# Scaling Effective Lagrangians in a Dense Medium

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By using effective chiral Lagrangians with a suitable incorporation of the scaling property of QCD, we establish the approximate *in-medium* scaling law, $m_\sigma^*/m_\sigma \approx m_N^*/m_N \approx m_\rho^*/m_\rho \approx m_\omega^*/m_\omega \approx f_\pi^*/f_\pi$. This has a highly nontrivial implication for nuclear processes at and above nuclear-matter density. Some concrete cases are cited in this paper.

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One of the most exciting new directions in nuclear physics is to study how nuclear phenomena change as the environment changes. Thus relativistic heavy-ion experiments are to address the state of nuclear matter at high temperature and/or density and high-energy high-duty-cycle electron machines are to probe the properties of individual hadrons in close encounter with other strongly interacting matter. These are the processes that reflect the change of the strong-interaction vacuum as density and temperature are "dialed." Given a fundamental theory of strong interactions, i.e., QCD, one should, in principle, be able to calculate all the observables unambiguously as the environment is modified. It is possible that this feat will be eventually accomplished but it is highly unlikely that it will come soon enough to make contact with either on-going experiments or experiments to come in the near future. In this Letter, we propose, focusing on the density effect, to approach this problem from an effective-Lagrangian point of view, starting from low-energy effective theories based on spontaneously broken chiral symmetry that have been phenomenologically successful in describing low-energy and low-density hadronic interactions.

Briefly our strategy is follows. We start with a known structure of effective Lagrangians at low energy and zero density (i.e., free space), dictated by symmetries and other constraints of QCD (e.g., chiral symmetry with its current algebra and anomaly, trace anomaly, etc.). We are interested in how this theory evolves as density (or temperature) is increased. Embedding a hadron in dense matter is equivalent to changing the vacuum, thereby modifying quark and gluon condensates in QCD variables. Our first key assumption is that as the condensates change, the symmetries of the Lagrangian remain more or less *intact*, while the relevant scale is changed in a prescribed way that we will explain below. This means that we will have, as density increases, the same Lagrangian but with the masses and coupling constants of the theory *modified according to the symmetry constraints of QCD.* Some of the steps we take may appear to be *ad hoc* and drastic but we will argue that there is strong evidence from experiments that our scheme is *supported* by nature.

To illustrate our point, we start with the original Skyrme Lagrangian[1] consisting of the current-algebra term characterized by a dimensional constant $f_\pi$ and the quartic stabilizing term characterized by a dimensionless constant $\epsilon$. (More precise definitions of these quantities will be given later.) It is convenient to work with physical quantities by taking $f_\pi$ to be the pion decay constant (experimentally $\approx 93$ MeV) and the axial-vector coupling constant $g_A$ in place of $\epsilon^2$.[2,3] If we accept that baryons arise as solitons from the Skyrme Lagrangian, which seems to be fairly well established by now, we can use simple scaling arguments[3,4] to show that, modulo overall constants, the size and mass of the baryon go as

$$\langle r^2\rangle \sim g_A/f_\pi^2, \quad M \sim g_A^{1/2} f_\pi . \tag{1}$$

We now ask what happens to these quantities when the Skyrmion is embedded in a dense medium. As in Refs. 3 and 4, we will argue that as long as there is no phase transition that changes symmetries of the "vacuum" (or more precisely the ground state), the leading modification in the theory is in the basic constants of the Lagrangian, i.e.,

$$\langle r^2(\rho)\rangle^* \sim g_A^*(\rho)/f_\pi^{*2}(\rho), \quad M^*(\rho) \sim [g_A^*(\rho)]^{1/2} f_\pi^*(\rho) . \tag{2}$$

Here we indicate the density-dependent quantities by asterisks. (From now on we will omit the density dependence in quantities with asterisks, unless explicitly required.) We identify (2) as the quasiparticle size and mass, respectively, for single baryons in the medium. Further (residual) interactions, say, between quasiparticles (or quasi-Skyrmions), can be introduced in a way analogous to the zero-density case using a given effective Lagrangian as specified more precisely below.

Now what about mesons? Chiral effective theories contain, in the large-$N_c$ limit, Goldstone bosons, ordinary mesons, and a $U_A(1)$ boson called $\eta'$, in addition to the baryons. The $\eta'$ decouples from the rest of the world in the large-$N_c$ limit and does not concern us for our purpose. In considering the properties of mesons in a dense medium, it is more convenient to introduce explicit degrees of freedom associated with other mesons than the Goldstone bosons. The most important of them all are the strong vector mesons (e.g., $\rho,\omega,\ldots$) and the glueballs. (In considering baryons, it is legitimate to "integrate out" these other meson degrees of freedom, as, e.g., in the Skyrme model.) A convenient framework to incorporate the strong vector mesons is through the hidden-gauge-symmetry strategy advocated by Bando, Kugo, and Yamawaki.[5] The glueballs that are needed for our purpose will be introduced through QCD anomalies.

Given a chiral Lagrangian which contains pseudoscalar (Goldstone) and vector (hidden-gauge) bosons, what is the effect of changing vacuum structure by density? We will argue that similarly to the baryon case, the leading effect is in the effective masses of the mesons and possibly their coupling constants.

The principal in-medium scaling law[6] conjectured in Ref. 7 which we would like to justify is

$$m_\sigma^*/m_\sigma \approx m_N^*/m_N \approx m_\rho^*/m_\rho \approx m_\omega^*/m_\omega\,, \tag{3}$$

where the masses without asterisks stand for free-space values. To do this, we follow the approach of Campbell, Ellis, and Olive[8] and introduce scale invariance into the effective Lagrangian. Their approach can be summarized as follows. The standard chiral Lagrangian lacks the trace anomaly which is an essential ingredient of QCD. (The axial anomaly, another ingredient of QCD, can be incorporated easily via the $\eta'$ field but we are not concerned with it here.) In order to understand what this is, we consider the scale transformation

$$x \to {}^\lambda x = \lambda^{-1} x, \quad \lambda \geq 0\,, \tag{4}$$

under which an arbitrary field $\phi$ transforms with the canonical mass dimension (called "scale dimension") $d_\phi$,

$$\phi(x) \to \lambda^{d_\phi}\phi(\lambda x)\,. \tag{5}$$

The conventional scale dimensions are 1 for scalar and vector fields and $\frac{3}{2}$ for fermion fields. An action with a Lagrangian density with scale dimension 4 is clearly invariant under scale transformation. This means that the classical QCD action in the chiral limit (i.e., with zero quark mass) is scale invariant. The quark mass term has scale dimension 3, and thus breaks scale invariance. Quantum mechanics introduces a profound modification to this structure. A well-known property of QCD is that the pure Yang-Mills action is not scale invariant at the quantum level due to dimensional transmutation. This can be stated in terms of the divergence of the dilatation current $D_\mu$ whose nonzero value signals the breaking of scale invariance or equivalently the nonvanishing trace of the energy-momentum tensor. Including the effect of quark masses, the trace anomaly is given (apart from an anomalous dimension contribution) by[9]

$$\partial^\mu D_\mu = \theta^\mu_\mu = \sum_q m_q \bar{q}q - \frac{\beta(g)}{g}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu}\,, \tag{6}$$

where $\beta(g)$ is the usual beta function of QCD. The quantum nature of the trace anomaly is evident from the dependence on the strong fine-structure constant $g^2/4\pi$. *Our second main assumption is this: In order to be consistent with the scale property of QCD, effective Lagrangians must reproduce faithfully the trace anomaly (6) in terms of effective fields.* This assumption leads to the universal scaling that we argue holds well in the Nambu-Goldstone phase of chiral symmetry.

Now the chiral field $U = e^{i\pi/f_\pi}$ with $\pi = \boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)$ has scale dimension zero.[10] A derivative brings in a scale dimension 1 and hence the current-algebra term $(f_\pi^2/4)\times\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)$ has scale dimension 2, the mass term $c\,\mathrm{Tr}(MU + \mathrm{H.c.})$, with $c$ a constant and $M$ the quark mass matrix, has scale dimension 0, and the Skyrme quartic term has scale dimension 4. Clearly this effective Lagrangian in its original form does not satisfy the trace condition (6). In order to restore consistency with the trace condition, it is convenient to define the scalar "glueball" field $\chi$,

$$\langle 0|(\mathrm{Tr}G_{\mu\nu}G^{\mu\nu})^{1/4}|0\rangle \propto \langle 0|\chi|0\rangle \equiv \chi_0\,, \tag{7}$$

where $|0\rangle$ stands for the vacuum at zero density. (The vacuum at nonzero density or, more properly, the ground state will be denoted by $|0^*\rangle$.) As defined, the $\chi$ field has scale dimension 1. In terms of this field, the second term of Eq. (6), i.e., the trace anomaly, can be reproduced by a potential term of the form

$$V(\chi) \propto \chi^4 \ln(\chi/e^{1/4}\chi_0)\,. \tag{8}$$

With the help of the $\chi$ field, the Skyrme Lagrangian can be rewritten so as to be consistent with the QCD scale property,

$$\mathcal{L} = \frac{f_\pi^2}{4}\left(\frac{\chi}{\chi_0}\right)^2 \mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger) + \frac{\epsilon^2}{4}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2 + \tfrac{1}{2}\partial_\mu\chi\partial^\mu\chi + c\left(\frac{\chi}{\chi_0}\right)^3 \mathrm{Tr}(MU + \mathrm{H.c.}) + V(\chi)\,, \tag{9}$$

where a scale-invariant kinetic-energy term for the glueball field is introduced. As written, the scale breaking resides entirely in the last line of Eq. (9), all the rest being scale invariant. It should be particularly noted that the Skyrme quartic term is scale invariant by itself. This is a key point for later discussions.[11]

One immediate consequence of the above discussion is that the quark condensate in a dense medium scales as

$$\langle 0^*|\bar{q}q|0^*\rangle/\langle 0|\bar{q}q|0\rangle = (\chi_*/\chi_0)^3, \quad \chi_* \equiv \langle 0^*|\chi|0^*\rangle\,. \tag{10}$$

This suggests defining an *effective* pion decay constant as

$$f_\pi^* = f_\pi \chi_* / \chi_0 , \tag{11}$$

so that

$$\langle 0^* | \bar{q} q | 0^* \rangle / \langle 0 | \bar{q} q | 0 \rangle = (f_\pi^* / f_\pi)^3 \tag{12}$$

which is the relation deduced in Ref. 7. Let us assume that as density increases, the potential develops a minimum at $\chi = \chi_*$. This suggests expanding the $\chi$ field as

$$\chi = \chi_* + \chi' , \tag{13}$$

with $\chi'$ a fluctuating field. In the vacuum characterized by $\chi_*$, the Lagrangian for the chiral field can be written as

$$\mathcal{L}_U = \frac{f_\pi^{*2}}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{\epsilon^2}{4} \mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 + c \left( \frac{f_\pi^*}{f_\pi} \right)^3 \mathrm{Tr}(MU + \mathrm{H.c.}) + \cdots , \tag{14}$$

with

$$U(x) = \exp(i \pi^* / f_\pi^*), \quad \pi^* \equiv \pi \chi_* / \chi_0 . \tag{15}$$

The ellipsis stands for other fields including the $\chi'$ field with which we are not directly concerned here. We have thus effectively assigned a scale dimension 1 to the $f_\pi^*$ and to the pion field $\pi^*$. There is no explicit $\sigma$ field of the linear $\sigma$ model (i.e., the $\bar{q}q$ scalar in the Nambu-Jona-Lasinio model). However, a strong mixing is expected between the glueball field $\chi'$ and two pions in the scalar channel so as to give the effective $\sigma$ with mass $m_\sigma \sim 560$ MeV needed in nuclear physics. Thus in nature we should identify the "observed" lowest-mass scalar to be a mixture of a quark-containing scalar and the glueball scalar. This immediately suggests the scaling

$$m_\chi^* / m_\chi \approx f_\pi^* / f_\pi \rightarrow m_\sigma^* / m_\sigma \approx f_\pi^* / f_\pi , \tag{16}$$

with the $\sigma$ field now understood to be the effective *nuclear physics* scalar.

The Lagrangian (14), together with (1) and (2), implies that the nucleon mass scales as

$$m_N^* / m_N \approx (g_A^* / g_A)^{1/2} f_\pi^* / f_\pi . \tag{17}$$

Since, modulo loop corrections, $g_A$ is scale invariant (because the coefficient $\epsilon^2$ of the Skyrme quartic term to which $g_A$ is related is scale invariant), we may set $g_A^*/g_A \approx 1$. Alternatively we may invoke the phenomenological observation that the in-medium constant $g_A^*$ saturates rapidly in light nuclei at 1 and stays constant as density (or mass number) increases.[12] In some problems[13] the density dependence of $g_A^*$ is important but not at densities higher than nuclear matter $\rho_0$. The rapid variation in $g_A$ between $\rho = 0$ and $\rho = \rho_0$ is a loop effect as suggested in Ref. 4, indicating a possible new (lower) scale *induced* in nuclei. Accounting for such effects is an important subject for classical nuclear physics but not relevant at the mean-field order we are concerned with. We thus establish that

$$m_N^* / m_N \approx f_\pi^* / f_\pi . \tag{18}$$

This result was also obtained using *in-medium* QCD sum rules.[14]

Before proceeding to complete the scaling relation (3), we pause to emphasize that Eq. (18) is extremely practical. It tells us that knowledge of the effective mass of the nucleon $m_N^*$ supplies the order parameter $f_\pi^*$ of the broken-symmetry regime at finite density. Essentially every nuclear physicist knows what $m_N^*$ is at (at least) nuclear matter density although there is no consensus on its exact value. Quite conservatively (from the standpoint of the difference of $m_N^*$ from $m_N$),

$$m_N^*(\rho_0) / m_N \approx 0.8 . \tag{19}$$

This, coupled with Eqs. (18) and (12), has the remarkable consequence that

$$\langle 0^* | \bar{q} q | 0^* \rangle / \langle 0 | \bar{q} q | 0 \rangle \approx \tfrac{1}{2} \text{ at } \rho = \rho_0 ; \tag{20}$$

that is, the quark condensate has dropped by $\sim 50\%$ already in the middle of nuclei.

Let us now consider the vector-meson masses. In a generalized model with vector mesons (i.e., the hidden-gauge-symmetric theory[5]), one has the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation

$$m_V^2 = 2 g^2 f_\pi^2 , \tag{21}$$

where $g$ is the hidden-gauge coupling. We will ignore $O(1/N_c)$ correction and set $m_V = m_\rho = m_\omega$. In terms of the original Skyrme model, we can identify the gauge coupling $g^2$ with $(8\epsilon^2)^{-1}$ and hence find,[15] thanks to the scale invariance of the Skyrme quartic term,

$$m_V^* / m_V \approx f_\pi^* / f_\pi . \tag{22}$$

This completes the relation (3). [A striking omission from Eq. (3) is the scaling for Goldstone bosons, in particular for the pion which figures importantly in nuclear physics. The Lagrangian (14) implies that the pion mass $m_\pi^*$ scales as $(f_\pi^*)^{1/2}$ which differs from Eq. (3). The story turns out to be much more intricate in the case of Goldstone bosons. As discussed in Ref. 16, there is an intricate effect associated with the explicit chiral-symmetry-breaking term which turns out to restore approximately, at least at the high temperature relevant in high-energy heavy-ions collisions, the universal scaling to the pion.]

We stress that our scaling relation is a *mean-field* relation that emerges at the tree level of the effective Lagrangian (14), the key point of our discussion being that (14) with scaled masses be taken as the effective Lagrangian to calculate physical observables according to the chiral perturbation scheme as, e.g., discussed in Ref. 17.

We see a lot of consequences of these dropping masses in nature, some of which are discussed in Refs. 13, 14, and 18–21. In fact, quite detailed recent work by Hosaka and Toki[22] who investigated effective matrix elements of the in-medium two-body interaction in the $s,d$ shell shows that a uniform decrease in various masses yields $G$-matrix elements which are consistent with empirical matrix elements. Perhaps more significantly, they are precursors to QCD phase transitions: From Eqs. (1) and (2), combined with the preceding arguments, we see that the nucleon radius increases rapidly beyond $\rho=\rho_0$,[23] i.e.,

$$\langle r^2\rangle^*/\langle r^2\rangle \approx f_\pi^2/f_\pi^{*2}\,. \tag{23}$$

This rapid increase in radius with higher density may be interpreted as deconfinement as suggested in Ref. 8. Implications of the scaling presented in this paper on QCD phase transitions in heavy-ion collisions were recently discussed by Brown, Bethe, and Pizzochero.[24]

We are living in a "swelled" world with all the masses dropping as density increases, how have the many calculations in conventional nuclear physics held up for so long in such a drastically different scenario? The answer to this puzzle is that to the extent that there is a common scale, it can be scaled out, leaving relatively small effects. To put it more precisely, to the extent that $m_i^*/m_i$ equals a common $\lambda(\rho)$ for any $i$, we have to leading order (in the mean-field sense defined above)

$$H(m_i^*,r)\approx\lambda(\rho)H(m_i,x)\,, \tag{24}$$

with $m_i^*r=m_ix$ defining $x$. Since $\lambda(\rho)\geq 0.8$ in nuclei, the energy is shifted little, say, less than 20%. But the distance swells by $\lambda(\rho)^{-1}$, i.e., $r=(m_i/m_i^*)x$. This "swelling" must of course show up in certain processes. Indeed one such case is the spin-orbit interaction in nuclei: It has been pointed out[21,25] that the spin-orbit interaction in nuclei most sensitively displays the density-dependent masses, giving us a factor $(m/m^*)^3$, as is obvious from its dimensionality in $r$.

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