Coexistence of Self-Induced Transparency Soliton and Nonlinear Schrödinger Soliton

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A self-induced transparency (SIT) soliton can coexist with a nonlinear Schrödinger (NLS) soliton. This mixed state is called an SIT-NLS soliton. The phase change of the new soliton is governed solely by the NLS component and the pulse delay is determined solely by the SIT component when a detuning from the resonance is zero. It is shown for the first time that a stable 2π-($N=1$) SIT-NLS soliton exists and that high-order SIT-NLS solitons always split into multiple 2π-($N=1$) SIT-NLS solitons.

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When radiation is guided by a fiber waveguide structure, diffraction effects are eliminated and this makes it possible for the radiation to interact with the fiber medium with resonance effects over long distances.[1] Experiments on self-induced transparency (SIT) solitons have been reported by many authors.[2-4] From a purely scientific point of view, it would be desirable to perform an SIT experiment in an environment in which diffraction effects can be completely ignored.

More importantly, SIT offers the possibility of pulse shaping[5] and standardization that is different from the nonlinear Schrödinger (NLS) soliton formation.[6] Since some of the energy released in the reshaping of a pulse remains in the medium, and eventually decays via absorption processes, pulse shaping by an SIT soliton may yield cleaner pulses than those produced by NLS soliton formation. Fundamental work on the possibility of an SIT-NLS soliton was reported by Maĭmistov and Manykin in 1983.[7] However, its detailed physical mechanisms have not yet been clarified and even its existence has not been proven.

In the present paper, we show that SIT-NLS solitons exist and present a detailed physical interpretation of their nature.

The polarization currents due to SIT and self-phase modulation (SPM) appear as a sum on the right-hand side of Maxwell's equations. The slowly varying electric field $\tilde{E}(z,t)$ is given by

$$\frac{\partial \tilde{E}}{\partial z}+\frac{1}{v_g}\frac{\partial \tilde{E}}{\partial t}=-\frac{\omega\mu}{2k}\int \tilde{J}_s e^*\, dS+i\frac{1}{2}k''\frac{\partial^2 \tilde{E}}{\partial t^2}\,, \tag{1}$$

where $\tilde{J}_s$ is the slow envelope function, in space and time, of the nonlinear polarization density and e is the radial field distribution. $\tilde{J}_s$ consists of two parts. One is due to the SIT contribution and the other is the SPM contribution. The total nonlinear polarization is

$$-\frac{\omega\mu}{2k}\int \tilde{J}_s e^*\, dS=-\tfrac{1}{2}i\omega\left(\frac{\mu}{\varepsilon}\right)^{1/2}Np_{12}\langle v_1 v_2^*\rangle\frac{S}{(A_{\rm eff})^{1/2}} -i\frac{\omega}{c}\frac{n_2}{A_{\rm eff}}|\tilde{E}|^2\tilde{E}\,. \tag{2}$$

Here v_1 and v_2 are the wave functions in a two-level system,[8,9] N is the particle density, p_{12} is the matrix element of the two-level system, S is the cross section of the SIT core, $A_{\rm eff}$ is the cross section of the mode, and n_2 is the nonlinear index.

Introducing the transformation of variables $t-z/v_g=s$ and $z=z$, we obtain the following equation as an SIT-NLS equation:

$$i\frac{\partial \tilde{E}}{\partial z}=\frac{1}{2}\omega\left(\frac{\mu}{\varepsilon}\right)^{1/2}Np_{12}\langle v_1 v_2^*\rangle\frac{S}{(A_{\rm eff})^{1/2}} +\frac{\omega}{c}\frac{n_2}{A_{\rm eff}}|\tilde{E}|^2\tilde{E}-\frac{1}{2}k''\frac{\partial^2 \tilde{E}}{\partial s^2}\,. \tag{3}$$

Here k'' should be negative for the generation of the NLS soliton.

The wave functions v_1 and v_2 are rewritten in the form of $F=|v_2|^2-|v_1|^2=\rho_{22}-\rho_{11}$ and $M=v_1v_2^*=\rho_{21}$. This makes it possible to understand intuitively the way in which the phase rotation of the dipole changes with the existence or nonexistence of the NLS soliton. These are given by

$$\frac{\partial F}{\partial s}=\frac{1}{2}\left(\frac{2p_{21}}{\hbar i}\right)(\tilde{E}^*M-\tilde{E}M^*)\,, \tag{4}$$

$$\frac{\partial M}{\partial s}+2i\xi\Omega M=\left(\frac{2p_{21}}{\hbar i}\right)\tilde{E}F\,, \tag{5}$$

where ξ is the normalized detuning and Ω is the normalization factor.[8] Equations (3)-(5) are a set of the general nonlinear pulse equations for SIT and NLS. To normalize the coupled soliton equation, we use the following transformations:

$$\tilde{E}=\tilde{A}/(\tfrac{1}{2}\varepsilon_0 cnA_{\rm eff})^{1/2},\quad u=\tilde{A}/(P_{0({\rm NLS})})^{1/2}, \quad x=s/\tau_s,\quad q=z/Z_0\,, \tag{6}$$

where

$$\kappa P_{0({\rm NLS})}=\frac{1}{Z_0},\quad \kappa=\frac{\omega}{c}\frac{n_2}{A_{\rm eff}},\quad Z_0=\frac{\tau_s^2}{|k''|}\,.$$

Thus we obtain

$$\frac{\partial F}{\partial x}=\tfrac{1}{2}i(2W)(uM^*-u^*M)\,, \qquad (7)$$

$$\frac{\partial M}{\partial x}+2i\xi\Omega M=-2iWuF\,, \qquad (8)$$

$$-i\frac{\partial u}{\partial q}=\frac{Z_0}{(P_{0(\mathrm{NLS})})^{1/2}}\frac{1}{2}\omega Np_{12}\left(\frac{A_{\mathrm{eff}}}{2\varepsilon_0 cn}\right)^{1/2}\langle M\rangle+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+|u|^2u\,, \qquad (9)$$

where we put $S=A_{\mathrm{eff}}$ for simplicity, and $\omega t-kz$ is replaced with $kz-\omega t$ for convenience, so that $i\,\partial u/\partial q \rightarrow -i\,\partial u/\partial q$. W satisfies

$$P_{0(\mathrm{NLS})}=W^2\tfrac{1}{2}\varepsilon_0 cn(\hbar/p_{21}\tau_s)^2A_{\mathrm{eff}}\,. \qquad (10)$$

Since $\frac{1}{2}\varepsilon_0 cn(\hbar/p_{21}\tau_s)^2A_{\mathrm{eff}}$ is equal to $P_{0(\mathrm{SIT})}$,

$$W^2=P_{0(\mathrm{NLS})}/P_{0(\mathrm{SIT})}\,. \qquad (11)$$

It should be noted that $2u$ satisfies 2π pulses when $P_{0(\mathrm{NLS})}=P_{0(\mathrm{SIT})}$. Thus one obtains

$$n_2=\frac{c|p_{21}|^2|k''|}{\omega\hbar^2}\quad\text{or}\quad |k''|=\frac{n_2\omega\hbar^2}{c|p_{21}|^2}\,. \qquad (12)$$

These results indicate that when W is equal to unity, $2u$ in (7) and (8) and the SIT part of (9) completely describe SIT and u satisfies the NLS soliton.

For the SIT soliton, the phase rotation $\phi(q)$ in the z direction can be taken as an arbitrary value. Therefore, it is possible to replace M with $Me^{i\phi(q)}$. This transformation is useful in understanding how the phase change in the SIT soliton is affected by the NLS soliton. We assume here a normalized SIT-NLS solution of the following form since both the SIT and the NLS equations give rise to a sech pulse solution:

$$u=2\eta\,\mathrm{sech}\,2\eta(x-\delta q)e^{i\alpha q}\,. \qquad (13)$$

Substituting (13) into (9), we obtain

$$\alpha=\tfrac{1}{2}(2\eta)^2 \qquad (14)$$

from the sech(x) term. This means that the phase factor is determined only by the NLS part.

The phase term is given by

$$\phi(q)=-\pi/2+\alpha q \qquad (15)$$

from the tanh(x)sech(x) term for homogeneous broadening. The phase difference at $q=0$ between the dipole and the input field is $-\pi/2$, which is the inherent nature of the dipole transition. Furthermore, the z dependence of the phase of the dipole is solely determined by the nonlinear phase change due to the NLS soliton. One also obtains

$$(2\eta)^2\delta=\frac{Z_0}{(P_{0(\mathrm{NLS})})^{1/2}}\frac{1}{2}\omega Np_{12}\left(\frac{A_{\mathrm{eff}}}{2\varepsilon_0 cn}\right)^{1/2}, \qquad (16)$$

which means that for a zero detuning the pulse delay is determined solely by the SIT component.

Here we show numerically that the stable SIT-NLS soliton exists. The Runge-Kutta method was used to calculate the time dependences of (7) and (8). These SIT equations couple with the main Eq. (9) through the z dependence. Here for simplicity, the coefficient of $\langle M\rangle$ is replaced with δ. The three-point differential method[10] was used rather than the beam propagation method,[11] and $\langle M\rangle$ is replaced with M by assuming a homogeneous broadening. The initial conditions are $\rho_{22}=0$, $\rho_{11}=1$, $\rho_{12}=0$, and $u=N\,\mathrm{sech}(x)$. The midpoint method was adopted rather than the conventional Euler method to increase accuracy in the propagation.[11]

As given in (10) and (11), if W is not equal to unity, in other words if the 2π SIT does not correspond to the $N=1$ NLS soliton, a stable SIT-NLS soliton cannot exist. That is, an arbitrary u is acceptable for (9) since there is no explicit expression of u for M. However, regarding the SIT soliton propagation, $W=1$ is the only acceptable condition for (7) and (8) which will maintain 2π pulses.

The propagation of stable SIT-NLS solitons is shown in Fig. 1. Figure 1(a) is a 2π SIT soliton with $\delta=1$. With the addition of the NLS component as shown in Fig. 1(b), a stable 2π-($N=1$) SIT-NLS soliton propagates. The delay for the 2π-($N=1$) SIT-NLS soliton is exactly the same as that for a 2π SIT soliton. This

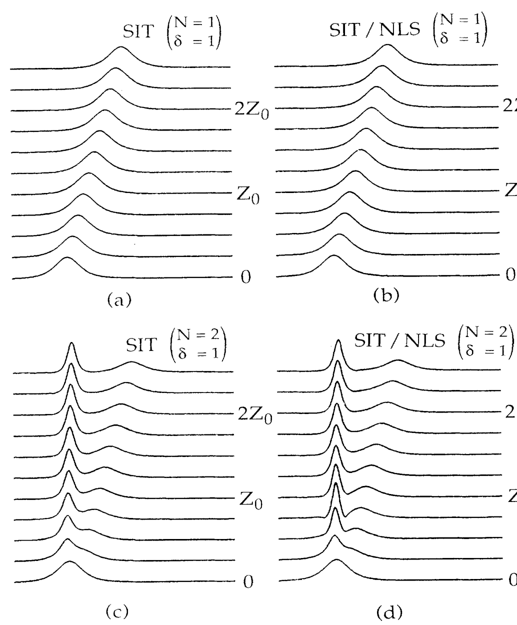

FIG. 1. Stable 2π-($N=1$) and 4π-($N=2$) SIT-NLS soliton propagations for $\delta=1$. (a) 2π SIT, (b) 2π-($N=1$) SIT-NLS, (c) 4π SIT, (d) 4π-($N=2$) SIT-NLS solitons. The delays in (a) and (c) are the same as those in (b) and (d), respectively. Z_0 is the normalized distance.

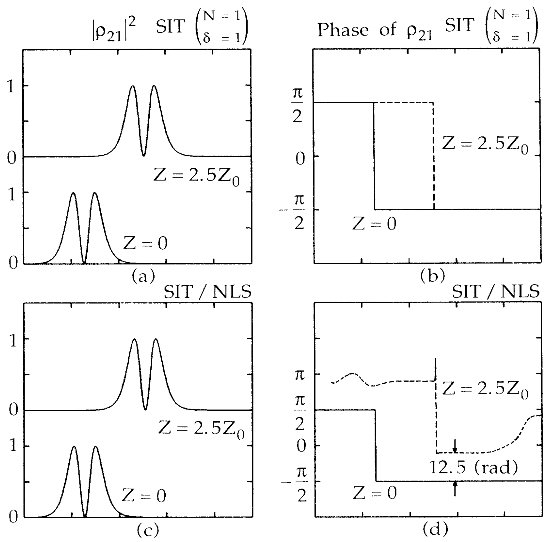

FIG. 2. Comparison of dipole changes for a 2π SIT soliton and a 2π-($N=1$) SIT-NLS soliton. $|\rho_{21}|^2$ and the phase of ρ_{21} for 2π SIT are shown in (a) and (b), respectively. Those for a 2π-($N=1$) SIT-NLS soliton are shown in (c) and (d). A nonlinear phase change which arises from the NLS part appears in (d).

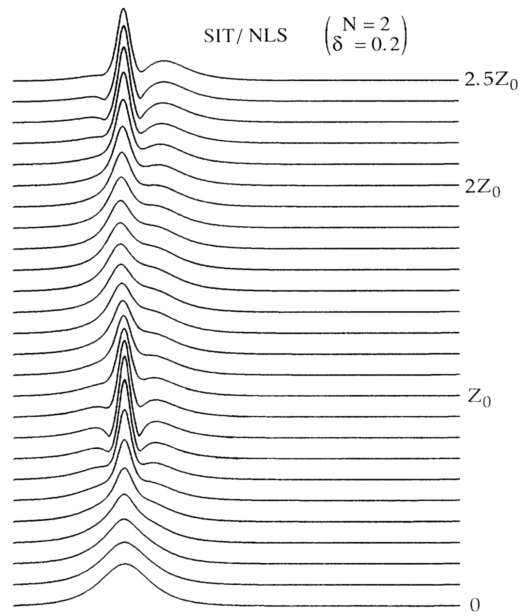

FIG. 3. Interaction between the SIT part and the NLS part in a 4π-($N=2$) SIT-NLS soliton with $\delta=0.2$. A small δ gives a longer interaction length. A wave-form change due to the $N=2$ NLS soliton is clearly seen, accompanying a pulse splitting on the right wing of the main pulse due to the SIT effect.

agrees with the theory described earlier in this paper.

Wave-form changes for a 4π SIT soliton and a 4π-($N=2$) SIT-NLS soliton are shown in Figs. 1(c) and 1(d). The 4π-($N=2$) SIT-NLS soliton eventually splits into two 2π solitons, even in the presence of the NLS component. The soliton cannot preserve only the NLS soliton property. When we look closely at transient wave-form changes during the first Z_0, a wave-form change caused by the NLS part can be found by comparison with Fig. 1(c). However, this does not influence the pulse delay due to the SIT effect. For 2π SIT and 2π-($N=1$) SIT-NLS solitons, the population ($\rho_{22}-\rho_{11}$) changes from -1 to 1 by the π pulse and it returns to -1 by the remaining π-pulse component. For 4π SIT and 4π-($N=2$) SIT-NLS solitons, the population changes twice between -1 and 1.

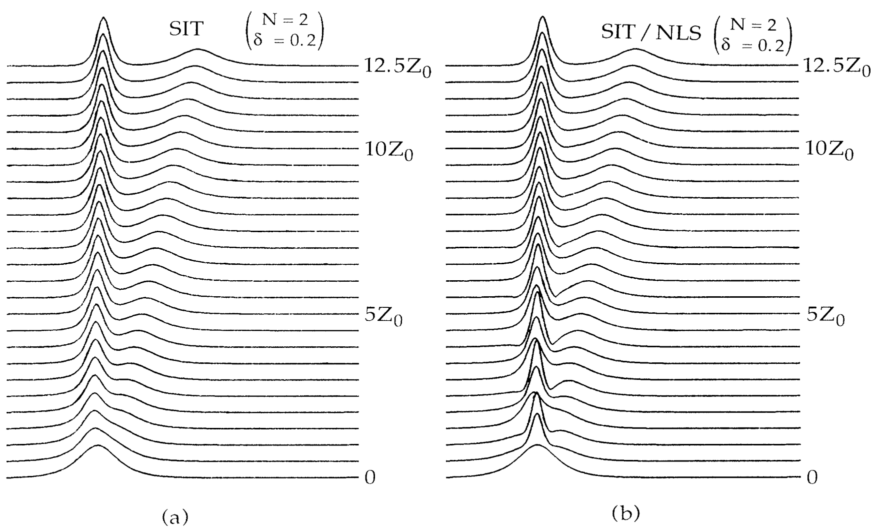

FIG. 4. Comparison of the pulse delays of a 4π SIT soliton and a 4π-($N=2$) SIT-NLS soliton with $\delta=0.2$. The delay for the SIT-NLS is identical to that of the SIT soliton.

We investigated $|\rho_{21}|^2$ and the phase of ρ_{21} as shown in Figs. 2(a) and 2(b), respectively. There is a $-\pi/2$ phase change in the field amplitude peak, which agrees with our result given in (15). This is the inherent phase difference between the electric field and the phase of the dipole moment. However, it should be noted that there is no phase change due to the pulse propagation as shown by the dashed line of Fig. 2(b). Although $|\rho_{21}|^2$ is the same as that of 2π SIT for 2π-($N=1$) SIT-NLS solitons, the phase of ρ_{21} is different from that of 2π SIT, as shown in Fig. 2(c). The phase rotation is determined by the nonlinear phase change due to the NLS soliton part. From (13) and (14) $\phi(z)=\frac{1}{2}(2\eta)^2 z/Z_0=1.25$ rad for $\eta=\frac{1}{2}$ and $z=2.5Z_0$, which agrees well with the present numerical result.

In high-order SIT-NLS solitons, $|\rho_{21}|^2$ changes very rapidly between 0 and 1, and the phase of ρ_{21} also changes between $\pi/2$ and $-\pi/2$. The phase shifts of the 2π solitons split from 4π-($N=2$) SIT-NLS and 6π-($N=3$) SIT-NLS solitons are also 1.25 rad at $z=2.5Z_0$.

Figure 3 shows the wave-form changes of a 4π-($N=2$) SIT-NLS soliton with $\delta=0.2$, in which the soliton period is shorter than the absorption length. In one normalized distance from $z=0$ to $z=Z_0$, it is clearly seen that an $N=2$ NLS soliton is excited, and that the hump on the left-hand side of the pulse peak is growing, resulting in the pulse splitting due to the SIT component. In this case, the wave-form changes due to the $N=2$ NLS soliton property dominate in the early stage, but eventually the pulse splits into two 2π solitons.

To investigate the delay difference between the SIT soliton and the SIT-NLS soliton, 4π SIT and 4π-($N=2$) SIT-NLS solitons with $\delta=0.2$ are propagated over $12.5Z_0$, as shown in Fig. 4. As a result, it is found that the delay is the same as that of a 2π SIT soliton. This is surprising because when the NLS term has no effect on the delay even the SIT-NLS soliton experiences a strong interaction between the SIT and the NLS solitons over long distances.

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