Coexistence of Regular and Chaotic Scattering in Heavy-Ion Collisions

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Classical dynamics of heavy-ion scattering is investigated in the case of a collision between a supposed spherical nucleus, ^{28}Si, and a deformed one, ^{24}Mg, at energies above the Coulomb barrier. Evidence of regular and irregular motion is found. The chaotic behavior justifies the presence of Ericson's fluctuations observed for this reaction, while the presence of regular motion embedded in the chaotic region could be the crucial point to explain the nature of the observed isolated resonances, once the semiclassical theory is applied.

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Chaotic scattering in classical dynamics has been recognized[1] to be a common feature of simple nonintegrable systems with a few degrees of freedom. This phenomenon in unbounded systems is the counterpart of the chaotic dynamics encountered in bounded systems. In the simple case of potential scattering the chaotic regime is characterized by deflection functions which oscillate on any scale of the initial boundary conditions. These oscillations are associated with trajectories which remain trapped for an exceedingly long time inside the interaction region. For very long times these trajectories approach and stay close to a set of unstable trapped orbits which form a fractal set in phase space, the so-called "strange repeller," and from which unbounded orbits must finally escape. In this sense the deflection function has a fractal structure. Another feature which often appears in chaotic scattering is the occurrence of "islands" of regular scattering embedded also in the irregular region: Usually these islands persist on all scales of the initial conditions. Semiclassical quantization of systems which display chaotic scattering leads to an S matrix which belongs to the random-unitary-matrix ensemble[1] and it has been argued that the corresponding fluctuations in the cross sections can be a model of Ericson's fluctuations[2,3] which have been well known in nuclear reactions for many years.

In this paper, we show that in the classical treatment of the collision between a deformed and a spherical nucleus, chaotic scattering is present with all the abovementioned features, provided the following two conditions are met: (i) The scattering is close to the grazing condition and (ii) the ions are sufficiently light. In the present treatment, other degrees of freedom different from rotational ones are excluded and absorption is neglected. Though the classical treatment is not fully justified, it can shed light on the dynamics of ion-ion collisions and it is the basis for the semiclassical approach. As a typical case we take the system $^{28}\text{Si}+{}^{24}\text{Mg}$. For simplicity we consider planar geometry. For a given total angular momentum projection L_{tot} along an axis perpendicular to the reaction plane the Hamiltonian of the system can be written as

$$H=\frac{p^2}{2m}+\frac{I^2\hbar^2}{2\mathcal{I}}+\frac{(L_{\text{tot}}-I)^2\hbar^2}{2mr^2}+V(r,\theta)\,, \qquad (1)$$

with

$$V(r,\theta)=\frac{Z_1Z_2e^2}{r}+\frac{Z_1e^2Q_0}{2r^3}P_2(\cos\theta)+U_N(r,\theta)\,, \qquad (2)$$

where m is the reduced mass, p the radial relative momentum, r the relative distance between the mass centers of the two nuclei, θ the rotational angle, Z_1 and Z_2 the charge of the supposed spherical (^{28}Si) and deformed (^{24}Mg) nucleus, Q_0 the intrinsic quadrupole moment of ^{24}Mg, I the angular momentum of the rotator, $\mathcal{I}$ the corresponding moment of inertia, and P_2 the Legendre polynomial of order 2. The ion-ion potential U_N is chosen to be the proximity one,[4] which is well suited for calculating the interaction between deformed nuclei.[5] That is,

$$U_N(r,\theta)=4\pi\gamma\mathcal{R}b\Phi(s(\theta))\,, \qquad (3)$$

$$s(\theta)=\frac{r-R_1-R_2(\theta)}{b}\,, \qquad (4)$$

$$R_2(\theta)=R_2^0[1+\alpha_{20}Y_{20}(\theta)]\,, \qquad (5)$$

$$\mathcal{R}=\frac{R_1R_2^0}{R_1+R_2^0}\left[1-\frac{2R_1}{R_1+R_2^0}\alpha_{20}Y_{20}(\theta)\right]\,, \qquad (6)$$

where Φ is the proximity function, γ is the surface tension, $b=1$ fm is the diffuseness parameter, R_1 is the radius of the spherical nucleus ^{28}Si, $R_2(\theta)$ is the radius which describes the quadrupole surface of ^{24}Mg, and α_{20} is the deformation parameter. The values of the different parameters are reported in Table I. Our Hamiltonian has three degrees of freedom, i.e., the relative distance r, the polar angle ϕ, and the rotational angle θ, and two constants of motion, i.e., the total energy E_{tot} and the total angular momentum L_{tot}. Neglecting the θ dependence of the full ion-ion potential, the Hamiltonian is separable and thus integrable, because the internal an-

TABLE I. The values used in the calculation for the surface tension γ (Ref. 4), the radii of the nuclei (Ref. 4), the deformation parameter α_{20} (Ref. 6), the intrinsic quadrupole moment Q_0 (Ref. 6), and the moment of inertia $\mathcal{J}$ (Ref. 7).

γ (MeV fm^{-2})	R_1 (fm)	R_2^0 (fm)	α_{20}	Q_0 (fm^2)	$\mathcal{J}\hbar^{-2}$ (MeV^{-1})
11.959	3.39	3.21	0.423	57	2.378

gular momentum I and the orbital one L are conserved separately. The θ dependence of the ion-ion potential introduces a symmetry breaking, causing the conservation of L_{tot} only. If the perturbation caused by the θ-dependent coupling term is small enough, the system is expected to be a mixed one.[8]

In Fig. 1 is shown the ion-ion potential for three values of relative angular momentum. The dashed curve refers to $\theta=0°$ while the solid one refers to $\theta=90°$. Integrating the equations of motion one gets the behavior shown in Figs. 2 and 3 for the final angular momentum of the rotator and for the final scattering angle. Because of the above-mentioned conservation of the total angular momentum, i.e., $L(t)+I(t)=L_{tot}$, the final scattering angle ϕ_f is obtained by integrating

$$\phi_f=\int \frac{L_{tot}-I}{mr^2}\,dt\,. \qquad (7)$$

The initial angular momentum of the rotator was $I_i=0$.

The integration was carried out starting from an initial distance of $r=18$ fm, up to the internal region, and then back until the distance was again 18 fm. There was also a time limit corresponding to $T_{max}=10^4$ fm/c, in case the trajectory was trapped inside the interaction region. Very few trajectories, however, remain trapped for so long. There are wild fluctuations in some regions due to irregular scattering and a smooth behavior in others. Moreover, there are regions of regular scattering embedded in the irregular one.

The large regular region beyond 60° for $L_{tot}=16\hbar$ corresponds to scattering from the tail of the potential

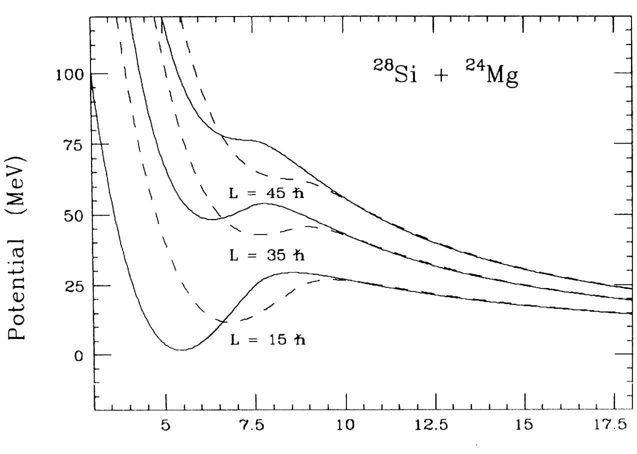

FIG. 1. The ion-ion potential $V(r,\theta)$ plus the centrifugal term of the Hamiltonian (1) for three different values of the relative angular momentum, $L=15\hbar$, $35\hbar$, and $45\hbar$, as a function of r and θ. The dashed curve refers to $\theta=0°$, while the solid one refers to $\theta=90°$.

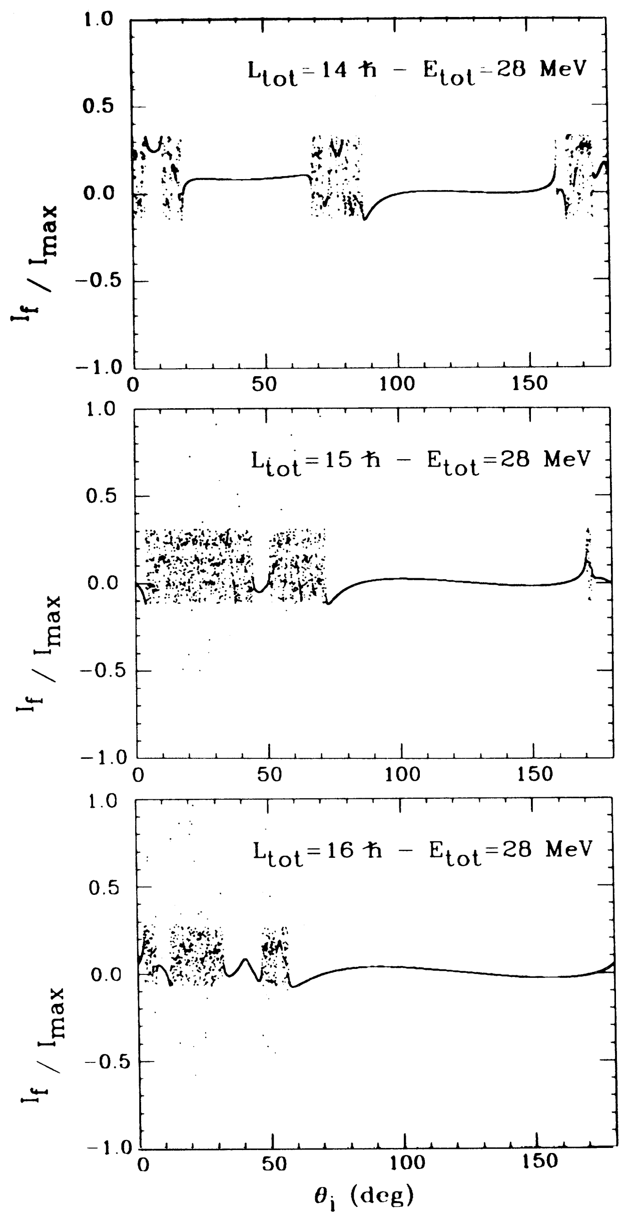

FIG. 2. For three different values of L_{tot}, i.e., $L_{tot}=14\hbar$, $15\hbar$, and $16\hbar$ and for $E_{tot}=28$ MeV, the final spin of the deformed nucleus ^{24}Mg divided by the maximum one, $I_{max}=(2\mathcal{J}E_{tot})^{1/2}$, as a function of the initial values of the rotation angle θ_i. The initial value of the spin was $I_i=0\hbar$. There is a symmetry of the reaction function around $\theta_i=180°$ because of the form of Y_{20}. See text for further details.

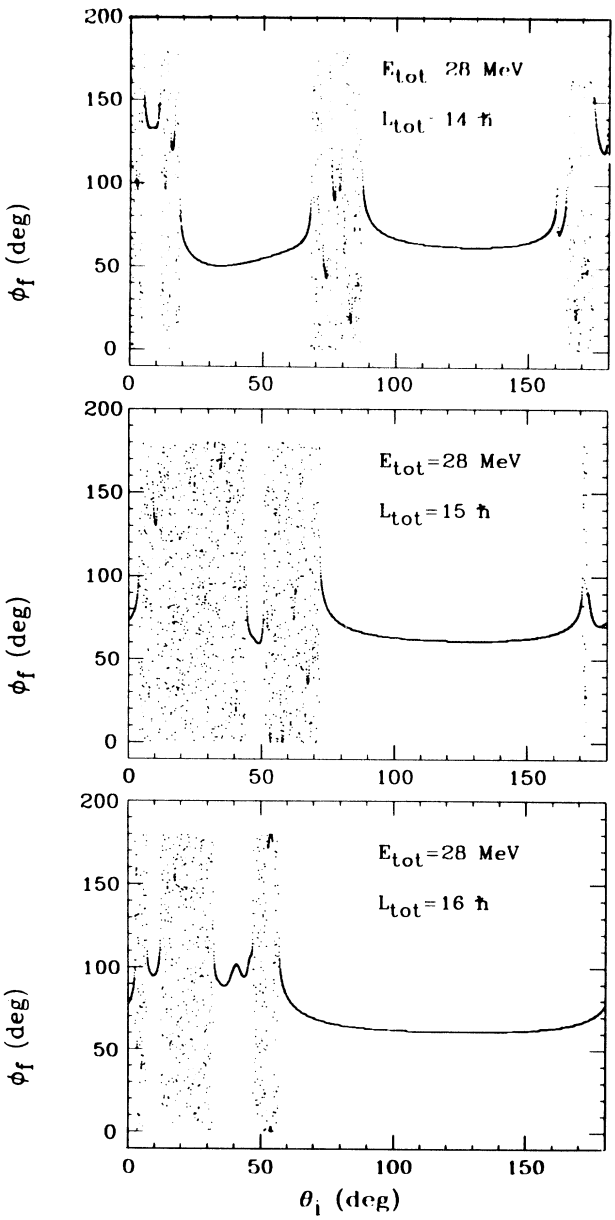

FIG. 3. The final scattering angle ϕ_f given by (7) as a function of θ_i for the initial relative angular momenta corresponding to those of Fig. 2.

which does not depend on the angle θ. In this region the Hamiltonian is quasi-integrable and the coupling term is nearly zero. Increasing the coupling term, the grazing condition is reached and symmetry breaking occurs: Our system shows a mixed behavior of regular and chaotic scattering. The islands of regular motion are very sensitive to the ion-ion potential and with increasing incident energy the chaotic scattering tends to disappear.

In Fig. 2 the final value of I_f never reaches the extreme values $\pm I_{\max} = \pm(2\mathcal{J}E_{\text{tot}})^{1/2}$. In general it is limited by two smaller values fixed by the shape of the potential, which determines the apparent band of points in the figures. Those isolated points which are outside this band are not real final values of I: They in fact correspond to trajectories trapped inside the pocket of the potential because the time limit was reached. In order to observe chaotic motion, the time associated with the rotation of the deformed nucleus should be comparable to the collision time. If the moment of inertia is too big, the change in the potential due to rotation would occur in a time too long compared to the collision time and the motion would be regular everywhere. This is one reason why light systems are favored. The other one is the fact that for these systems the quantal absorption is less than in heavier ones.

The simplest semiclassical quantization leads to the probability

$$P = \left| \sum_k (P_k)^{1/2} e^{i\Phi_k/\hbar} \right|^2, \tag{8}$$

where Φ_k is essentially the action integral, including the Maslov phase,[1,9] and P_k is the classical probability for the kth trajectory which gives the final classical state of the system. This treatment gives an excellent reproduction[5] of the full quantal result in the case of pure Coulomb excitation of rotational bands, provided it is analytically extended also into the classically forbidden region. The classical probability P_k is essentially given by the Van Vleck determinants[10] if the final variables are smooth functions of the initial boundary conditions. As shown in Ref. 1, in the case of irregular scattering, the averaged energy probability and the correlation function follow Ericson's fluctuation theory[2,3] provided the energy interval is properly chosen.

In the past, molecular resonances were found for light systems at energies above the Coulomb barrier.[11] The nature of these resonances has remained until now rather obscure. In particular, for the system $^{28}Si + ^{24}Mg$ isolated resonances embedded into a statistical noise (Ericson's fluctuations) have been discovered quite recently.[12,13] Qualitatively the nature of these isolated resonances can be explained with the presence of the above-discussed islands of regular trajectories inside the chaotic region. For those initial conditions which give regular scattering the system displays a quasi-integrable dynamics and the motion takes place on invariant surfaces. In the case of bounded systems generalized Bohr-Sommerfeld quantization rules inside the invariant surfaces give the semiclassical eigenenergies.[9] In the present case of unbounded scattering dynamics, similar considerations, applied to quasiclosed trajectories which explore the same phase-space region many times, should give the position of the isolated resonances. The proposed explanation is an alternative to the one invoking quasibounded quantal states, inside the pocket of a spherical potential, which decay by tunneling, and also can have some connection with resonances due to quantal coupled channels. However, for a quantitative analysis a careful study of the ion-ion potential and of the corresponding quantal scattering problem is required. This is beyond the aim of this paper.

Summarizing, we have shown that a classical description of a collision between a deformed and a spherical nucleus sufficiently light can show regions of regular and irregular motion around the grazing condition. This justifies the presence of Ericson's fluctuations and can be crucial for explaining the nature of experimental isolated resonances once the semiclassical theory is applied. Further investigation concerning the semiclassical limit is needed to reach a final conclusion. In particular, one should consider also the quantal absorption due to nonrotational degrees of freedom, which, however, for light systems should affect only the finest details of the dynamics.

The discussion has been restricted to planar geometry only, but the main qualitative features should hold also in the more general case. ^{28}Si is in reality a deformed system. We expect that taking into account also the ^{28}Si deformation would only increase the complexity of the dynamics without changing the qualitative chaotic behavior discussed. In order to investigate different systems, preliminary calculations for the scattering of spherical ^{16}O on the deformed ^{28}Si nucleus have also been performed, with results similar to those discussed above. In Ref. 14 a restricted coupled-channel treatment for this system has been considered. In principle, the phenomena discussed here for rotational degrees of freedom could also appear when considering vibrational ones. Some work is in progress in the latter case. It is also interesting to investigate the connections of the arguments discussed so far with the coherent fluctuations found in more dissipative heavy-ion reactions.[15]

[1]R. Blümel and U. Smilansky, Phys. Rev. Lett. **60**, 477 (1988); U. Smilansky, preliminary notes, Miniworkshop on Quantum Chaos, Trieste, 4–6 June 1990 (unpublished).

[2]T. Ericson, Phys. Rev. Lett. **5**, 430 (1960).

[3]D. Brink and R. Stephen, Phys. Lett. **5**, 77 (1963).

[4]J. P. Blocki, J. Randrup, C. F. Tsang, and W. J. Swiatecki, Ann. Phys. (N.Y.) **105**, 427 (1977).

[5]R. A. Broglia and A. Winther, *Heavy Ion Reactions,* Frontiers in Physics Vol. 1 (Benjamin, New York, 1981).

[6]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Menlo Park, CA, 1976), Vol. 2.

[7]E. Browne, J. M. Dairiki, and R. E. Doebler, in *Table of Isotopes,* edited by C. M. Lederer and V. S. Shirley (Wiley-Interscience, New York, 1978).

[8]I. C. Percival, in *Proceedings of the Royal Society of London,* edited by M. V. Berry, I. C. Percival, and N. O. Weiss (Princeton Univ. Press, Princeton, 1987).

[9]M. V. Berry, in *Chaotic Behaviour of Deterministic Systems,* Proceedings of the Les Houches Summer School, 1981, edited by G. Looss, R. H. G. Helleman, and R. Stora (North-Holland, Amsterdam, 1981).

[10]D. M. Brink, *Semiclassical Methods in Nucleus-Nucleus Scattering* (Cambridge Univ. Press, New York, 1985).

[11]K. A. Erb and D. A. Bromley, in *Treatise on Heavy Ion Science,* edited by D. A. Bromley (Plenum, New York, 1984), Vol. 3.

[12]A. H. Wuosmaa, S. Saini, P. H. Kutt, S. F. Pate, R. W. Zurmühle, and R. R. Betts, Phys. Rev. C **36**, 1011 (1987).

[13]A. Sarma and R. Singh, Z. Phys. **337**, 23 (1990).

[14]G. Pollarolo and R. A. Broglia, Nuovo Cimento **81A**, 278 (1984); V. N. Bragin, G. Pollarolo, and A. Winther, Nucl. Phys. **A456**, 475 (1986).

[15]G. Pappalardo, Nucl. Phys. **A488**, 395c (1988).