Nonperturbative Character of Electron-Positron Pair Production in Relativistic Heavy-Ion Collisions

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Coupled-channel calculations for relativistic heavy-ion collisions are performed in order to determine the electromagnetic electron-positron pair production yield with subsequent capture of the electron into the K shell of the target. We consider heavy projectiles impinging on Pb^{82+} and U^{92+} with small impact parameters at a bombarding energy of 1.2 and 2 GeV/nucleon, respectively. The dependence of the positron yields on the charge number of the projectile exhibits a strong nonperturbative feature. The corresponding result obtained within first-order perturbation theory is exceeded by about 2 orders of magnitude.

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During recent years the interest in electron-positron pair production in relativistic heavy-ion collisions increased steadily due to the progress in the design of heavy-ion accelerators and experimental facilities. For example, electromagnetic production of lepton pairs is considered as a competing process to the hadronic production of Drell-Yan pairs. At ultrarelativistic energies e^+-e^- pair production with capture of the electron is of definite interest for heavy-ion colliders since the ionic charge state is altered by this process and thus the ion gets lost from the beam.[1]

At present, most theoretical calculations are based on different variants of perturbation theory[2-5] or (in the high-energy limit) on the equivalent-photon method.[6,7] A review over the recent development in this field has been presented by Eichler.[8] Since at moderate relativistic velocities the pair-creation probabilities are small, perturbation theory has been considered to be rather reliable for the theoretical description. The main difference between the various approaches based on perturbation theory consists in the use of different wave functions for the electron and positron. At higher projectile energies the equivalent-photon method, also called the Weizsäcker-Williams method, is frequently employed. Because of the Lorentz contraction the electromagnetic field of the heavy ions can be treated as a pulse of photons. Thus, the electromagnetic production of particles is calculated utilizing the photoproduction cross section folded with the number of photons contained in the pulse.

In this Letter we want to demonstrate that coupled-channel calculations result in a large enhancement by about 2 orders of magnitude of the positron yield compared with the outcome of perturbation theory. Recently, a similar result has been reported by Strayer *et al.*[9] who solved the time-dependent Dirac equation using B splines on a spatial lattice. Using model potentials they calculated μ^+-μ^- pair-creation probabilities in relativistic heavy-ion collisions with capture of the produced μ^- in the ground state. Strayer *et al.* got several orders of magnitude more μ^+-μ^- pairs compared with corresponding results from perturbation theory.

To determine the pair-creation probabilities in relativistic ion-atom collisions we solve the time-dependent Dirac equation for the electron field,

$$i\partial_t \psi_s(\mathbf{r},t) = [\boldsymbol{\alpha}\cdot\mathbf{p} + \beta - eV_T(\mathbf{r}) + e\boldsymbol{\alpha}\cdot\mathbf{A}_P(\mathbf{r},t) - eV_P(\mathbf{r},t)]\psi_s(\mathbf{r},t)\,, \qquad (1)$$

where $\mathbf{A}_P$ and V_P denote the electromagnetic potentials of the impinging projectile.

We assume that the projectile moves classically on a straight line parallel to the z axis; i.e., we employ the semiclassical approximation. The potentials $\mathbf{A}_P$ and V_P can be calculated by performing a Lorentz transformation from the projectile's rest frame into the rest frame of the target. This procedure yields the Liénard-Wiechert potentials

$$V_P(\mathbf{r},t) = \gamma Z_P e/r' \qquad (2)$$

and

$$\mathbf{A}_P(\mathbf{r},t) = v_P\gamma(Z_P e/r')\mathbf{e}_3\,, \qquad (3)$$

with Z_P being the charge number of the projectile and

$$r' = [(x-b)^2 + y^2 + \gamma^2(z - v_P t)^2]^{1/2}\,, \quad \gamma = 1/(1-v_P^2)^{1/2}\,. \qquad (4)$$

x,y,z refer to a target-centered coordinate system. b denotes the impact parameter and v_P the projectile velocity. Equation (1) is solved by expanding ψ_s in the complete orthonormal set of eigenstates $\phi_k(\mathbf{r}) \times\exp(-iE_k t)$ of the Hamiltonian of the target system,

$$H_0=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta-eV_T(\mathbf{r})\,. \quad (5)$$

Insertion of the expansion

$$\psi_j(\mathbf{r},t)=\sum_k a_{jk}(t)\phi_k(\mathbf{r})\exp\{-iE_k t\} \quad (6)$$

into the Dirac equation and projection leads to the infinite system of first-order coupled differential equations for the occupation amplitudes $a_{jk}(t)$,

$$\dot{a}_{jf}(t)=-i\sum_k a_{jk}(t)\langle\phi_f|\hat{H}_P|\phi_k\rangle\exp\{i(E_f-E_k)t\}\,. \quad (7)$$

Since we are interested in electron-positron pair creation, in addition to the positive-energy continuum we also have to include the negative-energy continuum for the description of the positron.

Both continua are discretized by the use of relativistic wave packets[10]

$$\phi_{E_k}(\mathbf{r})=\frac{1}{\sqrt{\Delta E}}\int_{E_k-\Delta E/2}^{E_k+\Delta E/2} dE\,\phi_E(\mathbf{r})\,. \quad (8)$$

The matrix elements $\langle\phi_f|\hat{H}_P|\phi_i\rangle$ are calculated numerically in coordinate space. For details we refer to Mehler *et al.*[11]

The coupled-channel equations (7) are integrated numerically with the initial conditions

$$\psi_j(\mathbf{r},t)\underset{t\to-\infty}{\longrightarrow}\phi_j(\mathbf{r})\exp\{-iE_j t\}\,, \quad (9)$$

or equivalently

$$a_{jf}\underset{t\to-\infty}{\longrightarrow}\delta_{jf}\,. \quad (10)$$

The amplitudes a_{jf} determine the motion of a single electron during the collision. Since the negative-energy continuum is initially fully occupied by electrons, we have to deal with a many-electron system. Neglecting the electron-electron interaction the electrons are influenced by each other only through the Pauli principle. These effects can be taken into consideration by using the framework of second quantization.[12] It results that the number of electrons excited into an initially vacant level ϕ_P reads

$$N_p=\sum_{r<F}|a_{rp}|^2,\quad p>F\,, \quad (11)$$

and the number of holes in an initially occupied state ϕ_q is

$$N_q=\sum_{s>F}|a_{sq}|^2,\quad q<F\,, \quad (12)$$

where F is the Fermi level of the system, which is $E_F=-mc^2$ in the case of pair production. Employing the time-reversal symmetry, the number of electrons excited into the $1s$ state can be simplified to

$$N_{1s}=\sum_{r<F}|a_{1s,r}|^2\,. \quad (13)$$

Thus, for the calculation of the pair-production probability with capture of the electron into the ground state only a single coupled-channel integration for an electron initially occupying the $1s$ state is required. Using Eq. (11) the ionization probability for a single K-shell electron can be calculated by the same integration when contributions from the negative-energy continuum are neglected.

We performed this calculation for several collision systems $Z_P+\mathrm{Pb}^{82+}$ at a bombarding energy of 1.2 GeV/nucleon with various charge numbers of the projectile up to $Z_P=100$. Impact parameters up to $b=500$ fm were considered. Our basis set consists of 334 basis states (comprising the magnetic substates) including the 22 lowest bound states. The positive- and negative-energy continua are discretized by wave packets with an energy width $\Delta E=0.2$ and Dirac angular momentum quantum numbers $\kappa=\pm1,\pm2$. The energy covers the range from 1.1 to 3.5 and from -1.1 to -3.5 in natural units. This basis set includes the wave functions which have been found to be dominant in first-order perturbation theory for the impact-parameter range considered in this Letter. However, the inclusion of even more states, in principle, could change our results slightly.

In Fig. 1 we show the time evolution of the occupation probabilities for $Z_P=82$ and $b=15$ fm. In Figs. 1(b) and 1(c), the lines labeled "bound states" represent the sum over all bound states. The lines labeled "continuum ($E>+1$)" indicate the sum over all ionization channels, $\sum_{E_i>1}|a_{1s,E_i}|^2$, while the curves labeled "continuum ($E<-1$)" depict the contribution of the negative-energy continuum, $\sum_{E_i<-1}|a_{1s,E_i}|^2$. Thus, the value of the latter curves at $t\to\infty$ equals the probability for the creation of an electron-positron pair during the collision with capture of the electron into the $1s$ state.

Figure 1(a) displays the occupation probabilities obtained in first-order perturbation theory. The results of coupled-channel calculations are presented in Figs. 1(b) and 1(c). In Fig. 1(b), continuum-continuum interactions have been neglected, while in Fig. 1(c) all interactions have been incorporated. We notice that at the distance of closest approach at $t=0$, very strong contributions to the ionization as well as to the positron channels occur. In perturbation theory, the content of the positron channels decreases by 3 orders of magnitude on the outgoing path of the trajectory. This decrease strongly deviates from corresponding coupled-channel calculations. Here, the capture probability remains almost at the maximum value and thus the probability exceeds the perturbative probability by a factor of 50. This effect is observed in both coupled-channel calculations.

Additionally, we repeated the calculations for several projectiles for the impact parameter $b=193$ fm, which is half the Compton wavelength of the electron. The re-

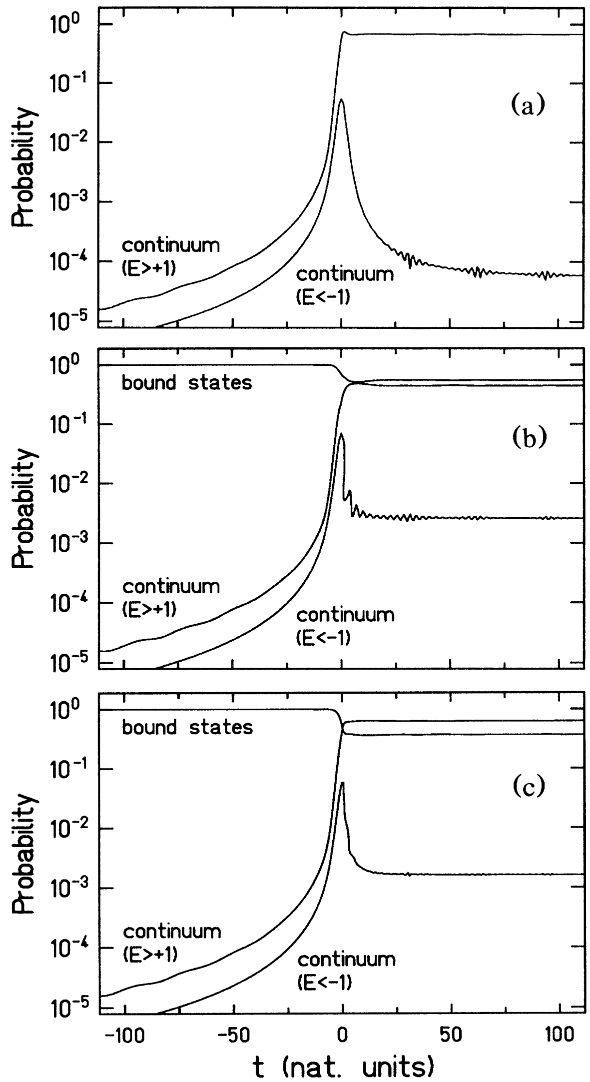

FIG. 1. Time evolution of the occupation probabilities of various channels in a Pb+Pb collision at 1.2 GeV/nucleon. The considered impact parameter is $b=15$ fm. The curves represent the sum over all bound states, the sum over the positive-energy continuum ($E>+1$), and the sum over the negative-energy continuum ($E<-1$), respectively. The sum over the negative-energy continuum is equal to the probability for pair creation with capture of the electron into the 1*s* state. (a) Perturbation theory, (b) coupled-channel calculation, continuum-continuum interactions neglected, and (c) coupled-channel calculation, all couplings included. The exponential function in Eq. (7) is responsible for the small oscillations and modulations on the outgoing branch of the lower curves.

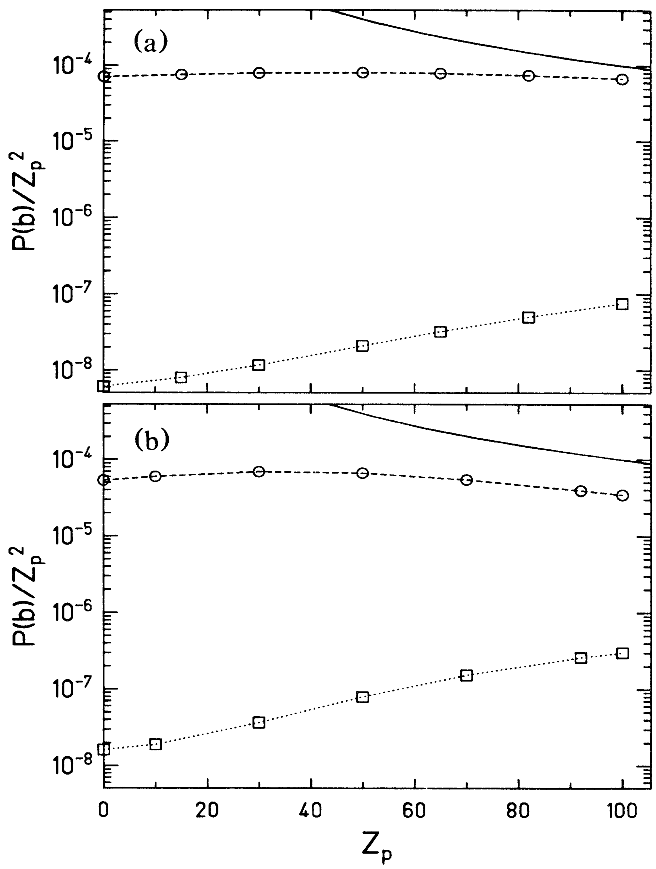

FIG. 2. (a) Z_P dependence of the pair-creation probability with capture of the electron into the K shell and the ionization probability of the K-shell electron in the collision $Z_P+\mathrm{Pb}^{82+}$ at 1.2 GeV/nucleon. The considered impact parameter is $b=193$ fm. To demonstrate the nonperturbative behavior, the probability is divided by Z_P^2. Dashed line with circles, ionization; dotted line with squares, pair creation; solid line, unitarity limit. (b) Same as in (a) for the collision $Z_P+\mathrm{U}^{92+}$ at 2 GeV/nucleon.

sults are shown in Fig. 2(a). The pair-creation probabilities as well as the ionization probabilities have been divided by Z_P^2, which is the scaling behavior in perturbation theory. Since the scaled outcome of first-order perturbation theory is independent of Z_P, the data point at $Z_P \to 0$ is obtained from perturbation theory. In this limit of vanishing projectile potential this result exactly equals the value obtained from coupled-channel calculations. The data for higher Z_P are the outcome of coupled-channel calculations. We note an approximate exponential increase of the pair-creation probability over the perturbative yield. At $Z_P=100$ this increase amounts to 1 order of magnitude. In contrast to this, the ionization probability is nearly constant as a function of Z_P.

In order to confirm the results and to test the computer code, comparative calculations have been performed using a slightly different formalism.[13] In this approach the wave packets were chosen to be time dependent. Furthermore, the method for the evaluation of the matrix elements is based on a more analytical integration over the electronic coordinates. The total e^+-e^- yield was found to be in good agreement in both calculations.

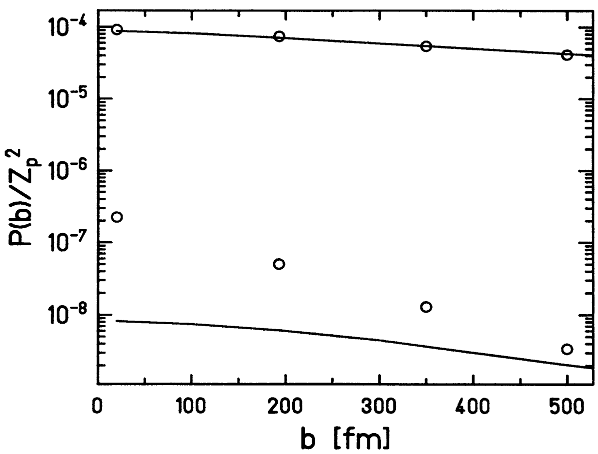

FIG. 3. Impact-parameter dependence of the pair-creation probability with capture of the electron into the K shell (lower curve) and of the ionization probability of the K-shell electron (upper curve). The considered collision system is Pb+Pb at 1.2 GeV/nucleon. Solid line, perturbation theory; circles, coupled-channel calculation.

Since this second independent program can be used also at higher energies, we want to present results for the collision system Z_P+U^{92+} at $E_{\text{lab}}=2$ GeV/nucleon and for the same impact parameter. The basis set for this calculation consists of the K and L shells for the bound states and wave packets of width $\Delta E=0.4$ at six energies for the positive continuum ($E_j=1.2,\ldots,3.2$) and ten energies for the negative continuum ($E_j=-1.2,\ldots,-4.8$) with $\kappa=-1,+1,-2$.

Within this basis set the Z_P dependence of the electron-positron yield was studied. The result is displayed in Fig. 2(b). For projectile charges up to $Z_P=10$ the pair-creation probability exhibits an almost perturbative behavior, while for larger values of Z_P the increment is approximately exponential. For the symmetric system at $Z_P=92$ the probability for pair production is larger by more than 1 order of magnitude compared with first-order perturbation theory.

The comparison with the corresponding results for the lower bombarding energy of 1.2 GeV/nucleon indicates that this effect remains observable with increasing collision energy. Our calculations cannot be extended to collision energies of more than about 10 GeV/nucleon, since perturbation theory indicates that higher angular momentum contributions become more and more important with increasing bombarding energy. However, coupled-channel calculations using still much larger basis systems are practically not feasible.

Figure 3 indicates that the large difference between coupled-channel calculations and perturbation theory occurs only at relatively small impact parameters. The K-shell ionization and pair-creation probability are plotted as functions of the impact parameter b for the system Pb+Pb at 1.2 GeV/nucleon. Apparently, the difference between perturbation theory and coupled-channel calculations decreases exponentially with increasing impact parameter and practically vanishes at $b\approx 500$ fm. For the total cross section for pair creation with capture we find in perturbation theory $\sigma_{\text{pair}}=0.30$ b while the coupled-channel calculations yield $\sigma_{\text{pair}}=1.48$ b. Thus, the total cross section is underestimated in perturbation theory by a factor of 5.

We conclude that the strong increase of the pair-creation probability compared to perturbation theory for high-Z projectiles is observable only for small impact parameters. Thus, for a clear experimental detection the impact parameter should be determined, e.g., by the measurement of the target recoil or the projectile deflection.

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