Comment on "Soliton Solutions to the Gauged Nonlinear Schrödinger Equation on the Plane"

There has recently been considerable interest[1-3] in the derivation of static self-dual solutions of wave equations which contain a Chern-Simons term. Such calculations have been carried out within the framework of the photonless gauge theory formulated by this author.[4] The latter happens to be particularly convenient because of the fact that the vector potential in this model is explicitly calculable in terms of the various charge fields.

The results of Refs. 1–3 are derived by introducing a function ω which satisfies

$$\nabla\times\nabla\omega=\delta(\mathbf{x})\,. \tag{1}$$

While Eq. (1) does not appear explicitly in Ref. 3, it is essential to the consistency between Eqs. (9a) and (22). [Note also the discussion between Eqs. (20) and (21) as well as that in Ref. 5.] It is the purpose of this Comment to point out, first, that Eq. (1) would require a nonstandard interpretation in the context of distribution theory, and, second, that the equation is not essential to the results obtained.

To this end one notes that standard distribution theory implies that (1) should be interpreted by smearing out (1) with a test function $f(\mathbf{r})$ which satisfies $\nabla\times\nabla f=0$ everywhere and which goes to zero at $\mathbf{r}=\infty$. (A possible choice might be e^{-r^2}.) Integration by parts yields

$$0=\int d^2r\,\omega\nabla\times\nabla f=f(0)\,,$$

i.e., a contradiction.

However, compatibility with distribution theory can be restored by avoiding introducing the function ω. To this end one writes the self-duality condition of Ref. 3 as

$$\partial\Psi/\partial z_\pm=ie(A_x\mp iA_y)\Psi\,,$$

where $z_\pm=x\pm iy$. In the radiation gauge $eA_i=-\epsilon_{ij}\times\nabla_j\chi$ so that

$$\partial\Psi/\partial z_\pm=\pm(\partial\chi/\partial z_\pm)\Psi$$

which yields the solution $\Psi=e^{\pm\chi}C(z_\mp)$, where $C(z_\mp)$ is an arbitrary function of its argument. Thus $e\nabla\times\mathbf{A}=-e^2\Psi^*\Psi\equiv-e^2\rho$ becomes

$$\pm\nabla^2\ln[\rho/|C(z_\mp)|^2]=-2e^2\rho\,. \tag{2}$$

Since $\ln|C(z_\mp)|^2$ is a solution of Laplace's equation everywhere except at points where $C(z_\mp)$ vanishes, the interpretation of (2) and the significance of $C(z_\mp)$ is apparent. One has only to solve (2) neglecting possible zeros of ρ. Once ρ is found for the case $C=1$ [and for the upper sign choice in (2)] it can be determined that if ρ has zeros of arbitrary multiplicity n_i at arbitrary points z_{-i}, then $C(z_-)$ must be

$$C(z_-)=\prod_i(z_- - z_{-i})^{n_i/2}$$

to within a normalization.

Using the solution[3]

$$\rho=(4/e^2)|f'(z_+)|^2/[1+|f(z_+)|^2]^2\,,$$

where $f(z_+)$ is arbitrary, one finds for $\mathbf{A}$ the result

$$eA_i=-\epsilon_{ij}\nabla_j\{\ln|f'(z_+)|-\ln[1+|f(z_+)|^2]-\ln|C(z_-)|\}\,,$$

while for Ψ one has

$$\Psi=\rho^{1/2}\exp[i\arg C(z_-)]\,.$$

Finally, upon specializing to the choice made in Ref. 3

$$f(z_+)=(z_+/r_0)^n\,,$$

there results

$$\rho(r)=\frac{4n^2}{e^2}\frac{1}{r^2}\left[\left(\frac{r_0}{r}\right)^n+\left(\frac{r}{r_0}\right)^n\right]^{-2}.$$

Since, for $r\to0$, this goes as $r^{2|n|-2}$, it follows that in this case

$$C(z_-)=z_-^{|n|-1}$$

and thus

$$\Psi(\mathbf{r})=\frac{2n}{e}\frac{1}{r}\left[\left(\frac{r_0}{r}\right)^n+\left(\frac{r}{r_0}\right)^n\right]^{-1}e^{i[|n|-1]\theta}.$$

This is the same form for Ψ as found in Ref. 3 and one thus sees that the final results of Jackiw and Pi (and Refs. 1 and 2) can be rigorously established. This could be of considerable importance in the future if studying such solutions develops into an active field of endeavor.

This work is supported in part by the U.S. Department of Energy Contract No. DE-AC02-76ER13065.

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Received 18 July 1990
PACS numbers: 11.15.−q, 03.65.Ge

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