential resistance $(dV/dI)_1$, and the extrapolated zero-voltage intercept I_0. The normal-like length L_{Anl} is related to $(dV/dI)_1$ by

$$L_{\mathrm{Anl}} = (L/R_n)(dV/dI)_1 , \qquad (1)$$

where L is the length of the sample and R_n its residual resistance.

From the V-I characteristics we evaluated the differential resistance $(dV/dI)_1$ and the ratio I_0/I_c. We plot them as a function of temperature in Fig. 2. Both quantities go through a maximum. For sample Sn 1 the maximum occurs at $\Delta T^{\max} = T_{c0} - T^{\max} = 21$ mK. For another sample, Sn 11, similar resonancelike temperature dependences are obtained[7] with a maximum at $\Delta T^{\max} = 27$ mK. The temperature difference ΔT always refers to the critical temperature T_{c0} of the sample considered. For each sample mentioned, $V_1(I_c)$ consists of two linear portions.[7] The change from the first linear part to the second one occurs at a current I_c corresponding to the maximum of the differential resistance $(dV/dI)_1$. In Figs. 3 and 4 we show $V_1(I_c)$ and $I_c^{2/3}(\Delta T)$ for sample Sn 1. The temperature of the maximum of $(dV/dI)_1$, $\Delta T = 21$ mK, pertains to a critical current of $I_c = (>71.2)^{3/2}$ μA $= 601$ μA, according to Fig. 4. Regarding Fig. 3, we see that there is a crossover of $V_1(I_c)$ from one linear part to the next one at a critical current of about 600 μA.

The basic mechanism of a phase-slip center is described by the model of Skocpol, Beasley, and Tinkham (SBT).[3] In this model the nonequilibrium quasiparticles, which are generated during the phase-slip cycle in the core region of the phase-slip center, diffuse into the bordering parts of the superconductor and their charge imbalance relaxes while they are traveling by random

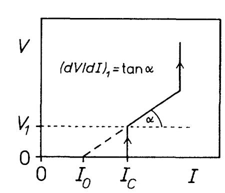

FIG. 1. Sketch of the first voltage step in the V-I characteristics of a Sn whisker.

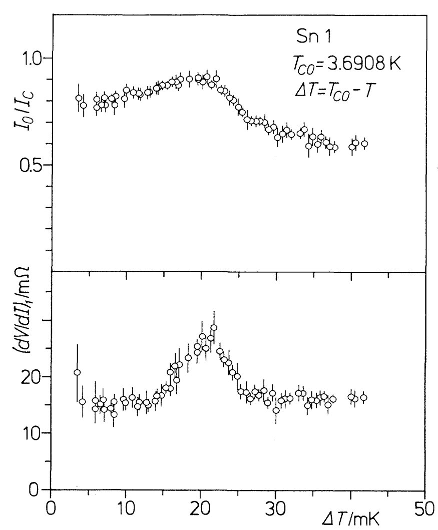

FIG. 2. Differential resistance $(dV/dI)_1$ and ratio I_0/I_c as a function of the temperature T for the first voltage step in the V-I characteristic of a Sn whisker (sample Sn 1).

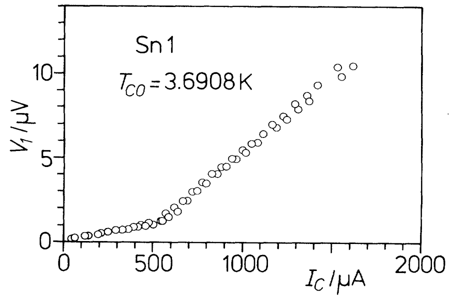

FIG. 3. Height V_1 of the first voltage jump in the V-I characteristic of a Sn whisker (sample Sn 1) as a function of the critical current I_c at which the first voltage jump occurs.

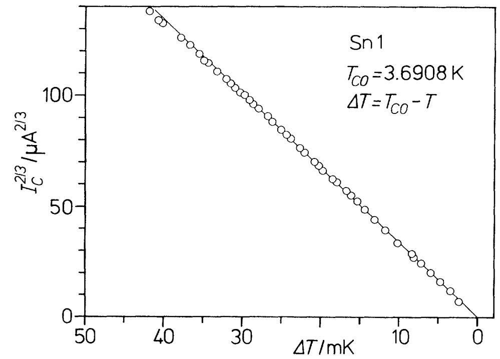

FIG. 4. Critical current raised to a power of $\frac{2}{3}$, $I_c^{2/3}$, as a function of the temperature T for a Sn whisker (sample Sn 1). $I_c^{2/3}(\Delta T)$ follows a straight line with a zero-current intercept T_{c0}, as predicted by the Ginzburg-Landau theory for a thin superconducting wire (Ref. 6).

walk. The normal-like length L_{An1} is identified with twice the charge-imbalance relaxation length, $\Lambda_{Q^*} = (\frac{1}{3} l v_F \tau_{Q^*})^{1/2}$. Here l is the mean free path of the electrons, v_F the Fermi velocity, and τ_{Q^*} the charge-imbalance relaxation time. This model cannot explain our experimental results. Since $(dV/dI)_1 \sim L_{\mathrm{An1}} \sim \Lambda_{Q^*} \sim \tau_{Q^*}^{1/2}$, the temperature dependence of the charge-imbalance relaxation time should govern the temperature dependence of the differential resistance. Chi and Clarke[8] and Clarke[9] calculated the time τ_{Q^*} for a wide temperature range. However, as a function of the temperature, the time τ_{Q^*} shows a minimum and not a maximum.

We, therefore, consider the Kadin, Smith, and Skocpol (KSS) model[4] which treats a phase-slip center as a source for charge-imbalance waves propagating along the superconductor just as electrical signals along a telegraph line. We found that the decay length of these waves has a maximum at a temperature which is in a good quantitative agreement with the temperature at which the maximum of $(dV/dI)_1$ is observed in our experiments. This good agreement suggests that we regard the appearance of the maximum of $(dV/dI)_1$ as experimental evidence for charge-imbalance waves excited by the phase-slip center, as will be discussed in the following.

The KSS model allows the calculation of the V-I characteristics generated by a phase-slip center. In the high-voltage dc limit ($I \gg I_c$) the KSS model reproduces the SBT result for the V-I dependence. In the general case, the V-I characteristics have to be calculated numerically by a, usually self-consistent, computing procedure. Moreover, KSS give an approximate solution of their model (see Fig. 13 of Ref. 4). For impressed current, as in our experiments, a V-I characteristic with a voltage jump at I_c and a straight-line behavior above I_c is only predicted for $\tau_{0R} < \tau_E$, where τ_E is the inelastic electron-phonon scattering time and τ_{0R} the supercurrent response time. In this case the transition is hysteretic, i.e., the dissipative phase-slip state is entered at I_c for increasing current while for decreasing current the superconducting state is recovered at a current which is smaller than I_c. For $\tau_{0R} \geq \tau_E$ the KSS model predicts a continuous transition at I_c with a slowly changing slope. The experiments, however, already show V-I characteristics with a sharp voltage jump, followed by a straight line, in the temperature range where $\tau_{0R} \geq \tau_E$. Therefore, it is not reasonable to evaluate the form of $(dV/dI)_1$ vs T in the framework of the KSS model and to compare it with our experimental results. Investigations of the hysteretic behavior of a phase-slip center show that in addition to charge-imbalance waves also Joule heating and quasiparticle overpopulation effects have to be considered for a quantitative description of the onset and width of the measured hysteresis.[6,10] To get realistic V-I characteristics, and thus the form of $(dV/dI)_1$ vs T, one probably has to consider these additional mechanisms in the KSS model. Nevertheless, qualitatively one would expect a maximum of L_{An1}, and thus of $(dV/dI)_1$, if the decay length of the charge-imbalance waves, k_l^{-1}, becomes maximal. The reason is that L_{An1} (see the SBT model) and k_l^{-1} both are a measure of the extent of the nonequilibrium region around a phase-slip center.

The damping of the charge-imbalance waves depends on their frequency, the Josephson frequency $\omega = (2e/\hbar)V$. Here e is the elementary charge, $\hbar = h/2\pi$ with h Planck's constant, and V the voltage across the phase-slip center. In the high-frequency limit ($\omega \gg \tau_{0R}^{-1}, \tau_E^{-1}$) the charge-imbalance waves have a decay length given by[4]

$$k_l^{-1} = 2\Lambda_{Q^*}(\tau_{0R}\tau_E)^{1/2}/(\tau_{0R}+\tau_E)\,. \tag{2}$$

While τ_E is temperature independent for a fixed sample, the time τ_{0R} depends on temperature. So, we changed the time t_{0R} by a variation of the temperature in our experiments.

The decay length k_l^{-1} has indeed a local maximum. The condition for this maximum depends on the temperature dependence of Λ_{Q^*}. Applying the SBT model to the experimental data in the region close to the critical temperature where the differential resistance is temperature independent yields a temperature-independent charge-imbalance relaxation length. In this case the length k_l^{-1} has a local maximum, if the function $(\tau_{0R}\tau_E)^{1/2}/(\tau_{0R}+\tau_E)$ in Eq. (2) is maximal, which happens for $\tau_{0R}=\tau_E$.

Now we determine the temperature where $\tau_{0R}=\tau_E$ is fulfilled. For this purpose we calculate τ_{0R} according to[10]

$$\tau_{0R}=(l/2v_F\chi)T_{c0}(T_{c0}-T)^{-1}, \tag{3}$$

where $\chi=(1+0.752\xi_0/l)^{-1}$ with ξ_0 the BCS coherence length. For τ_E we use Tinkham's estimate[10]

$$\tau_E=(\tau_\Theta/8.4)(\Theta/T_{c0})^3, \tag{4}$$

where $\tau_\Theta=(\rho_\Theta l_\Theta/\rho_{298\,\mathrm{K}}v_F)$ [(298 K)/Θ]. Here, Θ is the Debye temperature, $\rho_{298\,\mathrm{K}}$ the temperature-dependent part of the resistivity at room temperature, and $\rho_\Theta l_\Theta=\rho_n l$ with ρ_n the residual resistivity. We insert the following values:[10] $v_F=0.684\times10^6$ m/s, $\Theta=200$ K, $\rho_n l=10^{-3}$ $\Omega\,\mu\mathrm{m}^2$. For the BCS coherence length we take[2] $\xi_0=2980\times10^{-10}$ m. The resistivity $\rho_{298\,\mathrm{K}}$ depends on the orientation of the sample. It is $\rho_{298\,\mathrm{K}\,[101]}=11.10\times10^{-2}$ $\Omega\,\mu$m for a sample with [101] orientation and $\rho_{298\,\mathrm{K}\,[111]}=10.57\times10^{-2}$ $\Omega\,\mu$m for a sample with a [111] orientation.[2] Values for T_{c0} and l are given in Table I which contains further material parameters of the samples Sn 1 and Sn 11. We get $\tau_E=3.72\times10^{-10}$ s for sample Sn 1 and $\tau_E=3.91\times10^{-10}$ s for sample Sn 11, which are reasonable values for Sn as can be seen from the summary of experimental and theoretical results given in Ref. 6. Our values for τ_E are close to Yen and Lemberger's recent experimental result of $\tau_E=(3.57\pm0.36)\times10^{-10}$ s for Sn films.[11]

For sample Sn 1, the calculation yields $\tau_{0R}=\tau_E$ at $\Delta T=28.5$ mK, in reasonable agreement with the measured temperature $\Delta T^{\max}=21$ mK. For sample Sn 11 we obtain $\tau_{0R}=\tau_E$ at $\Delta T=27.3$ mK which is exactly the position of the maximum of $(dV/dI)_1$. This agreement between calculated and measured values indicates that charge-imbalance waves excited by the phase-slip center could indeed be the reason for the maximum observed in the differential resistance.

Our argument contains some critical points: Eq. (2) is only valid in the high-frequency limit. In the region of the maximum, the frequency ω is, no doubt, larger than τ_E^{-1} and τ_{0R}^{-1} but not so much larger. Moreover, it may be criticized that we assume Λ_{Q^*} to be temperature independent in Eq. (2) according to a comparison of the SBT model with our experimental results. The observed temperature-independent differential resistance is only partly understood, as discussed in Sec. 7.2 and Chap. 10 of Ref. 6. While whiskers of In and *In*-Pb show a divergence of the differential resistance in the direct vicinity of the critical temperature, a temperature-independent $(dV/dI)_1$, and thus $L_{\mathrm{An}1}$, is obtained for whiskers of Sn, Zn, Pb, several alloys, and, for temperatures not too close to T_{c0}, also for whiskers of In and *In*-Pb.[6] Applying the SBT model then yields a charge-imbalance relaxation length and time which are independent of temperature. However, no temperature-independent steady-state charge-imbalance relaxation time is known. Also the time-dependent Ginzburg-Landau (TDGL) theory[5,6] does not predict a temperature-independent normal-like length. In non-steady-state situations, KSS,[4] and also Lemberger,[12] showed that the dynamic charge-imbalance relaxation time is τ_E rather than τ_{Q^*} (see also Secs. 5.4 and 5.7 of Ref. 6). Moreover, Baratoff[13-15] calculated the behavior of phase-slip centers in a filament beyond the local equilibrium range of the TDGL theory (see also Sec. 5.9 of Ref. 6). For temperatures not too close to T_{c0}, Baratoff found a temperature-independent normal-like length, which is roughly given by $2\Lambda_E$, where $\Lambda_E=(\frac{1}{3}lv_F\tau_E)^{1/2}$. In Chap. 10 of Ref. 6 we concluded, with some caution, that the observation of a temperature-independent normal-like length indicates that the sample has left the temperature range of the local equilibrium approximation. Our assumption that Λ_{Q^*} in Eq. (2) is temperature independent is an attempt to consider this fact in the decay length of the charge-imbalance waves. Finally, as done by KSS, we only consider charge-imbalance relaxation due to inelastic electron-phonon scattering. Since Sn has an anisotropic energy gap, elastic scattering in principle also contributes to the charge-imbalance relaxation.[6,9,11,16] This contribution becomes large at low temperatures but can be neglected compared to the inelastic electron-phonon contribution very close to T_{c0}. Since for both samples investigated the maximum of the differential resistance is observed at a temperature not lower than 30 mK below T_{c0}, the effect of elastic scattering on the charge-imbalance relaxation should be small.[9,11,16]

Investigations performed with microbridges do not yield any local maximum of the differential resistance.[17-20] This result does not contradict our interpre-

TABLE I. Material parameters of the samples. L is the length, A the cross-sectional area, l the electron mean free path, $R_{298\,\mathrm{K}}$ the resistance at room temperature, R_n the residual resistance, and T_{c0} the critical temperature of the sample. The experimentally observed maximum of the differential resistance occurs at the temperature difference $\Delta T^{\max}=T_{c0}-T^{\max}$.

Sample	L (μm)	Orientation	A ($\mu\mathrm{m}^2$)	l (μm)	$R_{298\,\mathrm{K}}$ (Ω)	R_n (Ω)	T_{c0} (K)	$\Delta T^{\max}$ (mK)
Sn 1	750^{+63}_{-25}	[101]	$1.35^{+0.11}_{-0.05}$	3.697	61.7	0.15	3.6908	21
Sn 11	663^{+38}_{-25}	[111]	$0.639^{+0.036}_{-0.024}$	3.731	109.9	0.278	3.6892	27

tation. We demonstrate this by discussing measurements on tin microbridges: The tin microbridges differ from the tin whiskers in their much shorter electron mean free path l. In addition, the charge-imbalance relaxation length Λ_{Q^*} measured for tin microbridges is always temperature dependent. Because of these two distinctions the maximum of the decay length k_l^{-1} of the charge-imbalance waves in a microbridge appears at temperatures very close to the critical temperature T_{c0}. For $\Lambda_{Q^*} \sim \Delta T^{-1/4}$, as usually observed for microbridges, we obtain a local maximum of the decay length k_l^{-1} at $\tau_{0R} = 3\tau_E$. This $\Delta T^{-1/4}$ dependence of Λ_{Q^*} is found, for example, for microbridge TN 2 (with $l = 0.18$ μm and $\tau_E = 1.4\times 10^{-10}$ s) measured by Aponte and Tinkham,[20] and also for microbridges in Ref. 19. Calculating τ_{0R} with Eq. (3) and setting $\tau_{0R} = 3\tau_E$, we obtain for TN 2 a local maximum of k_l^{-1} at $\Delta T/T_{c0} = 7\times 10^{-4}$; that means $\Delta T = 2.6$ mK, taking $T_{c0} = 3.72$ K, valid for polycrystalline tin. In such a close region to T_{c0} there does not exist any measurement for that microbridge. Also for the tin microbridges in Ref. 19 with a still shorter electron mean free path there does not exist any measurement where the maximum of the decay length should appear.

Since the first observation of collective excitations in a superconductor by Carlson and Goldman,[21] there have been many studies of this phenomenon,[6] but only a few experiments give evidence for collective excitations caused by a phase-slip center.[22-24] In the present work we found a maximum of the differential resistance of the V-I characteristic generated by a phase-slip center. This maximum appears at a temperature where the decay length of charge-imbalance waves in the specimen is maximal. This result suggests further evidence for collective excitations arising from a phase-slip process.

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