Experimental Study of Color Degree of Freedom of Gluons in e^+e^- Annihilation at $\sqrt{s}$ around 60 GeV

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We have studied four-jet angular distributions using data collected with the VENUS detector at the KEK e^+e^- collider TRISTAN at c.m. energies between 54 and 64 GeV. The observed angular distributions are consistent with QCD, but are inconsistent with the Abelian gluon model at the 5% significance level. We have further obtained 95%-confidence experimental bounds on the fundamental parameters of the SU(N_c) local color symmetry.

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The theory of strong interactions between quarks is by now well established in the form of quantum chromodynamics (QCD). Quarks carry a color degree of freedom, which is constrained to be three; the strong interactions between these colored quarks are mediated by an octet of colored gauge particles called gluons.[1] Direct evidence for the existence of the gluon came about from radiation of a hard jet by either the produced quark or antiquark in e^+e^- annihilation; measurements of the topologies of three-jet events demonstrated that the gluon must be a vector boson.[2]

A key property of QCD is the nonzero color degree of freedom of gluons, which results in a self-interaction of gluons, the so-called three-gluon ($3G$) coupling. The fact that QCD calculations including this non-Abelian nature describe the data well[3] lends indirect support to QCD. It is, however, possible to construct an Abelian form of strong-interaction theory in which the gluon carries a zero color degree of freedom and self-interaction is forbidden. Thus, the nonzero color degree of freedom of gluons must be confirmed experimentally.

The reaction $e^+e^- \to$ four partons is a theoretically and experimentally clean example in which the $3G$ coupling is directly manifested in the lowest-order diagram. The Feynman diagrams contributing to this process are shown in Fig. 1 where (a) contains QED-like diagrams,

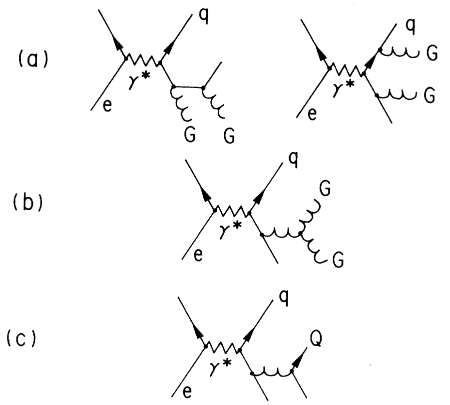

FIG. 1. The Feynman diagrams contributing to the four-parton final states: (a) gluon emissions from quark lines, (b) gluon-pair creation by emitted gluons, the 3G coupling, and (c) quark-pair creation by emitted gluons.

(b) shows gluon-pair creation by gluons (the 3G coupling), and (c) illustrates quark-pair creation by gluons. A number of authors have proposed sensitive measurements of the 3G coupling,[4-7] which involve the angles between the primary quark pairs ($q\bar{q}$) and the secondary gluon or quark pairs (GG or $Q\bar{Q}$). Such schemes are based on the fact that in a $q\bar{q}G$ three-parton intermediate state the gluon is polarized; if it splits into a GG pair, the pair tends to lie in the $q\bar{q}$ plane, while if it splits into a $Q\bar{Q}$ pair, the pair tends to be perpendicular to the $q\bar{q}$ plane, due to the spin of the quarks and gluons.

In this paper, we present data from measurements of the angular distributions of four-jet events, definitions and predictions of QCD and the Abelian gluon model, as well as a comparison with the predictions. We further present experimental bounds on the color degree of freedom of gluons without assuming a specific model.

Experimental data.—The data used for this analysis were collected with the VENUS detector[8] at the KEK e^+e^- collider TRISTAN. The accumulated luminosity was 42.4 pb^{-1} between center-of-mass energies ($\sqrt{s}$) of 54 and 64 GeV with an average energy of 58.7 GeV (we generated Monte Carlo events at $\sqrt{s}=58$ GeV).

Four-jet events produced via a single photon or Z^0 annihilation were selected by requiring the following selection criteria.

(1) The total energy in electromagnetic calorimeters within $|\cos\theta| < 0.89$ is greater than 5 GeV. The calorimeters cover $|\cos\theta| < 0.99$ (the angle θ is measured from the beam axis, the z direction).

(2) The number of charged tracks is 5 or more. Charged tracks are only accepted if they originate within 2 cm from the collision point in the x-y plane and 20 cm in the z direction; further, p_t must be more than 0.2 GeV/c and within $|\cos\theta| < 0.85$.

(3) The sum of the momenta of the charged tracks and the energies of the energy clusters ($E_{\rm vis}$) is greater than $0.5\sqrt{s}$.

(4) The sum of the momenta of the charged tracks and the energies of the energy clusters along the beam direction divided by $E_{\rm vis}$ is within 0.4.

Up to this point the selection criteria were used to select multihadronic events; 5477 events were accepted.

(5) The number of jets is four. Tracks and clusters are clustered into a jet according to a jet-clustering algorithm[9] if $d_{ij}=2p_ip_j\sin(\theta_{ij}/2)/(p_i+p_j)$ is less than 2.5 GeV ($=d_{\rm join}$). The energies of the clusters are required to be greater than 0.3 GeV.

(6) The direction of all the jets is within $|\cos\theta_j| < 0.95$ and the reconstructed energy of the jets is greater than 3 GeV. Since the directions of the jets are rather well determined, the jet energies were reconstructed from the directions using energy-momentum conservation and assuming massless partons. The angular resolutions of the jets from the original partons were estimated to be $\sigma=4°$ and the jet-energy resolutions to be improved from 20% to 10% using Monte Carlo simulations.

(7) When the jets are ordered according to their energies as $E_1 > E_2 > E_3 > E_4$, the angles between jet 1 and jet 2 (θ_{12}), and between jet 3 and jet 4 (θ_{34}), are less than 170°. This condition is set so as to avoid ambiguity in defining the planes or angles while considering the jet-angular resolution mentioned above.

After the selection, 345 four-jet events were accepted for analysis.

TABLE I. Transitions contributing to the four-parton final states along with the group weight factors as a function of C_F, N_c, and T_R, and fractions of each transition in QCD and in the Abelian gluon model at $y_{\rm min}=0.02$.

Transitions	Group weight factors	Fractions (%) QCD	Abelian gluon
$q\bar{q}GG$	C_F^2, $C_F(C_F-N_c/2)$	28.4	65.5
$q\bar{q}(G)GG$	C_FN_c	66.2	0
$q\bar{q}Q\bar{Q}$	C_FT_R, $C_F(C_F-N_c/2)$	5.4	34.5

QCD and the Abelian gluon model.—A complete perturbative calculation of the order α_s^2 for the production of the four-parton final state was presented by Ellis, Ross, and Terrano (ERT).[10] The differential cross sections of the four-parton final state comprise three transitions: a transition containing the diagrams and their interference terms in Fig. 1(a) (referred to as $q\bar{q}GG$); a transition containing the diagram in Fig. 1(b), the 3G coupling, and interference terms with the diagrams in Fig. 1(a) [$q\bar{q}(G)GG$]; and a transition containing the diagram in Fig. 1(c) ($q\bar{q}Q\bar{Q}$). The gauge structure is defined by the group constants of the SU(N_c) local color symmetry, (C_F, N_c, T_R), where N_c is the color degree of freedom of gluons. Each transition has relevant angular distributions with specific group weight factors as a function of C_F, N_c, and T_R, which are tabulated in Table I. The non-Abelian gluon model, QCD, is defined with

$C_F = \frac{4}{3}$, $N_c = 3$, and $T_R = \frac{1}{2} n_f$, where n_f is the number of quark flavors allowed (we take 5 at our center-of-mass energies), while the Abelian gluon model has $C_F = 1$, $N_c = 0$, and $T_R = 3n_f$.[1]

We obtained the fractions for the $q\bar{q}GG$, $q\bar{q}(G)GG$, and $q\bar{q}Q\bar{Q}$ transitions in QCD and in the Abelian gluon model by integrating the differential cross sections over the phase space for partons with two- and three-parton scaled invariant mass squared, $y_{ij} = m_{ij}^2/s$, $y_{ijk} = m_{ijk}^2/s$, being greater than $y_{\min}$. At $y_{\min} = 0.02$, the $q\bar{q}Q\bar{Q}$ fractions are 5.4% and 34.5% for QCD and the Abelian gluon model, respectively. Other fractions are summarized in Table I.

Monte Carlo event generation.—We generated four-partonic configurations according to the ERT matrix element in a LUND Monte Carlo simulation program[11] (LUND72) with $y_{\min} = 0.02$ and with initial-state radiations. We fragmented quarks and gluons with the string-fragmentation scheme in LUND72. The generated events were then passed through a VENUS detector simulation program and analyzed using the same selection criteria as for the experimental data.

To investigate the validity of the fragmentation scheme, we compared the experimental distributions of jet energies, transverse momenta, pseudorapidity, and multiplicity of charged particles in a jet with those of four-parton final states together with a background of two- and three-parton final states. The fragmentation is parametrized using the symmetric LUND fragmentation function, $f(z) = [(1-z)^a/z]\exp(-bm_t^2/z)$, and the Gaussian width s of the transverse-momentum distributions for primary hadrons. We found that a set of values ($a = 1.0$, $b = 0.7$, $s = 0.40$) described the data well and the default values ($a = 0.5$, $b = 0.9$, $s = 0.35$) worse, even though their difference was small. We used the former values and took the latter into consideration regarding systematic errors.

To estimate the background we generated the two- and three-parton final states (referred to as $q\bar{q}G$) as well as the four-parton final states separately with $y_{\min} = 0.02$. Their ratio was fixed at a value describing the experimental two-, three-, and four-jet ratios up to selection (4) (R_2:R_3:R_4:R_5 = 0.446:0.418:0.118:0.018). The four-jet events which we finally accepted were estimated to contain $(10 \pm 5)\%$ $q\bar{q}G$ background.[12]

When the jets are ordered according to their energies as $E_1 > E_2 > E_3 > E_4$, the probabilities that jet 1, 2, 3, or 4 is the gluon were estimated to be 0.16, 0.38, 0.64, and 0.81, respectively, in the $q\bar{q}(G)GG$ transition. We assigned jets 1 and 2 as being primary quarks, $q\bar{q}$, and jets 3 and 4 as being secondary gluons, GG, or quarks, $Q\bar{Q}$. Since the jet energy plays an important role, an improvement of the jet-energy resolution in selection (6) was very effective.

Angular distributions.—To observe that the GG pair of $q\bar{q}(G)GG$ lies in the $q\bar{q}$ plane and that the $Q\bar{Q}$ pair of $q\bar{q}Q\bar{Q}$ is perpendicular to the $q\bar{q}$ plane, Korner, Schierholz, and Willrodt[4] (KSW) proposed the angle $\phi_{\rm KSW}$ between the plane formed by the momentum vectors, qG or qQ, and the plane formed by $\bar{q}G$ or $\bar{q}\bar{Q}$; Nachtmann and Reiter[5] (NR) proposed the angle $\cos\theta^*_{\rm NR}$ between the momentum-vectorial difference of $q\bar{q}$ and of GG or $Q\bar{Q}$; and Bengtsson and Zerwas[6] (BZ) proposed the angle χ between the plane formed by $q\bar{q}$ and GG or $Q\bar{Q}$.

Using the above-mentioned jet assignment we simulated the angular distributions of $q\bar{q}GG$, $q\bar{q}(G)GG$, $q\bar{q}Q\bar{Q}$, and $q\bar{q}G$ in QCD. We found little difference in $\phi_{\rm KSW}$ and did not consider the angle further. The χ and $\cos\theta^*_{\rm NR}$ angular distributions are shown in Figs. 2(a) and 2(b) for transitions $q\bar{q}GG$, $q\bar{q}(G)GG$, $q\bar{q}Q\bar{Q}$, and $q\bar{q}G$. As expected, $q\bar{q}(G)GG$ prefers parallel and $q\bar{q}Q\bar{Q}$ perpendicular configurations. We found that only $q\bar{q}Q\bar{Q}$ has different distributions and $q\bar{q}(G)GG$ and $q\bar{q}GG$ have quite similar ones. This fact implies that discrimination of $q\bar{q}Q\bar{Q}$ is insufficient to separate the $q\bar{q}(G)GG$ transitions.

The experimental χ and $\cos\theta^*_{\rm NR}$ angular distributions are plotted in Figs. 2(c) and 2(d), together with the predictions of QCD and the Abelian gluon model, with 10% $q\bar{q}G$ background.

The systematic errors involved in each bin of the distributions came from uncertainties in the Monte Carlo statistics of 10^4 four-jet events (1.6%), the fragmentation parameters (3.0%), the momentum and energy scale errors of $\pm 10\%$ (2.0%), the $q\bar{q}G$ fraction of $(10 \pm 5)\%$ (1.1%), the $q\bar{q}Q\bar{Q}$ fraction of $(34.5 \pm 1.5)\%$ (2.1%), and

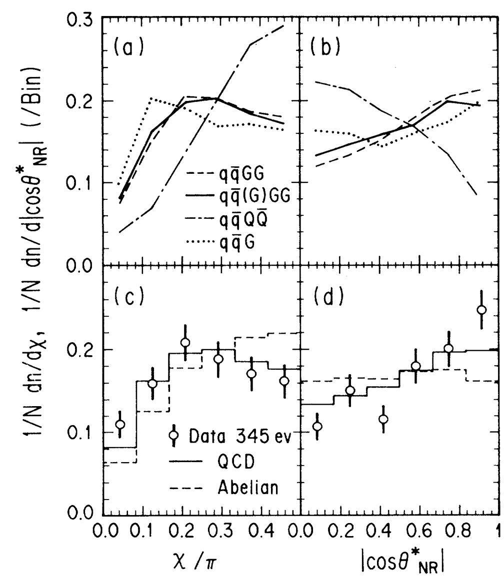

FIG. 2. The χ and $\cos\theta^*_{\rm NR}$ angular distributions: (a),(b) distributions of transitions $q\bar{q}GG$, $q\bar{q}(G)GG$, $q\bar{q}Q\bar{Q}$, and $q\bar{q}G$; (c),(d) distributions of the experimental data with statistical errors only, QCD, and the Abelian gluon model.

a variation of $y_{\min}$ of 0.02 ± 0.003 (3.1%). The last two terms were introduced due to the fact that $d_{\text{join}} = 2.5$ GeV corresponded to a $y_{\min}$ of about 0.02. Combining these errors quadratically, the systematic error per bin was estimated to be 6%.[13]

Combining the statistical and the systematic errors quadratically, the predictions of QCD and the Abelian gluon model were tested against the experimental data. χ^2 for QCD was 3.90, while for the Abelian gluon model it was 20.31, with the number of degrees of freedom (N_{DF}) $=5$ in the χ distribution; χ^2 for QCD was 9.76, while for the Abelian gluon model it was 27.09, with $N_{\text{DF}} = 5$ in the $\cos\theta^*_{\text{NR}}$ distribution. The observed distributions are consistent with QCD but are inconsistent with the Abelian gluon model at the 5% significance level ($\chi^2 = 11.07$). The AMY Collaboration presented the first experimental analysis concerning the angular distributions and a comparison with QCD and the "Abelian model." Their analysis was based on the $q\bar{q}Q\bar{Q}$ fraction of 51% in the Abelian model,[14] which fraction is inconsistent with the present analysis.

Within the ERT framework, the group constants (C_F, N_c, T_R) of the SU(N_c) local color symmetry are free parameters and can be determined experimentally. Angular distributions of the four-parton final states are functions of the ratios N_c/C_F and T_R/C_F. Thus, using the χ and $\cos\theta^*_{\text{NR}}$ distributions, we obtained 95%-confidence experimental bounds on the parameters in the N_c/C_F-T_R/C_F plane, as shown in Fig. 3. Also shown in this figure are the QCD ($N_c/C_F = 2.25$, $T_R/C_F = 1.875$) and the Abelian gluon model ($N_c/C_F = 0$, $T_R/C_F = 15$) points, and contours of the $q\bar{q}(G)GG$ and $q\bar{q}Q\bar{Q}$ fractions. The bounds were found to be rather insensitive to N_c, because $q\bar{q}GG$ and $q\bar{q}(G)GG$ give essentially the same χ and $\cos\theta^*_{\text{NR}}$ distributions. We rejected any

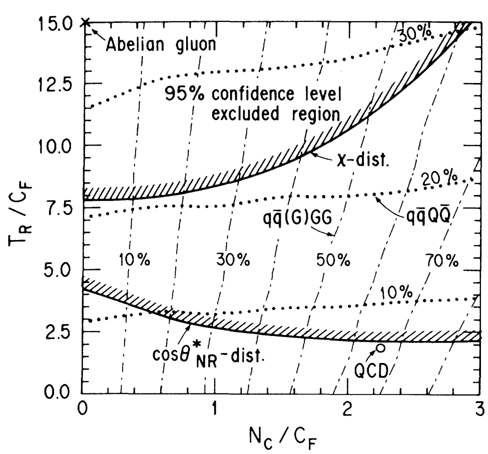

FIG. 3. Experimental bounds on the group constants N_c/C_F and T_R/C_F. The shaded area is the 95%-confidence-level excluded region with the χ or $\cos\theta^*_{\text{NR}}$ distribution. Shown together are the QCD and the Abelian-gluon-model points and contours of the $q\bar{q}(G)GG$ and $q\bar{q}Q\bar{Q}$ fractions.

$N_c = 0$ model with the $q\bar{q}Q\bar{Q}$ fraction being greater than 21.4% with the χ distribution.

In conclusion, we have measured the angular distributions in the four-jet events collected with the VENUS detector at TRISTAN at $\sqrt{s}$ between 54 and 64 GeV. QCD and the Abelian gluon model have been defined explicitly with the group constants (C_F, N_c, T_R) of the SU(N_c) local color symmetry, where N_c is the color degree of freedom of gluons. The $q\bar{q}Q\bar{Q}$ fractions have been calculated to be 5.4% and 34.5% at $y_{\min} = 0.02$ for QCD and the Abelian gluon model, respectively. The observed χ and $\cos\theta^*_{\text{NR}}$ angular distributions are consistent with QCD but are inconsistent with the Abelian gluon model at the 5% significance level. Taking the group constants as free parameters, we have obtained the 95%-confidence experimental bounds on the fundamental parameters in the N_c/C_F-T_R/C_F plane. The bounds are found to be relatively insensitive to N_c and any $N_c = 0$ model with the $q\bar{q}Q\bar{Q}$ fraction being greater than 21.4% is rejected. Because $q\bar{q}GG$ cannot be separated from $q\bar{q}(G)GG$ with the proposed angles, we do not prove the existence of $3G$ coupling ($N_c \neq 0$). Rather we have shown that the $q\bar{q}Q\bar{Q}$ fraction is substantially less than the 34.5% predicted by the Abelian gluon model.

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[12]The amount was also confirmed from the fitting to the angular distributions, χ and θ_{34}, and the result was (12 ± 16)%.

[13]The present analysis is based on the $O(\alpha_s^2)$ matrix element.

The $O(\alpha_s^3)$ contributions are yet to be calculated. Though the leading-log approximation (parton shower) may approximate the higher-order corrections, its reliability for generating angular distributions is yet to be established. We are unable to estimate the uncertainty arising from higher-order corrections; however, a part of the uncertainty is expected to be included in the uncertainty of the fragmentation of partons evaluated in the analysis.

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