Quantum Dynamics and Tunneling of Domain Walls in Ferromagnetic Insulators

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It is shown how large domain walls, containing $\geq 10^{10}$ spins, can behave as quantum objects at low temperatures. They move quantum diffusively, and exhibit macroscopic tunneling from defect pinning centers. The dissipation is calculated and shown to be very small; it does not involve the usual Caldeira-Leggett environmental couplings. The theory can also be used to treat smaller single-domain-wall behavior.

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Despite great advances in recent years in our understanding of mesoscopic and macroscopic quantum phenomena, it is still widely assumed that very special conditions must prevail [such as phase coherence or off-diagonal long-range order (ODLRO)] for really macroscopic effects to exist. Indeed, macroscopic quantum tunneling[1] (MQT), involving some 10^9 Cooper pairs in the relevant "instanton," has only been convincingly observed in SQUID rings.[2]

Here it is shown that a detailed theory of magnetic domain-wall motion, *incorporating all relevant dissipative couplings to the environment,* predicts that very large walls (with $> 10^{10}$ spins) should show both MQT and quantum diffusive behavior at millikelvin temperatures. This is contrary to the usual opinion,[3,4] which holds that magnetic phenomena can involve, at most, coherence amongst $\sim 10^4$ spins. Although this is indeed true for grain tunneling,[4] it is not true for walls.

The reason for this can be seen if one goes to a microscopic description which isolates the relevant macroscopic coordinate from all other degrees of freedom.[1] Any other description would naively indicate coherence effects to fall off with N (the number of spins involved in the coherent wall motion) as $e^{-\alpha N}$, with $\alpha \sim 1$. Just such a collective coordinate appears in domain-wall tunneling; it is the "wall center" coordinate $Q(t)$.

However, as has been made clear in recent years, these other degrees of freedom usually have a prohibitive dissipative effect on macroscopic quantum phenomena. These dissipative effects have never been considered for domain-wall tunneling (although the effect of phonons has been considered for grain tunneling[4,5] within a Caldeira-Leggett context). In fact, the domain wall has a variety of dissipative couplings to magnons, photons, impurities, and defects, as well as phonons. Unfortunately almost all work on these couplings has been phenomenological (see, however, Ref. 3, and also Ref. 6), and none of it has addressed the problems discussed here. Yet it cannot be overemphasized that a realistic theory must treat *all* of the "dangerous" dissipative mechanisms—otherwise one would have no justification for believing that quantum phenomena could persist on anything but microscopic or mesoscopic scales (i.e., $N \lesssim 10^4$). This general argument in fact applies to almost any large-scale quantum phenomenon; macroscopic systems almost always behave classically precisely because environmental couplings can so easily destroy coherence.[1]

Thus a fully microscopic derivation is given here of the effects of these couplings on the quantum dynamics of a large domain wall. We assume a magnetic insulator, which removes the very strong dissipation from itinerent electrons. Although most of this paper is concerned with macroscopic walls, it is clear that the theory may also be applied to smaller-scale processes, and this will be discussed at the end.

The main result of this paper is that in magnetic insulators the environmental couplings to the wall do *not* seriously affect its quantum motion. However, to arrive at such a result involves an analysis quite different from the usual Caldeira-Leggett method—this is because there is no coupling between the wall, and either its magnetic or phononic environments, which is linear in the environmental variables.

We start with a lattice Lagrangian

$$L = s\sum_j \frac{d\phi_j}{dt}\cos\theta_j(t) + L_{s\phi} - H_M - H_{EM} - H_{\rm imp} - H_{\rm def}\,, \quad (1)$$

$$H_M = \tfrac{1}{2}\sum_{\langle ij\rangle} J_{ij}\mathbf{s}_i\cdot\mathbf{s}_j + \tfrac{1}{2}K\sum_j s_j^x s_j^x + H_{\rm dip}\,, \quad (2)$$

where the lattice spins $\mathbf{s}_j = s\hat{\mathbf{n}}_j$, and $\hat{\mathbf{n}}_j = (\theta_j, \phi_j)$; $H_{\rm dip}$ is the usual dipolar interaction,[7] while $L_{s\phi}$ includes phonons and magnetoacoustic couplings, and H_{EM} the remaining magnetostatic terms, as well as the interaction H_{sA} with photons:

$$H_{sA} = \int d^3r \sum_{\mathbf{k}\lambda}\left(\frac{\hbar\omega_{\mathbf{k}\lambda}}{2\epsilon(\omega)}\right)^{1/2}[\hat{\mathbf{e}}_{\mathbf{k}\lambda}\cdot\mathbf{M}(\mathbf{r},t)] \times[\alpha_{\mathbf{k}\lambda} - \alpha^{\dagger}_{-\mathbf{k}\lambda}]\,. \quad (3)$$

Here $\alpha_{\mathbf{k}\lambda}$, the photon polarization $\hat{\mathbf{e}}_{\mathbf{k}\lambda}$, frequency $\omega_{\mathbf{k}\lambda}$, and permittivity $\epsilon(\omega)$ are defined for photons *in the magnet.*

We have for simplicity assumed an easy $\hat{\mathbf{x}}$-axis magnet, and for purposes of calculation we will assume a planar Bloch wall (whose experimental realization will be discussed below). Then the magnetization density $\mathbf{M}(\mathbf{r},t)$ will take the semiclassical profile $\mathbf{M}_0(\mathbf{r},t)$ for a wall moving along the $\hat{\mathbf{z}}$ axis, perpendicular to the wall plane, with

$$\begin{aligned} M_0^x(\mathbf{r},t) &= |M|\tanh[\tilde{z}(t)/\lambda]\,, \\ M_0^y(\mathbf{r},t) &= |M|(1-\dot{Q}^2/8c^2)\operatorname{sech}[\tilde{z}(t)/\lambda]\,, \\ M_0^z(\mathbf{r},t) &= |M|(\dot{Q}/2c)\operatorname{sech}[\tilde{z}(t)/\lambda]\,, \end{aligned} \tag{4}$$

where $|M| = 2\hbar\gamma s/l_0^3$, for lattice constant l_0, and $Q(t)$ is the wall center z coordinate [and $\tilde{z}(t) = z - Q(t)$]; we assume $\dot{Q} \ll c \equiv 4\pi\gamma\lambda$, where $\lambda = (J/K)^{1/2} \gg l_0$ is the wall thickness, thereby allowing a continuum treatment. $\gamma = \mu_B/\hbar$ is the gyromagnetic ratio. The impurity and defect terms H_{imp} and H_{def} are discussed below.

If we now look at quantum fluctuations of the magnon and photon fields around $M_0(\mathbf{r},t)$, we obtain a new effective Lagrangian coupling the wall, with kinetic energy $\frac{1}{2}M_W\dot{Q}^2$ (the Döring mass $M_W = 2A_W/\mu_0\gamma^2\lambda$ for a wall of area A_W), to new canonically transformed magnon and photon baths. Thus, to $O(1/s^3)$, the magnon part has the Lagrangian

$$L_{\text{mag}} = \int d^3r \left\{ \lambda^2\Delta_0(\nabla b)^2 - \Delta_0\left[1-\operatorname{sech}^2\left(\frac{\tilde{z}}{\lambda}\right)\right] b^\dagger b - i\hbar\gamma K\left(\frac{32s}{l_0^3}\right)^{1/2} \operatorname{sech}\left(\frac{\tilde{z}}{\lambda}\right)\left[\tanh\left(\frac{\tilde{z}}{\lambda}\right) - \lambda\nabla\right](b^\dagger - b)b^\dagger b \right\}, \tag{5}$$

where the new magnons [created by $b^\dagger(\mathbf{r},t)$] describe magnetization fluctuations about $M_0(\mathbf{r},t)$, and have both bulk branches (with energy gap $\Delta_0 = 4\hbar^2\gamma^2 sK/l_0^3$) and wall or "Winter magnon" branches.[8] We may derive a similar expression for the magnetoacoustic coupling to the wall.[9]

Now it is of paramount importance that, in the absence of defects, neither the magnon nor the phonon bath couple to the wall coordinate Q (their energy is independent of Q) to any order in $1/s$. Hence at $T=0$ (and effectively for $kT \ll \Delta_0$, see below), and $\dot{Q} \ll c$, *they have no dissipative effect on wall motion.*

We note also that there is no coupling of any kind to the wall below second order in the bath coordinates, as mentioned previously—this therefore takes us outside the scope of the Caldeira-Leggett model Lagrangian, in analyzing wall-phonon or wall-magnon interactions.

Moreover, we can rule out any appreciable effects from photons or impurities by a consideration of energy scales. Notice first that for the kind of soft magnetic insulator we consider here, $\lambda \sim 10^3$ Å, and the Walker velocity $c \sim 10^3$ m s^{-1}. Thus the frequency scale ω_0 associated with its free motion is much less than $c/\lambda \sim 10^9$ Hz; and we shall see that typical instanton frequencies (inverse bounce times) $\Omega_0 \sim 10^8$ Hz in tunneling.

Now from (3) we may easily derive a Caldeira-Leggett spectral function[1] for wall-photon interactions of the form

$$J_A(\omega) = \frac{|M|\lambda A_W}{6\pi^2\hbar\epsilon(\omega)}\left(\frac{\omega}{c}\right)^3, \tag{6}$$

which for $\omega \simeq \omega_0$ or Ω_0 gives an utterly negligible effect on tunneling (and even less on free motion) even for quite gigantic walls. The argument is more delicate for impurities or resonant scatterers of characteristic energy scale $\hbar\Omega_I$; calculations similar to those in Ref. 10 show their dissipative effect to be $\sim(\omega/\Omega_I)^2$ for $\omega < \Omega_I$, so it must be ensured that any magnetic impurities with $\Omega_I \sim \Omega_0$ or ω_0 are absent from the sample.[11]

Thus, at low T, the wall moves as a quantum object. Indeed, its real-time quantum diffusive motion can be derived by applying time-dependent perturbation theory to the equation of motion for the wall density matrix, using (5) and the magnetoacoustic coupling. The explicit calculation at finite T is tedious but straightforward,[9] and gives an Ohmic friction coefficient $\eta(T)$ for the motion (still assuming $\dot{Q} \ll c$), where[6]

$$\eta(T) = \frac{A_W}{16\pi^2\gamma^2\lambda^3}\left(\frac{kT}{\Delta_0^2}\right)e^{-\Delta_0/kT} + \beta_\phi A_W\left(\frac{kT}{\hbar\Theta_D}\right)^3, \tag{7}$$

where β_ϕ is a complicated function of magnetoelastic coefficients,[10] and $\beta_\phi A_W$ is very small for realistic walls.

Some features of (7) deserve comment. The magnon scattering term is just what one would expect from second-order perturbation theory for inelastic scattering of magnons having a gap Δ_0 (coming from the uniaxial anisotropy). The third-order term in (5) gives a correction to (7) of higher order ($\sim T^3 e^{-\Delta_0/kT}$). The phonon term in (7), of order $(T/\Theta_D)^3$, is quite different; it comes from the phase shift experienced by the phonons passing through the wall, and was first considered (in one dimension) by Wada and Schreiffer.[6] Real inelastic phonon transitions are impossible for a wall moving at less than the sound velocity.

Now, since $\Delta_0 \ll \hbar\Theta_D$, Eq. (7) implies quantum motion when $kT < \Delta_0$. This motion will be difficult to see for two reasons. First, a large wall is quite heavy (a wall containing $N \sim 10^{10}$ spins, with $\lambda \sim 10^3$ Å and $A_W \sim 10^{-8}$ m^2, has $M_W \sim 4\times10^{-17}$ kg), so wave-packet spreading will be slow. Second, the walls tend to be trapped by defects in the magnet. This, however, is a blessing in disguise, for it allows a test of the much more spectacular MQT of a wall off a pinning defect in an applied field.

The easiest way to do this experimentally is to use a small applied field H to push the wall off the defect. Such an idea is of course not new (although it has never been previously advocated for *macroscopic* walls). The question now becomes: What would the tunneling rate be, and under what conditions could it be observed?

To treat this problem we use the spherical defect model of Sparks, Loudon, and Kittel;[12] for defect radius $R_0 \ll \lambda$, the actual shape of the defect is irrelevant to the form of the pinning potential, itself given by $H_{\rm def} = -\frac{1}{2}\mu_0 \int d^3r\, \mathbf{M}(\mathbf{r})\cdot \mathbf{H}_{\rm dm}(\mathbf{r})$, where $\mathbf{H}_{\rm dm}(\mathbf{r})$ is the demagnetization field around the impurity (which we assume to be at the origin).[13]

Calculation of $H_{\rm def}$ produces a number of terms, of which the leading term [to $O(R_0/\lambda)$] is the *static* potential

$$H_{\rm def} \sim -U_0 \,\mathrm{sech}^2(Q/\lambda)\,, \tag{8}$$

where $U_0 = 8\pi\mu_0 |M|^2 R_0^3/3$. If we now apply a field $H_0 = (1-\epsilon)H_c$ along $\mathbf{x}$ (where H_c is the coercive field for the defect), then we will obtain tunneling. The calculation of the tunneling rate $\Gamma = Ae^{-B}$ at $T=0$ is a standard problem in instanton physics, and we find [using a total potential given by adding to (8) the field-induced potential $-\frac{1}{2}\mu_0 \mathbf{H}\cdot d\mathbf{M} = -(\mu_0 \hbar\gamma s/l_0^3) H A_W Q$]

$$A = \left[\frac{24}{\pi} (\mu_0 g s)^2 \gamma^3 H_c \left(\frac{H_c}{\hbar\gamma l_0^3} \right)^{1/2} N \right]^{1/2} \epsilon^{7/8}\,, \tag{9}$$

$$B = \left(\frac{4gs}{15} \right)^{1/2} \left(\frac{l_0^3 H_c}{\hbar\gamma} \right)^{1/2} N \epsilon^{5/4}\,, \tag{10}$$

with a crossover to thermally activated hopping around a temperature

$$T_c \simeq \left(\tfrac{96}{25}\right)^{1/2} (g\hbar\gamma\mu_0 s H_c N)\epsilon^{3/2}/B\,, \tag{11}$$

where $N = \lambda A_W / l_0^3$ is the number of spins in the wall. Again putting $N = 10^{10}$, and $H_c = 1$ G, with $\epsilon = 10^{-3}$, one finds a tunneling rate $\Gamma \sim 10^3$ Hz below $T_c \sim 15$ mK. A wall of this size involves more spins than there are Cooper pairs involved in MQT in SQUIDs, and so this is a genuine macroscopic tunneling phenomenon. Moreover, we have eliminated as unimportant all sources of dissipation in the problem (apart from one-site defect dissipation, see below), so the results (9)–(11) should be reasonably accurate.

Thus it is suggested that a search for domain-wall MQT be carried out at low T, in very pure ferromagnetic insulators. At this point it is useful to discuss the connection between the above theory and a practical experiment. A sample with negligible magnetostatic effects would be desirable, such as the Kittel-Galt "picture-frame" geometry,[14] or perhaps a straight wall slicing the length of a whisker and moving across it (notice that in both of these configurations the wall area A_W is naturally limited by the sample geometry). Rather sensitive magnetometry would be required, using either SQUIDs or "Aharonov-Bohm" electron-diffraction techniques,[15] or some other relatively "noninvasive" probe of domain-wall position. Finally, one would like to have a material which can be made in very pure crystalline form, and almost free of defects, for which yttrium iron garnet, or one of its relatives, is the obvious candidate.

The implications of a discovery of such domain-wall MQT would be quite interesting. The essential reason why it can work here is that the relevant collective coordinate $Q(t)$ represents a "quantum soliton" center of mass, and it is in the nature of solitons that they do not dissipate energy into other modes of their field. Thus one might expect to find similar phenomena in other systems where large solitons exist. Notice that this is more important than ODLRO or superconducting phase coherence.

If the MQT was *not* seen, one would naturally suspect some hidden source of dissipation, not considered here. In fact, the only other dissipative coupling in our original Lagrangian comes from a *dynamic* term in $H_{\rm def}$, which couples $Q(t)$ to the wall magnons. This term is linear in $Q(t)$, and can be treated using the usual Caldeira-Leggett techniques—its effects are somewhat analogous to those of ripplons on ion scattering off ^{4}He II liquid. We do not give the details here because they are very complicated, and, moreover, numerical estimates of their effects indicate them to be negligible. They will be described in detail elsewhere.[9]

I conclude by noting that the theory here can be equally applied to *mesoscopic* walls. Indeed, there is some rather striking evidence for such behavior in disordered hard magnets, involving $N \sim 400$ spins.[16] In these systems, H_c and ϵ are much larger than the values considered above for MQT, and so the corresponding crossover temperatures are higher ($T_c \sim 1$–2 K). Nevertheless, it is clear from the analysis here that if one were able to go to lower temperatures, the tunneling of larger domain walls would become observable (the experiments of Ref. 16 never went below 1.4 K). The analysis is considerably complicated by the presence of a very large number of domain walls in these disordered samples, so that the relation between theory and experiment is rather indirect; I intend to return to this problem elsewhere.[17]

For this reason it is clearly desirable that further experimental work be done on *pure insulators,* since the comparison between theory and experiment will be much easier. In any case, the results of this paper show that a rigorous treatment of both mesoscopic and macroscopic domain-wall motion can be given, including apparently all relevant dissipative effects, and that quantum coherence phenomena associated with such motion should be observable up to a macroscopic level.

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[2]See, e.g., J. M. Martinis *et al.*, Phys. Rev. B **35**, 4682 (1987), and references therein.

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[5]A. Garg and G.-H. Kim. Phys. Rev. Lett. **63**, 2512 (1989).

[6]J. F. Janak, Phys. Rev. **134**, A411 (1964), derived a similar result for the first term in Eq. (7). His expansion is rather different from that used here—a detailed comparison will be given in Ref. 9. For readers familiar with the usual theory (Ref. 1) of MQT, and who are surprised that we recover Ohmic dissipation in a theory lacking the usual Caldeira-Leggett dissipative coupling with spectral function $J(\omega)=\eta\omega$, we note that (a) Ref. 1 showed that this $J(\omega)$ gave Ohmic dissipation, but they did *not* show that all Ohmic dissipation comes from their kind of model (indeed, they gave a counterexample of sliding friction); and (b) the quantum dynamics of solitons is not, in general, described by a Caldeira-Leggett Lagrangian. The soliton-phonon scattering mechanism leading to the $(T/\Theta_D)^3$ term in (7) was first discussed by Y. Wada and J. R. Schreiffer, Phys. Rev. B **18**, 3897 (1978), for one-dimensional systems. They found a friction $\sim T^2$; the extra power of T comes from summing over phonon directions. An equivalent mechanism exists for wall-magnon scattering, but gives a very small contribution.

[7]See, e.g., A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, *Spin Waves* (North-Holland, Amsterdam, 1968).

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[9]The rather lengthy calculations which lie behind some of the results in this paper will be given in more detail in a longer paper.

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[11]Actually, in a pure magnet like yttrium iron garnett this is hardly a problem. I have not considered coupling to nuclear spins in this paper.

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[13]This is because of the dipolar nature of $H_{\rm dm}(\mathbf{r})$ for $|\mathbf{r}|\gg R_0$. Somewhat surprisingly, Eq. (8) does not seem to have been derived previously from microscopic theory, although it follows in a straightforward (although tedious) way from this coupling.

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[17]Note that the experiments of Ref. 14 were done on conducting systems, and the heating effects they saw show how important is the dissipative coupling to conduction electrons.