$\mu \rightarrow e+\gamma$ and $\tau \rightarrow \mu+\gamma$ Decays in String Models with E_6 Symmetry

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E_6 string models for which (i) E_6 breaks to $[SU(3)]^3$ at the Planck scale, (ii) $[SU(3)]^3$ breaks to $SU(3)\times SU(2)\times U(1)$ at an intermediate scale M_I triggered by a $(\text{mass})^2$ $m^2<0$ where $m \lesssim 1$ TeV, and (iii) the model possesses a matter-parity invariance lead to the new-physics signals of $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decays which may be accessible experimentally. The outgoing lepton is nearly 100% right handed and the $\tau \rightarrow \mu\gamma$ branching ratio about 10^5 times larger than the $\mu \rightarrow e\gamma$. Possible ways of detecting these decays are discussed.

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Though superstring theory has been the subject of a large amount of study for nearly a decade, it has only been recently that progress has been made in investigating its experimental consequences.[1-3] Part of the difficulty lies in the existence of a huge number of possible vacuum states which are *a priori* degenerate. At present, there is no firm theoretical principle to choose between these possibilities. An alternate phenomenological requirement that we will impose here is that the theory reduce in form to the standard model at low energies. The requirement that the standard model emerge at the W mass scale as a consequence of the string vacuum chosen at the Planck scale $M_{\rm Pl}=2.4\times10^{18}$ GeV is a strong constraint which severely limits the choice of vacuum state.[4]

One promising class of models that leads to a successful phenomenology arises from compactifying the ten-dimensional string on a Calabi-Yau manifold. This leads to string models which at the Planck scale possess the symmetry

$$E_6\times(N=1 \text{ supergravity})\times G', \qquad (1)$$

where G' is an appropriate hidden-sector group. We consider in this paper models obeying Eq. (1). The symmetry (1) also occurs in orbifold compactifications and four-dimensional string models and thus our analysis has a wide range of applicability. However, the analysis given here may not apply to other model building strategies for string theory.

A natural consequence of Eq. (1) is that the massless states at $M_{\rm Pl}$ lie in the **27**, $\overline{\mathbf{27}}$, and singlet representations of E_6. The $[SU(3)]^3\equiv SU(3)_C\times SU(3)_L\times SU(3)_R$ content of the ith **27** generation is

$$L^l_{ri}(1,3,\bar 3)\oplus Q^r_{ai}(3,\bar 3,1)\oplus(Q^c)^r_{ai}(\bar 3,1,3),$$

where $a,l,r=1,2,3$ label the $SU(3)_{C,L,R}$ states. The lepton nonet is $L=[l=(\nu,e);e^c;H;H';\nu^c;N]$ while the quark and conjugate quark nonets are $Q=[q^a=(u^a,d^a);H_3=D^a]$ and $Q^c=[u^c_a,d^c_a;H'_3=D^c_a]$. Here l, H, H', and q^a are $SU(2)_L$ doublets, D^a,D^c_a are color triplets, ν^c is an $SU(5)$ singlet, and N an $O(10)$ singlet.

The following basic theorem was established in Ref. (1): For any string model obeying Eq. (1) with (i) breaking of $E_6\rightarrow[SU(3)]^3$ at scale $\sim M_{\rm Pl}$, (ii) intermediate-scale breaking of $[SU(3)]^3\rightarrow SU(3)\times SU(2)\times U(1)$ at scale M_I, where $M_{\rm Pl}>M_I\gtrsim10^{15}$ GeV, triggered by a mass $m\lesssim1$ TeV, and (iii) vacuum expectation values (VEVs) of N_i,ν^c_i obeying $\sum\langle N_i\rangle\langle\nu^c_i\rangle=0$, then there are always at least two new non-E_6-singlet light chiral multiplets (in addition to the three generations of light states of the standard model), and in some cases four new non-E_6-singlet light chiral multiplets. In addition, there may also be a number of light E_6-singlet multiplets.

Within the class of models defined at the beginning of the paper, these hypotheses are quite natural. Hypothesis (i) can arise naturally from flux breaking at compactification. Note that since the further breaking at M_I can only occur through N_i and ν^c_i VEV formation (to preserve the standard model group at M_I) other alternatives to (i) such as breaking to $SU(6)\times U(1)$ or no breaking would not reduce E_6 below $SU(5)$. Thus hypothesis (i) appears to be the only viable way of recovering the standard model at low energy. Hypothesis (ii) occurs naturally when m is the soft supersymmetry-breaking mass. Here $m\lesssim1$ TeV is required on phenomenological grounds. Hypothesis (iii) holds when matter-parity[5,6] invariance holds, which is needed to maintain adequate proton stability.[5,7]

Hypothesis (iii) implies one may choose a basis in generation space where $\langle N_i\rangle=\langle N_1\rangle\delta_{i1}$ and $\langle\nu^c_i\rangle=\langle\nu^c_2\rangle\delta_{i2}$. In this basis the two new non-E_6-singlet states guaranteed to be light are[1]

$$n_i=(N_1+\bar N_1)/\sqrt2,\quad \hat\nu^c_2=(\nu^c_2+\bar\nu^c_2)/\sqrt2, \qquad (2)$$

while the additional possible light states are $n_2=\cos\theta N_2+\sin\theta\,\bar{v}_1^c$ and $\bar{n}_2=\cos\theta\bar{N}_2+\sin\theta\,v_1^c$, where $\tan\theta=\langle v_2^c\rangle/\langle N_1\rangle$.

Low-energy interactions.—The theorem stated above is of interest since first the assumptions are relatively mild (there are known string manifolds where they hold) and second they are a string prediction of the existence of *new low-energy physics* not found in the standard model. In Refs. 1 and 2 the interactions of the new low-energy states with the standard-model particles were worked out for the case of $M_2\equiv CU$ matter parity[5] and where the light standard-model quarks and leptons lie in the M_2-odd sector (as is the situation of known three-generation models[8]). Here $UN_i=+N_i$, $Uv_i^c=-v_i^c$, and C divides generations into even and odd sectors, e.g., $CL_i=\pm L_i$.

N_1 (v_1^c) is C-even and matter-parity-even (-odd) while N_2 (v_2^c) is C-odd and matter-parity-even (-odd). The symmetry-breaking pattern at the intermediate scale dictates that the mass terms involving the M_2-odd fields in the leptonic sector have the form $\xi_a M_{ab}^{(0)}\xi_b$, where $\xi_a=(\lambda_L^{(-)},l_n,\bar{H}_r,H_r')$ and $\xi_b'=(\lambda_L^{(+)},\bar{l}_n,H_r,\bar{H}_r')$ and where l_n are C-even primary lepton fields, H_r ($\bar{H}_r'$) are the C-odd primary Higgs (mirror Higgs) fields, and $\lambda_L^{(\pm)}=\pm(\lambda_{4\pm}^L,\lambda_{6\pm}^L)$ with $\lambda_{4\pm}^L=(\lambda_4^L\pm i\lambda_5^L)/\sqrt{2}$ and λ_i^L the $SU(3)_L$ gauginos. The matrix $M_{ab}^{(0)}$ is rectangular with dimensionality $n_a\times n_b$, where $n_b=n_a+3$, which guarantees the existence of three massless lepton generations. These three generations of light leptons are related to the primary ξ_a fields by projection matrices $U^\dagger$ and $\bar{U}^\dagger$ defined in Eq. (8). The light Higgs doublet which enters in the $SU(2)_L\times U(1)_Y$ breaking arises from the M_2-even sector. Here the mass matrix has the form $\eta_a M_{ab}^{(e)}\eta_b'$, where $\eta_a=(\bar{l}_r,H_n,\bar{H}_n')$ and $\eta_b'=(l_r,\bar{H}_n,H_n')$. Here l_r ($\bar{l}_r$) are the primary lepton (mirror lepton) C-odd fields, H_n ($\bar{H}_n$) are the primary Higgs (mirror Higgs) C-even fields, etc. The light Higgs doublet H' is given by[1] $H'=H_1'$ while the light Higgs doublet H is related to the primary η_a fields by the projection matrices V and $\bar{V}$ defined by Eq. (9). One finds for the low-energy effective superpotential[1,2]

$$W^{\rm eff}=\{\lambda_{pp'}^{(l)}H'e_p^c l_{p'}+\lambda_{pp'}^{(u)}Hq_pu_{p'}^c+\lambda_{pp'}^{(d)}H'q_pd_{p'}^c\}+[(\lambda_{ap}Hl_p\phi_a+\lambda_pHl_pn_2+\bar{\lambda}_pHl_p\bar{n}_2)+\Phi_a m_{a\beta}\Phi_\beta+W_{\rm seesaw}]+\lambda_{abc}\phi_a\phi_b\phi_c\,. \quad (3)$$

Here $p=1,2,3$ labels the three light generations of the standard model, and $\Phi_a=(\hat{v}_2^c,n_1;n_2,\bar{n}_2,\phi_a)$ is the array new light fields where ϕ_a are E_6 singlets. In the superpotential of Eq. (3), the curly brackets contain the interactions of the standard supersymmetric model, i.e., the interactions of quarks and leptons with Higgs fields. The remaining terms of Eq. (3) are new, generated in the E_6 string models. They involve interactions of the SU(2) $\times$U(1)-singlet fields $n_2,\bar{n}_2$ and of the E_6-singlet fields ϕ_a with Higgs bosons and leptons. In addition, there is a gauge interaction contribution to the Lagrangian arising from $SU(3)_L$ gaugino couplings:

$$\mathcal{L}_{\rm gaugino}=g_LU_{p(-)}^\dagger l_p\gamma^0[2^{-1/2}(-cv_2^c+sn_1)\tilde{e}^\dagger-(s\bar{V}_{1H}'Hn_2^\dagger+c\bar{V}_{2H}H\bar{n}_2^\dagger)+(s\bar{n}_2H'^\dagger+\tilde{\lambda}_{pp'}e_{p'}^cH^\dagger)]+\text{H.c.}\,, \quad (4)$$

where $s\equiv\sin\theta$, $c\equiv\cos\theta$ ($\tan\theta=\langle v_2^c\rangle/\langle N_1\rangle$), $\tilde{e}$ is the selectron, and fields with daggers are Bose. The interactions of Eq. (4) are all new. They involve couplings of the SU(2)$\times$U(1)-singlet fields n_1 and $\hat{v}_2^c$ with leptons and sleptons as well as additional couplings on n_2 and $\bar{n}_2$ fields.

The coupling constants in Eqs. (3) and (4) are obtained from the string-predicted coupling constants and the unitary matrices that project onto the light sector. Thus the lepton part of the superpotential for the $(\mathbf{27})^3$, $(\overline{\mathbf{27}})^3$, and ϕ_a couplings is

$$W_3=\lambda_{ijk}^3(-H_iH_j'N_k-H_iv_j^cl_k+H_i'e_j^cl_k)+\bar{\lambda}_{ijk}^3(-\bar{H}_i\bar{H}_j'\bar{N}_k-\bar{H}_i\bar{v}_j^c\bar{l}_k+\bar{H}_i'\bar{e}_j^c\bar{l}_k)+m_{ab}\phi_a\phi_b+\lambda_{abc}\phi_a\phi_b\phi_c+\lambda_{aij}\phi_a\,27_i\overline{27}_j\,. \quad (5)$$

One finds[2]

$$-\lambda_p=c\lambda_{n2r}^3U_{pr}^\dagger V_{nH}+s\bar{\lambda}_{1rr'}^3\bar{U}_{pr}^\dagger\bar{V}_{r'H}\,,$$
$$-\bar{\lambda}_p=s\lambda_{1mm'}^3U_{pm}^\dagger V_{m'H}+c\bar{\lambda}_{2rn}\bar{U}_{pr}^\dagger\bar{V}_{nH}'\,, \quad (6)$$

and for the E_6-singlet couplings

$$\lambda_{ap}=\lambda_{ams}[U_{pm}^\dagger\bar{V}_{sH}+V_{mH}\bar{U}_{ps}^\dagger]+\lambda_{asm}U_{ps}^\dagger\bar{V}_{mH}'\,. \quad (7)$$

Here $n\equiv(1,m)$ are C-even and $r\equiv(1,s)$ are C-odd indices. The unitary transformations $U_{pr}^\dagger$, etc., are defined from the projections onto the light lepton doublets,[2]

$$\lambda_L^{(-)}=l_pU_{p(-)}^\dagger+\chi_aU_{a(-)}^\dagger,\quad l_n=l_pU_{pn}^\dagger+\chi_aU_{an}^\dagger\,,$$
$$\bar{H}_r=l_p\bar{U}_{pr}^\dagger+\chi_a\bar{U}_{ar}^\dagger,\quad H_r'=l_pU_{pr}^\dagger+\chi_aU_{ar}^\dagger\,, \quad (8)$$

where the doublets $\lambda_L^{(\pm)}=\pm(\lambda_{4\pm},\lambda_{6\pm})$ with $\lambda_{4\pm}=(\lambda_4\pm i\lambda_5)2^{-1/2}$ are $SU(3)_L$ gauginos and χ_a are superheavy-mass states. Similarly, the V_{nH}, etc., project onto the light H doublet:[2]

$$H_n=V_{nH}H+V_{na}\eta_a,\quad \bar{H}_n'=\bar{V}_{nH}'H+\bar{V}_{na}'\eta_a\,,$$
$$\bar{l}_r=\bar{V}_{rH}H+\bar{V}_{ra}\eta_a\,, \quad (9)$$

where η_a are superheavy. The unitary matrices can be characterized in terms of two parameters $\varepsilon\equiv\tan\theta=\langle v_2^c\rangle/\langle N_1\rangle$ and δ^2 defined by $\lambda_{12r}^3=\delta^2\tilde{\lambda}_{12r}$ with $\tilde{\lambda}_{12r}\sim\lambda_{m2r}^3$. The analysis of the charged-lepton mass matrix in Ref. 2 showed that δ^2/ε was quite small since $m_e/m_\tau=(\delta^2/\varepsilon)/(1+r^2)^{1/2}$, where $r\sim1$. A fit to m_e, m_μ, and m_τ suggests $r\sim3.5$, and hence $\delta^2/\varepsilon\sim10^{-3}$. On the

TABLE I. Dependence of unitary transformations of Eqs. (6) and (7) on δ^2 and ε for different light lepton generations $p=1,2,3$ (Ref. 2).

p	$U^{\dagger}_{p(-)}$	$U^{\dagger}_{p1}$	$U^{\dagger}_{pm}$	$\bar{U}^{\dagger}_{pr}$	$U^{\dagger}_{p2}$	$U^{\dagger}_{ps}$
1	$\delta^2\varepsilon$	ε	δ^2/ε	δ^2	1	δ^2
2,3	ε^2	0	1	ε	0	ε
	V_{1H}	V_{mH}	$\bar{V}'_{nH}$	$\bar{V}_{2H}$	$\bar{V}_{sH}$	
	1	δ^2	δ^2	δ^2/ε	$\delta^3\varepsilon$	

other hand,[9] $\varepsilon \propto (\lambda^1_{\rm NR}/\lambda^2_{\rm NR})^{1/n}$, $n \geq 2$, where $\lambda^{1,2}_{\rm NR}$ are two nonrenormalizable coupling constants, and hence one expects[10] $\varepsilon \sim 1$. The ε and δ^2 dependence for the different lepton generations is given in Table I.

$\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays.—The curly brackets in Eq. (3) are the supersymmetric standard-model interactions while the remaining terms of Eqs. (3) and (4) represent the new physics arising from the new low-mass states. Note that these states interact with leptons and Higgs bosons only and not with quarks. Each of these interactions violates lepton number and allows for decays such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$ (see Fig. 1). The amplitude for these decays is scaled by $\lambda_\mu\lambda_e$ and $\lambda_\tau\lambda_\mu$. Using Eqs. (3), (4), (6), and (7) and Table I, one obtains the values for $\lambda_\mu\lambda_e$ and $\lambda_\tau\lambda_\mu$ of Table II.

Since δ^2/ε is small, we see that the dominant contribution is expected to come from $\hat{\nu}^c_2$ and n_1 interactions when $10^{-3} < \varepsilon < 1$. Further, as discussed above, the basic theorem of Ref. 1 guarantees the existence of these particles in the low-energy spectrum and hence the existence of the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays.

One may parametrize the $\mu \to e\gamma$ decay by the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{e}{4m_\mu} F^{\alpha\beta}\bar{\mu}\sigma_{\alpha\beta}(a_R^{(\mu)}P_R + a_L^{(\mu)}P_R)e + \text{H.c.}, \qquad (10)$$

where $\mu(x), e(x)$ are the lepton fields. We consider the contributions from $\hat{\nu}^c_2$ and n_1 and for simplicity we set

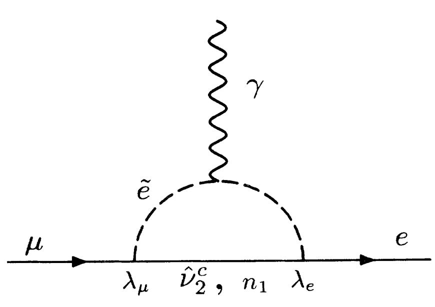

FIG. 1. Decay $\mu \to e\gamma$ arising from Eq. (4) via intermediate $\hat{\nu}^c_2$ and n_1 fermions. Similar decay diagrams via ϕ_a n_2, and $\bar{n}_2$ occur from Eq. (3) interactions.

TABLE II. The δ^2 and ε dependence of the leading part of $\lambda_\mu\lambda_e$ for the decay $\mu \to e\gamma$ and $\lambda_\tau\lambda_\mu$ for $\tau \to \mu\gamma$ when $\hat{\nu}^c_2$, n_1, ϕ_a, n_2, and $\bar{n}_2$ are loop particles (see Fig. 1).

	$\hat{\nu}^c_2$	n_1	ϕ_a	n_2	$\bar{n}_2$
$\mu \to e\gamma$	$c^2\delta^2\varepsilon^3$	$s^2\delta^2\varepsilon^3$	$\delta^6\varepsilon$	$c^2\delta^4\varepsilon$	$c^2\delta^2\varepsilon^5$
$\tau \to \mu\gamma$	$c^2\varepsilon^4$	$s^2\varepsilon^4$	$\delta^4\varepsilon^2$	$c^2\delta^4\varepsilon^2$	ε^6

their masses to a common value $\hat{m}$. One obtains

$$a_R^{(\mu)} \cong \frac{\alpha}{8\pi\sin^2\theta_W}\left[\frac{m_\mu}{\hat{m}}\right]^2 L(x)\left[\frac{m_e}{m_\tau}(1+r^2)^{1/2}\right]\varepsilon^4,$$
$$a_L^{(\mu)} = (m_e/m_\mu)a_R^{(\mu)} \ll a_R^{(\mu)}, \qquad (11)$$

where $x = m^2_{\tilde{e}}/\hat{m}^2$ and $L(x)$ is the loop integral,

$$L(x) = (1-x)^{-4}(\tfrac{1}{3} + \tfrac{1}{2}x - x^2 + \tfrac{1}{6}x^3 + x\ln x),$$

and $r \sim 3.5$ is defined above. The total decay rate is proportional to $a^2 \equiv \frac{1}{2}(a_R^2 + a_L^2)$. Values for a are given in Table III. The current experimental upper limit on the branching ratio for $\mu \to e\gamma$ is[11] $B(\mu \to e\gamma) \leq 5\times10^{-11}$, which corresponds to[12] $a \leq 2.4\times10^{-13}$. The MEGA experiment at Los Alamos should be sensitive to $a \lesssim 2\times10^{-14}$ while KAON at TRIUMF would be able to test $a \lesssim 5\times10^{-15}$. Thus if $\varepsilon \sim 1$, $\hat{\nu}^c_2$ masses up to 1 TeV could be probed by this decay. One also predicts from Eq. (11) that the emitted electron would be almost 100% right-handed polarized. Note also that if δ^2, ε were ~ 1, the decay rate would increase by a factor $\sim 10^6$ and the theory would be in serious disagreement with the present experiment for the entire range of $\hat{m}$. Thus the smallness of the δ^2/ε required from the m_e/m_τ analysis[2] is consistent with the observed bounds on the $\mu \to e\gamma$ decay.

A similar analysis for the $\tau \to \mu\gamma$ decay yields $a^{(\tau)} = (m_\tau^2/m_\mu^2)(\delta^2/\varepsilon)^{-1}a^{(\mu)}$ and hence the relation

$$B(\tau \to \mu\gamma) = \left[\frac{m_\tau}{m_\mu}\right]^5\left[\left[\frac{m_\tau}{m_e}\right]^2(1+r^2)^{-1}\right]\left[\frac{\Gamma_\mu}{\Gamma_\tau}\right]B(\mu \to e\gamma) \qquad (12)$$

or $B(\tau \to \mu\gamma) \simeq 2\times10^5 B(\mu \to e\gamma)$. Thus the theory predicts a definite relation between the two lepton-number-violating decays. The present bounds on $\mu \to e\gamma$ then imply $B(\tau \to \mu\gamma) \lesssim 1\times10^{-5}$ which can be compared with the direct experimental bound[11] of $B(\tau \to \mu\gamma)$

TABLE III. Values for a for $m_{\tilde{e}} = 60$ GeV and $r = 3.5$ for various values of $\hat{\nu}^c_2$ mass $\hat{m}$.

$\hat{m}$ (GeV)	a (theory)
60	$(2.3\times10^{-13})\varepsilon^4$
300	$(3.0\times10^{-14})\varepsilon^4$
1000	$(3.3\times10^{-15})\varepsilon^4$

$<6\times 10^{-4}$. The $\tau\rightarrow\mu\gamma$ decay is a very clean signal at e^+-e^- machines and a strong bound should be obtainable from them. Thus CLEO has a sample of about 10^6 τ's and should be able already to improve the existing bounds by a factor of 10 or more. τ/charm factories may be expected to accumulate a data sample of 10^8 τ's and a B factory perhaps as many as 10^8–10^9 τ's. Hadron colliders also are a copious source of τ leptons from B decays via $B\rightarrow\tau+X$. With the luminosity of $\mathcal{L}=10^{31}$ cm^{-2}s^{-1} expected for the 1991 run of the Fermilab Tevatron, one roughly estimates a data sample of 10^7 τ's. A similar analysis for the Superconducting Super Collider for a luminosity of $\mathcal{L}=10^{33}$ cm^{-2}s^{-1} (and a B-production cross section of $\sigma_B\sim 1$ mb) yields about 10^{12} τ's/yr. We note that a test of the $\tau\rightarrow\mu\gamma$ branching ratio at the 10^{-10} level corresponds, by Eq. (12), to testing $\mu\rightarrow e\gamma$ at the 5×10^{-16} level. Of course there are serious hadronic backgrounds that must be overcome to detect the $\tau\rightarrow\mu\gamma$ decay at a hadronic machine, and a Monte Carlo simulation of these is currently under investigation for the Tevatron.[13]

In conclusion, E_6 string models obeying the hypotheses of the theorem stated in the opening section possess a number of predictions that may be experimentally testable. In particular, they lead to new physics of lepton-number violation not found in the standard model, giving rise to $\mu\rightarrow e\gamma$ decay and the much larger $\tau\rightarrow\mu\gamma$ decay that may be accessible in future experiments. String theory also predicts specific features of these decays: The outgoing lepton is almost 100% right handed. This leads to a characteristic angular distribution of the outgoing lepton relative to the spin of the initial lepton even if the spin of the outgoing lepton is not measured. (We note that at MEGA the muons are almost 100% polarized and decay at rest, allowing a test of this prediction.) The ratio of the $\tau\rightarrow\mu\gamma$ and $\mu\rightarrow e\gamma$ decay rates is determined, the former being 10^5 times larger. Thus there are a number of specific features of these decays that could be used to single out E_6 string models from other possible non-standard-model theories. Finally, we note that the theory naturally relates the suppression of the $\mu\rightarrow e\gamma$ decay below existing experimental bounds to the smallness of m_e/m_τ. This ratio is also the reason neutrino masses are much smaller than charged-lepton masses[2] and governs the pattern of neutrino oscillations. A general discussion of neutrino masses and neutrino oscillations, which is another example of new-physics predictions from E_6 string models, will be given elsewhere.

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