# Neutrino Interactions in a Dense Plasma

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We investigate many-body corrections to neutrino scattering from electrons and nuclei. The Coulomb correlations of the electrons screen the weak vector neutral current. Electron screening is important only for electron neutrinos (or antineutrinos) and most effective for neutrino energies of several MeV or less. At electron densities relevant for supernovae, the total quasielastic $\nu$-$e$ cross section can be reduced by 50%, and elastic neutrino-nucleus scattering by up to 85%.

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The interactions of neutrinos in dense matter are of considerable interest in astrophysics. Neutrino interactions and neutrino transport are very important for supernovae and the cooling of young neutron stars.[1,2] For example, neutrino-electron scattering reduces neutrino energies, allowing them to escape more easily. This can keep a shock from forming a supernova.[3,4]

The neutrino cross sections are modified by correlations in the dense electron and hadron gas. Previous work on correlations had focused on nucleons, in a nonrelativistic framework.[5] In this paper, we concentrate on the correlations in a degenerate electron gas using a relativistic random-phase approximation (RPA). Electron correlations have often been omitted in the past because of the need for a relativistic many-body formalism.

In general, the relativistic many-body problem is both very hard and very interesting. New features such as the mixing of particle-hole and particle-antiparticle degrees of freedom arise. In the context of nuclear physics, there are the added complications of a strong-coupling field theory. A system of neutrinos and electrons, however, is one of the few cases where the interactions are known from the standard model, and the calculation does not involve any free parameters.

We use linear-response theory to calculate the quasielastic cross section of neutrinos from a dense electron gas. The typical momentum transfer is of the order of several MeV. Therefore, the correlations induced by the electromagnetic interaction between the electrons can be important. We work to lowest order in the weak-coupling constant $G$, but, by employing a relativistic RPA, to all orders in the fine-structure constant. We expect the RPA to describe the dominant contributions to the linear response of the electron gas.

For values of the momentum transfer much less than the $W$ mass, the direct ($Z^0$) and exchange ($W^{\pm}$) contributions to the matrix element $\mathcal{M}$ can be written as an effective four-point coupling

$$\mathcal{M}=(G/\sqrt{2})[\bar{\nu}\gamma_\mu(1-\gamma_5)\nu](\bar{e}J^\mu e) \tag{1}$$

($\nu$ and $e$ are neutrino and electron spinors, respectively), with the current

$$J^\mu=\gamma^\mu(c_V-c_A\gamma_5)\,, \tag{2}$$

where the vector and axial-vector couplings $c_V$ and $c_A$ in terms of the Weinberg angle $\theta_W$ ($\sin^2\theta_W\approx 0.223$) are given by

$$c_V=2\sin^2\theta_W\pm\tfrac{1}{2},\quad c_A=\pm\tfrac{1}{2}\,. \tag{3}$$

Here the upper sign is for $\nu_e$'s. The lower sign is valid for $\nu_\mu$'s or $\nu_\tau$'s, where only the direct term contributes.

The differential cross section per volume $V$ for scattering of neutrinos with initial energy $E_\nu$, final energy $E'_\nu$, from an electron gas is

$$\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu}=-\frac{G^2}{32\pi^2}\frac{E'_\nu}{E_\nu}\operatorname{Im}(L_{\mu\nu}\Pi^{\mu\nu})\,. \tag{4}$$

Here $L_{\mu\nu}$ is the neutrino tensor

$$L_{\mu\nu}=8[2k_\mu k_\nu+(k\cdot q)g_{\mu\nu}-(k_\mu q_\nu+q_\mu k_\nu)\mp i\epsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta]\,, \tag{5}$$

with $k$ the initial neutrino four-momentum and $q$ the four-momentum transfer. The sign in the last term is $-$ for neutrinos and $+$ for antineutrinos.

The polarization (without correlations) is defined by

$$\Pi^{\mu\nu}(q)=-i\int\frac{d^4p}{(2\pi)^4}\operatorname{tr}[G(p)J^\mu G(p+q)J^\nu]\,, \tag{6}$$

with the electron propagator for mass $m$ at Fermi energy $E_F=(k_F^2+m^2)^{1/2}$,

$$G(p)=(\not{p}+m)\left\{\frac{1}{p^2-m^2+i\epsilon}+\frac{i\pi}{E_p}\delta(p_0-E_p)\theta(E_F-E_p)\right\} \tag{7}$$

$[E_p=(|\mathbf{p}|^2+m^2)^{1/2}]$. Here the second term changes the $+i\epsilon$ boundary condition to $-i\epsilon$ for the occupied states in the Fermi sea. In this formulation, it is easy to include correlations by replacing the lowest-order polariza-

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tion [Eq. (6)] with a better approximation which includes the interactions between electrons.

Since the current is the sum of a vector and an axial-vector piece, we can decompose the polarization into vector $\Pi_V^{\mu\nu}$, axial-vector $\Pi_A^{\mu\nu}$, and mixed $\Pi_{VA}^{\mu\nu}$ contributions:

$$\Pi^{\mu\nu}=c_V^2\Pi_V^{\mu\nu}+c_A^2\Pi_A^{\mu\nu}+2c_Vc_A\Pi_{VA}^{\mu\nu}\,. \tag{8}$$

For $\Pi_V^{\mu\nu}$ the currents $J^\mu, J^\nu$ in Eq. (8) are replaced by $\gamma^\mu$ and $\gamma^\nu$, for $\Pi_A^{\mu\nu}$ they are $\gamma^\mu\gamma^5$ and $\gamma^\nu\gamma^5$, and for $\Pi_{VA}^{\mu\nu}$ they are $\gamma^\mu\gamma^5$ and $\gamma^\nu$. Each polarization can be written as the sum of a density-dependent piece and a density-independent (Feynman) piece. The Feynman piece is obtained by dropping the second term in Eq. (7) in both propagators in Eq. (6). This describes vacuum polarization and is explicitly included. Because of current conservation and translational invariance, the vector piece has only two independent components which we choose to be [in a frame with $q_\mu=(q_0,|\mathbf{q}|,0,0)$][6]

$$\Pi_T=\Pi_V^{22},\quad \Pi_L=-(q_\mu^2/|\mathbf{q}|^2)\Pi_V^{00}\,. \tag{9}$$

For $q_\mu^2<0$, the imaginary parts of the Feynman pieces vanish, and the density-dependent pieces of $\Pi_T$ and $\Pi_L$ are given in Ref. 7. The axial-vector and mixed pieces are found to be

$$\Pi_A^{\mu\nu}=\Pi_V^{\mu\nu}+g^{\mu\nu}\Pi_A,\quad \Pi_{VA}^{\mu\nu}=i\epsilon^{\mu\nu\alpha 0}q_\alpha\Pi_{VA} \tag{10}$$

(in the electron-gas rest frame), and the imaginary parts of $\Pi_A$ and $\Pi_{VA}$ are easily obtained from Eq. (6). It is straightforward to evaluate the contraction of neutrino and electron tensor. We find for the cross section per volume

$$\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE_\nu'}=\frac{G^2E_\nu' q_\mu^2}{4\pi^3E_\nu}\left(\frac{2E_\nu E_\nu'+\frac{1}{2}q_\mu^2}{|\mathbf{q}|^2}R_1+R_2+\frac{E_\nu+E_\nu'}{m}R_3\right), \tag{11}$$

and the three uncorrelated response functions (+ for neutrinos, − for antineutrinos),

$$R_1=(c_V^2+c_A^2)(\mathrm{Im}\Pi_T+\mathrm{Im}\Pi_L)\,, \tag{12}$$

$$R_2=(c_V^2+c_A^2)\mathrm{Im}\Pi_T-c_A^2\mathrm{Im}\Pi_A\,, \tag{13}$$

$$R_3=\pm 2c_Vc_A m\,\mathrm{Im}\Pi_{VA}\,. \tag{14}$$

We now consider the effect of electromagnetic correlations in the relativistic RPA. The perturbation of the system by the probe can couple to the correlated electrons and excite particle-hole and particle-antiparticle pairs. In the RPA, this amounts to the replacement of the polarizations $\Pi$ by the corresponding RPA polarizations $\tilde{\Pi}$, obtained as the solution of a Dyson equation, with the photon propagator determining the strength of the interaction.[6]

For the vector polarization, the Dyson equation reads

$$\tilde{\Pi}_V^{\mu\nu}=\Pi_V^{\mu\nu}-(e^2/q_\kappa^2)\Pi^{\mu}_{V,\alpha}\tilde{\Pi}_V^{\alpha\nu}\,. \tag{15}$$

With the transverse and longitudinal dielectric functions $\epsilon_T$ and $\epsilon_L$ defined by

$$\epsilon_T=1-(e^2/q_\kappa^2)\Pi_T,\quad \epsilon_L=1+(e^2/q_\kappa^2)\Pi_L\,, \tag{16}$$

the solution is given by

$$\tilde{\Pi}_T=\Pi_T/\epsilon_T,\quad \tilde{\Pi}_L=\Pi_L/\epsilon_L\,. \tag{17}$$

The correction terms for the mixed and axial-vector polarizations can be obtained analogously. Using these RPA polarizations, the cross section in the RPA is given by Eq. (11) with the response functions $R_i$ replaced by

$$\tilde{R}_1=c_V^2\left[\mathrm{Im}\left(\frac{\Pi_T}{\epsilon_T}\right)+\mathrm{Im}\left(\frac{\Pi_L}{\epsilon_L}\right)\right]+c_A^2\left[\mathrm{Im}\Pi_T+\mathrm{Im}\Pi_L+\frac{e^2|\mathbf{q}|^2}{q_\mu^2}\mathrm{Im}\left(\frac{(\Pi_{VA})^2}{\epsilon_T}\right)\right], \tag{18}$$

$$\tilde{R}_2=c_V^2\,\mathrm{Im}\left(\frac{\Pi_T}{\epsilon_T}\right)+c_A^2\left[\mathrm{Im}\Pi_T-\mathrm{Im}\Pi_A+\frac{e^2|\mathbf{q}|^2}{q_\mu^2}\mathrm{Im}\left(\frac{(\Pi_{VA})^2}{\epsilon_T}\right)\right], \tag{19}$$

$$\tilde{R}_3=\pm 2c_Vc_A m\,\mathrm{Im}\left(\frac{\Pi_{VA}}{\epsilon_T}\right) \tag{20}$$

(+ for neutrinos, − for antineutrinos).

We now consider a related issue, the screening of elastic neutrino scattering from nuclei (typically $\approx$ Fe) in a dense electron gas. This was also considered by Leinson, Oraevsky, and Semikoz[8] with very similar results. Elastic scattering has a large cross section and is believed to trap neutrinos into a neutrino sphere. However, a neutrino can first couple to an electron and then the electron will couple coherently to the nucleus through the large charge $Z$. The amplitude for this process interferes with direct neutrino-nucleus scattering. The differential cross section for elastic neutrino scattering with scattering angle $\theta$ from nuclei with charge $Z$ and neutron number $N$ is (without correlations)

$$\frac{d\sigma}{d\cos\theta}=\frac{G_{\mathrm{eff}}^2}{2\pi}E^2(1+\cos\theta)\,, \tag{21}$$

with the effective weak coupling $G_{\mathrm{eff}}$ and weak charge $C$ defined by

$$G_{\mathrm{eff}}=GC,\quad C=(2\sin^2\theta_W-\tfrac{1}{2})Z+\tfrac{1}{2}N \tag{22}$$

(see also Ref. 9). The screening due to the coupling to the electron gas is again evaluated in the RPA. We obtain a cross section of the same form as Eq. (21), but with the effective weak coupling $G_{\mathrm{eff}}$ of Eq. (22) re-

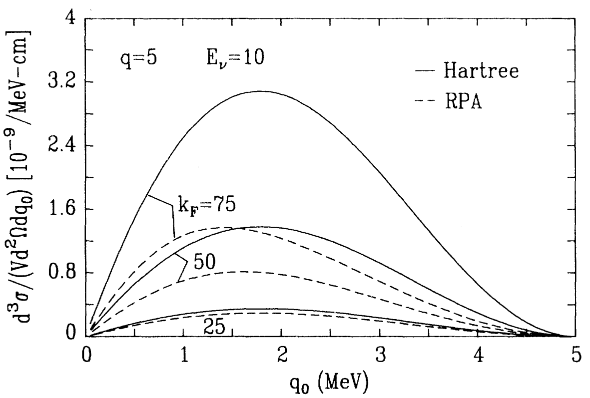


FIG. 1. Differential cross section for quasielastic scattering of $\nu_e$'s from an electron gas, for a $\nu_e$ energy of 10 MeV and momentum transfer $q=5$ MeV, at the electron Fermi momenta indicated (in MeV). The (solid) Hartree lines are obtained without RPA correlations.

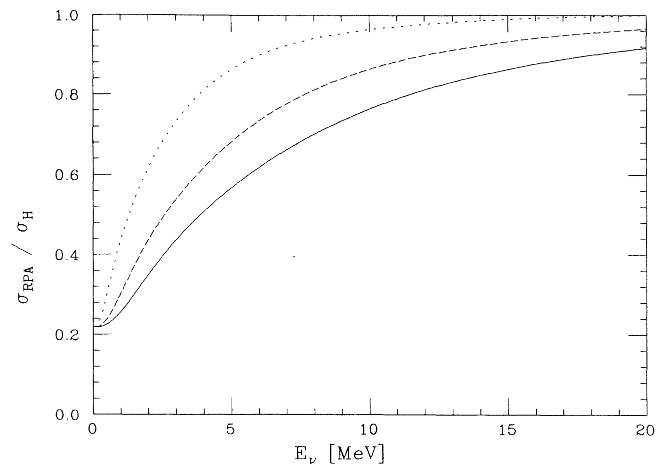


FIG. 2. Ratio of the total quasielastic cross section with and without correlations for $\nu_e$'s in an electron gas. The solid line is for $k_F=75$ MeV, the dashed line is for $k_F=50$ MeV, and the dotted line is for $k_F=25$ MeV.

placed by $\tilde{G}_{\rm eff}$ given by

$$\frac{\tilde{G}_{\rm eff}}{G_{\rm eff}}=1-\left(\frac{\epsilon_L(|\mathbf{q}|,q_0=0)-1}{\epsilon_L(|\mathbf{q}|,q_0=0)}\right)\frac{c_V Z}{C}. \tag{23}$$

In addition to the screening due to the electron gas, there will be screening from the correlated ions, and possibly from protons. The ion-ion correlations can be included by multiplying the right-hand side of Eq. (21) by the static structure factor $S(q)$.[10] We evaluate this in a hypernetted-chain approximation[11] for a nonrelativistic classical liquid of pure $^{56}$Fe ions interacting through screened Coulomb potentials, $U(r)=(Z_1Z_2e^2/4\pi r)\times\exp(-r/\lambda)$, with the Thomas-Fermi screening length $\lambda=[(e^2/\pi^2)k_FE_F]^{-1/2}$.

We now discuss the results. Figure 1 shows the differential cross section for scattering of $\nu_e$'s from an electron gas, for a typical astrophysical situation. The cross section is plotted as a function of the energy transfer $q_0$, for the electron Fermi momenta $k_F=25$, 50, and 75 MeV. For an electron fraction $Y_e\approx\frac{26}{56}$ this corresponds to densities of $\approx 0.25\times10^{12}$, $2\times10^{12}$, and $7\times10^{12}$ g/cm$^3$. The screening due to Coulomb correlations increases with the electron density. Screening is even more important for smaller values of the momentum transfer. The ratio of the total quasielastic cross sections with and without correlations is shown in Fig. 2 as a function of the $\nu_e$ energy. The screening is most effective for low energies and high densities. Since $c_V^2\approx 1$ for $\nu_e$, but $c_V^2\ll 1$ for $\nu_\mu$ or $\nu_\tau$ [Eq. (3)], there is very little screening for $\nu_\mu$'s and $\nu_\tau$'s. The cross sections for neutrinos and antineutrinos differ only by the sign in the small response function $R_3$ and are almost equal.

The screening of elastic $\nu_e$ scattering from Fe ions is shown in Fig. 3. The amplitude for a $\nu_e$ coupling first to an electron interferes destructively with direct $\nu_e$-$^{56}$Fe scattering. As a result, the differential cross section has a node as a function of $q$, and the total cross section has a minimum as a function of $E_\nu$. For intermediate values of the $\nu_e$ energy, this screening can reduce the cross section by a factor of 8. Again, for $\nu_\mu$ or $\nu_\tau$ the screening is small because of their small vector couplings. Therefore, the $\nu_e$ cross section is much smaller than the $\nu_\mu$ or $\nu_\tau$ cross section.

The ratio of $\nu$-$^{56}$Fe total elastic cross sections with and without ion-ion correlations (but no electron screening) is shown in Fig. 4. These reductions are even larger than those from electrons. Nonetheless, the electrons could produce significant further reductions and may differentiate between $\nu_e$ and $\nu_\mu$ or $\nu_\tau$. Further details of these

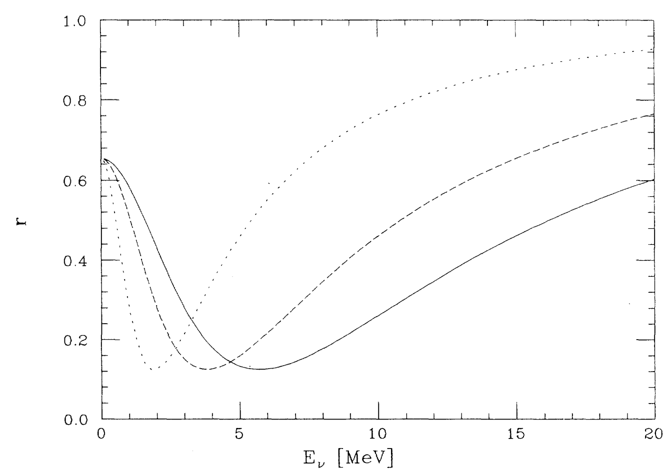


FIG. 3. Ratio $r=(\tilde{G}_{\rm eff}/G_{\rm eff})^2$ of the total elastic cross section with and without correlations for $\nu_e$'s from $^{56}$Fe nuclei [see Eq. (23)]. The solid line is for $k_F=75$ MeV, the dashed line is for $k_F=50$ MeV, and the dotted line is for $k_F=25$ MeV.

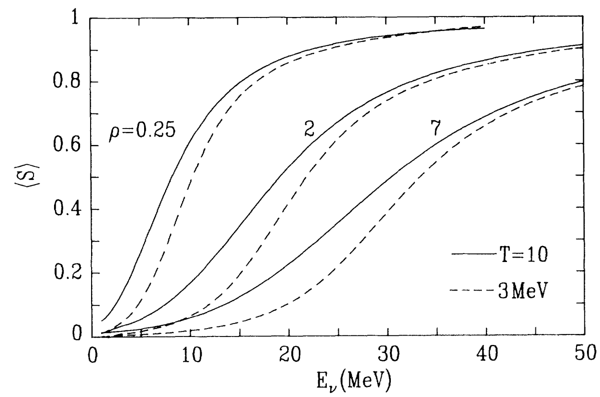


FIG. 4. Ratio of total $\nu$-$^{56}$Fe elastic cross section with and without ion-ion correlations vs $\nu$ energy at the indicated temperatures and densities in units of $10^{12}$ g/cm$^3$. Note that electron screening is not included.

ion calculations along with a treatment of the coupled electron-ion system will be presented later.[12]

To summarize, we have derived simple analytic formulas for electron screening of neutrino scattering from an electron gas or from nuclei. The results are exact within the relativistic RPA. We find the electron screening to be important only for $\nu_e$ or $\bar{\nu}_e$, and largest for high density and small neutrino energy. Both electron and ion screening can reduce $\nu$-nucleus elastic scattering by large amounts.

We expect all neutral vector currents to be Coulomb screened at low momentum transfer. One should examine how this screening affects the transport of low-energy neutrinos in supernovae and the cooling of neutron stars. Work on extending the relativistic formalism to finite temperature and to include hadrons is in progress.

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