# *CP* Violation via Electroweak Gauginos and the Electric Dipole Moment of the Electron

Pran Nath
*Department of Physics, Northeastern University, Boston, Massachusetts 02115*
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The effective Lagrangian which exhibits explicitly the transmission of the *CP*-violating phases induced via the electroweak gaugino masses below the scale of the spontaneous breaking of $SU(2)_L\times U_Y(1)$ electroweak gauge symmetry is deduced. The formalism is used to compute the one-loop electric dipole moment of the electron including the full set of neutralino states. Contributions of neutralinos other than the photino are found to be significant. It is shown that the current data exclude maximal *CP* violation in the electroweak sector except for selectron masses $\gtrsim 200$ GeV and photino masses $\gtrsim 750$ GeV.

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Over the past year an improvement on the limits of the electric dipole moment (EDM) of the electron by more than 2 orders of magnitude[1] over previous measurements[2] has been reported. The recent results of the Amherst group (Murthy *et al.*[1]) and of the Berkeley group (Abdullah *et al.*[1]) for $d_e$ are, respectively,

$$(-1.5\pm5.5\pm1.5)\times10^{-26}\,e\,\text{cm}\,, \tag{1a}$$

$$(-2.7\pm8.3)\times10^{-27}\,e\,\text{cm}\,. \tag{1b}$$

In view of this improvement in experiment and the possibility that the error in the most recent measurement Eq. (1b) may be further significantly reduced in the near future,[3] a fresh look at quantitative evaluations of the electron's EDM beyond the standard model becomes desirable. The existence of an electron EDM $d_e\gg10^{-38}$ $e$ cm (where $\sim10^{-38}$ $e$ cm is the value of $d_e$ expected in the standard model) is a clean test of new physics beyond the standard model, since unlike the neutron electric dipole moment which is plagued by large QCD correction factors[4] and uncertainties,[5] there are no corresponding QCD suppression factors or uncertainties in the evaluation of the electron's EDM.[6] Recently, several papers have reported a value of $d_e$ in the vicinity of the results of Eqs. (1) in non-standard-model frameworks.[7] In this Letter we shall be concerned with the electron's EDM in supersymmetric theories.[8] Although this problem has a long history,[9] a full analysis of the electron's EDM which exhibits its dependence on the several *CP*-violating phases that enter in the spontaneously broken supersymmetric theory has not been given. Thus in all of the previous analyses, quantitative discussions of contributions of the neutralino exchange other than that of the photino exchange have not been given. In this Letter we present a framework which deduces the low-energy effective Lagrangian below the scale of $SU(2)_L\times U_Y(1)$ spontaneous breaking with explicit exhibition of the effect of *CP* violation induced via weak gauginos. This effective Lagrangian is then used to compute the electron's EDM at the one-loop level. One finds that for large gaugino masses the contribution of the neutralino states other than the photino are comparable to that of the photino. It is shown that the current data do not allow maximal *CP* violation in the electroweak sector except for selectron masses $\gtrsim 200$ GeV or photino masses $\gtrsim 750$ GeV.

As is well known, in softly broken supersymmetric theories the *CP*-violating phases may be isolated in a few soft-supersymmetry-breaking parameters by a suitable redefinition of fields, e.g., in the Higgs-field mixing parameter ($\mu$), in the Poloni constant ($A$), and in the gaugino masses ($\tilde m$).[9] We assume the mass term in the gaugino sector to be $SU(2)_L\times U_Y(1)$ invariant and to have the form

$$L_m(\text{gaugino})=-\tfrac12\bar\lambda^0\tilde m_1e^{i\gamma_5\phi_1}\lambda^0-\tfrac12\bar\lambda^a\tilde m_2e^{i\gamma_5\phi_2}\lambda^a\,, \tag{2a}$$

where $\{\lambda^a(a=1,2,3),\lambda^0\}$ are weak gauginos for the $\{SU(2)_L,U_Y(1)\}$ gauge groups. One has in addition to Eq. (2a), the mass term for the Higgsinos and an off-diagonal mass term between gauginos and Higgsinos:[8,10]

$$\begin{aligned}L_m(\lambda-H)=&-\bar{\tilde H}{}^c\mu\exp(i\gamma_5\phi_\mu)\tilde H'\\&-ie_a\bar\lambda^a\langle\phi^\dagger\rangle(T^a/2)\chi+\text{H.c.}\,,\end{aligned} \tag{2b}$$

where $\phi=(H,H')$ are the $SU(2)_L$ Higgs doublets and $T^a$ are the Gell-Mann matrices. We introduce now the notation[8,10]

$$\begin{aligned}&\psi^0=\cos\theta_W\lambda^0+\sin\theta_W\lambda^3\,,\\&\psi^1=\sin\theta_W\lambda^0-\cos\theta_W\lambda^3\,,\\&\psi^2=i[-\sin\alpha(\tilde H_2'-\tilde H_2'^c)+\cos\alpha(\tilde H^2-\tilde H^{2c})]\,,\\&\psi^3=-i[\sin\alpha(\tilde H^2-\tilde H^{2c})+\cos\alpha(\tilde H_2'-\tilde H_2'^c)]\,,\end{aligned} \tag{3}$$

where $\tan\alpha=\langle H_2'\rangle/\langle H_2\rangle$ and $\theta_W$ is the Weinberg angle. We can write the mass matrix in the neutral gaugino-Higgsino sector as follows: $L_m=-\frac12\bar\psi(\tilde M+i\gamma_5\tilde\mu)\psi$, where

$$(\tilde M+i\gamma_5\tilde\mu)=\begin{pmatrix}A&B\\B&C\end{pmatrix},\quad B=\begin{pmatrix}0&0\\M_z&0\end{pmatrix},$$

and $A$ and $C$ are given by

$$A=\begin{pmatrix} \tilde{m}_\gamma e^{i\gamma_5\phi_\gamma} & \tilde{m}_{\gamma Z} e^{i\gamma_5\phi_{\gamma Z}} \\ \tilde{m}_{\gamma Z} e^{i\gamma_5\phi_{\gamma Z}} & \tilde{m}_Z e^{i\gamma_5\phi_Z} \end{pmatrix},$$
$$C=\begin{pmatrix} \mu e^{i\gamma_5\phi_\mu}\sin 2\alpha & \mu e^{i\gamma_5\phi_\mu}\cos 2\alpha \\ \mu e^{i\gamma_5\phi_\mu}\cos 2\alpha & -\mu e^{i\gamma_5\phi_\mu}\sin 2\alpha \end{pmatrix}, \tag{4}$$

and where $\tilde{m}_\gamma$, $\phi_\gamma$, etc., are defined by

$$\tilde{m}_\gamma e^{i\phi_\gamma}=\tilde{m}_1 e^{i\phi_1}\cos^2\theta_W+\tilde{m}_2 e^{i\phi_2}\sin^2\theta_W,$$
$$\tilde{m}_Z e^{i\phi_Z}=\tilde{m}_1 e^{i\phi_1}\sin^2\theta_W+\tilde{m}_2 e^{i\phi_2}\cos^2\theta_W, \tag{5}$$
$$\tilde{m}_{\gamma Z} e^{i\phi_{\gamma Z}}=2^{-1}(\tilde{m}_1 e^{i\phi_1}-\tilde{m}_2 e^{i\phi_2})\sin^2\theta_W.$$

The previous analyses[9] proceed by diagonalizing directly, via a biunitary transformation, the complex symmetric matrix

$$X=\begin{pmatrix} A' & B \\ B & C' \end{pmatrix},$$

where $(A',C')$ are $(A,C)$ with $\gamma_5$ replaced by unity in Eq. (4). This procedure does not make transparent the various $CP$-violating phases that enter in the formalism and to our knowledge has not been carried far enough to exhibit the transmission of these $CP$-violating phases below the scale of spontaneous breaking.

Here we give an alternative formulation which renders the parametrization of the $CP$-violating phases in the spontaneously broken theory explicit. In this procedure to effect the diagonalization of $L_m$, we first make the transformation $\psi^i=e^{i\gamma_5\beta_i}\chi^i$ on each of the $\psi^i$, which transforms $L_m$ to the form $L_m=-\frac{1}{2}\bar{\chi}(\tilde{M}'+i\gamma_5\tilde{\mu}')\chi$. Here $\tilde{M}'$ and $\tilde{\mu}'$ are symmetric and parametrically dependent on $\beta_i$. Next, we make an orthogonal rotation to diagonalize $\tilde{\mu}'$. This yields $L_m=-\frac{1}{2}\bar{\chi}'(P^T\tilde{M}'P+i\gamma_5\tilde{\mu}'_D)\chi'$, where $P$ is the orthogonal matrix that diagonalizes $\tilde{\mu}'$. We note that any further effort to diagonalize $P^T\tilde{M}'P$ will undiagonalize $\tilde{\mu}'_D$. At this point we utilize the freedom of the $\beta_i$ to set $\tilde{\mu}'_D$ proportional to the unit matrix, followed by a final orthogonal rotation $(Q)$ to diagonalize the $P^T\tilde{M}'P$ term, which then renders the entire mass matrix diagonal while retaining the diagonality of the kinetic terms, i.e., one has

$$L_m=-\tfrac{1}{2}\bar{\tilde{Z}}_k[\tilde{M}^0_k+(-1)^{\theta_k}i\gamma_5\tilde{\mu}_0]\tilde{Z}_k,$$
$$\tilde{Z}_k=(i\gamma_5)^{\theta_k}O^T_{kj}e^{-i\gamma_5\beta_j}\psi^j, \tag{6a}$$

where $M^0_k=|\lambda_k|$ and $\lambda_k$ are the elements of the diagonal matrix $\tilde{M}_D$ defined by $\tilde{M}_D=O^T\tilde{M}'O$, with $O=PQ$ an orthogonal matrix, and $\theta_k\equiv\theta(-\lambda_k)$.

The interaction Lagrangian which governs the couplings of the leptons with the neutralino states is given by[11]

$$L_{e\text{-}\tilde{Z}}=ie\sum_k(\bar{e}U_kP_L\tilde{e}_R+\bar{e}V_kP_R\tilde{e}_L)(-i\gamma_5)^{\theta_k}\tilde{Z}_k+\text{H.c.}, \tag{6b}$$

$$U_k=-O_{0k}e^{i\beta_0}-O_{1k}\tan\theta_W e^{i\beta_1},$$
$$V_k=-O_{0k}e^{-i\beta_0}+O_{1k}\cot 2\theta_W e^{-i\beta_1}. \tag{6c}$$

In Eqs. (6) $CP$ violation arises from two sources. First, there is a common $CP$-violating mass insertion $(-1)^{\theta_k}\times i\gamma_5\tilde{\mu}_0$ for all neutralino states. Second, there are $CP$-violating contributions arising from $CP$-violating phases $\beta_{0,1}$ in the vertices given by Eqs. (6b) and (6c). Equation (6) is the desired supergravity effective Lagrangian in the neutralino sector below the $SU(2)_L\times U_Y(1)$ electroweak-symmetry-breaking scale which exhibits explicitly the $CP$-violating phases for each of the neutralino mass eigenstates. Since we are in the mass-diagonal basis in Eqs. (6), we can compute directly the contribution of Eqs. (6) to the electric dipole moment of the electron at the one-loop level. The loop calculation proceeds by the exchange of neutralino and selectron fields and involves chirality flip. The mass matrix mixing the $LR$ selectron states has the form

$$\begin{pmatrix} \tilde{m}^2_L & \tilde{m}^2_{LR} \\ \tilde{m}^2_{RL} & \tilde{m}^2_R \end{pmatrix}, \tag{7a}$$

where

$$\tilde{m}^2_{LR}\equiv\Delta e^{i\phi_\Delta}=m_e[Ae^{-i\phi_A}m_{3/2}+(v'/v)\mu e^{i\phi_\mu}] \tag{7b}$$

and where $m_{3/2}$ is the gravitino mass, $Ae^{i\phi_A}$ is the (complex) Poloni constant, $\mu e^{i\phi_\mu}$ is the (complex) Higgs-field mixing parameter, and $v'/v=\langle H^1_2\rangle/\langle H^2\rangle$. The eigenvalues of the selectron mass matrix are $\tilde{M}^2_{L\pm}=\frac{1}{2}\{m^2_L+\tilde{m}^2_R\pm[(\tilde{m}^2_L-\tilde{m}^2_R)^2+4\Delta^2]^{1/2}\}$. We calculate the electric dipole moment of the electron at the one-loop level to be

$$d_e=e\frac{\alpha}{2\pi}\sum_k(-1)^{\theta_k}\{(O_{0k})^2\sin[(-1)^{\theta_k}\phi_k+\phi_\Delta-2\beta_0]+2^{-1}(O_{1k})^2(-1+\tan^2\theta_W)\sin[(-1)^{\theta_k}\phi_k+\phi_\Delta-2\beta_1]$$
$$+2^{-1}O_{0k}O_{1k}(-\cot\theta_W+3\tan\theta_W)\sin[(-1)^{\theta_k}\phi_k+\phi_\Delta-\beta_0-\beta_1]\}\tfrac{1}{2}\sin 2\Theta[J(x_{k-})-J(x_{k+})]\tilde{M}_k^{-1}, \tag{8}$$

where $\tan\phi_k=\tilde{\mu}_0/\tilde{M}^0_k$, $\tan 2\Theta=2\Delta/(\tilde{m}^2_L-\tilde{m}^2_R)$, $x_{k\pm}=\tilde{M}^2_k/\tilde{M}^2_{L\pm}$, $\tilde{M}_k=(\tilde{M}^{02}_k+\tilde{\mu}^2_0)^{1/2}$, and where $J(x)$ is the loop function

$$J(x)=2^{-1}(1-x)^{-3}(x-x^3+2x^2\ln x). \tag{9}$$

The result of Eq. (8) arises from $CP$-violating effects from both propagator insertions and from vertex corrections. Equation (8) is a one-loop result which shows the full dependence of $d_e$ on the $CP$-violating phases $\beta_0$, $\beta_1$,

TABLE I. Numerical evaluation of the electric dipole moment of the electron with $\phi_1=\phi_2=\phi$, $\mu=m_{\tilde{e}}=m_{3/2}$, $A=1$ and using the SU(5) relation on gaugino masses.

| $m_{\tilde{e}}=75$ GeV, $\phi_\mu=-\phi_A=\phi=\pi/4$ | | $m_{\tilde{\gamma}}=10$ GeV, $\phi_\mu=\phi_A=0$, $\phi=\pi/4$ | | $m_{\tilde{\gamma}}=25$ GeV, $m_{\tilde{e}}=75$ GeV, $\phi_\mu=\phi_A=0$ | |
|---|---|---|---|---|---|
| $m_{\tilde{\gamma}}$ (GeV) | $10^{26}d_e$ ($e$ cm) | $m_{\tilde{e}}$ (GeV) | $10^{26}d_e$ ($e$ cm) | $\phi$ | $10^{26}d_e$ ($e$ cm) |
| 1 | 2.64 | 50 | 48.26 | $\pi/2$ | 36.36 |
| 50 | 34.96 | 100 | 6.52 | $\pi/8$ | 6.71 |
| 750 | 0.74 | 200 | 0.67 | $\pi/128$ | 0.86 |

$\phi_k$, etc.

For an explicit numerical analysis we consider the limit when $\tilde{m}_{\gamma Z}=0$ and $\cos 2\alpha=0$.[12] In this limit the modes $\lambda^{\tilde{\gamma}}\equiv\psi^0$ and $\psi^3$ decouple and only the $2\times 2$ submatrix which couples $\psi^1$ and $\psi^2$ need be diagonalized. After the $e^{i\gamma_5\beta}$ transformation a direct inspection shows that conditions needed to diagonalize $\tilde{\mu}'$ are $\sin(\beta_1+\beta_2)=0$ and

$$\tan 2\beta_1=-(\tilde{m}_Z\sin\phi_Z-\mu\sin\phi_\mu)/(\mu\cos\phi_\mu+\tilde{m}_Z\cos\phi_Z)$$

and $\tilde{\mu}_0$ is

$$\tilde{\mu}_0=\mu\tilde{m}_Z\sin(\phi_\mu+\phi_Z)\times[\mu^2+\tilde{m}_Z^2+2\mu\tilde{m}_Z\cos(\phi_Z+\phi_\mu)]^{-1/2}. \tag{10}$$

The mass eigenvalues $\lambda_\pm$ of $\tilde{M}$ are $\frac{1}{2}(\bar{\mu}+\bar{m}_Z)\pm[M_Z^2+\frac{1}{4}(\bar{\mu}-\bar{m}_Z)^2]^{1/2}$, where $\bar{\mu}=\mu\cos(\phi_\mu+2\beta_2)$ and $\bar{m}_Z=\tilde{m}_Z\cos(\phi_Z+2\beta_1)$. In the limit where $\tilde{m}_L^2=\tilde{m}_R^2=\tilde{M}_L^2$ and $\Delta\ll\tilde{M}_L^2$, we can expand the difference of the loop integrals in Eq. (8) and we have in the above approximation

$$d_e=e\frac{\alpha m_e}{2\pi}\left[\left(Am_{3/2}+\frac{v'}{v}\mu\right)^2-\frac{4v'}{v}\mu Am_{3/2}\sin^2\left(\frac{\phi_A+\phi_\mu}{2}\right)\right]^{1/2}\tilde{M}_L^{-4}[\tilde{\mu}_\gamma L(x_\gamma)+\tilde{\mu}_+L(x_+)+\tilde{\mu}_-L(x_-)]\,, \tag{11a}$$

where $L(x)=dJ(x)/dx$, $\tilde{\mu}_\gamma=\tilde{m}_\gamma\sin(\phi_\gamma+\phi_\Delta)$,

$$\tilde{\mu}_\pm=(-1)^{\theta_\pm}\tilde{M}_\pm 2^{-1}O_{1\pm}^2(-1+\tan^2\theta_W)\sin[(-1)^{\theta_\pm}\phi_\pm+\phi_\Delta-2\beta_1]\,, \tag{11b}$$

$$O_{1\pm}=M_Z\{2M_Z^2+\tfrac{1}{2}(\bar{\mu}-\bar{m}_Z)^2\pm(\bar{\mu}-\bar{m}_Z)[M_Z^2+\tfrac{1}{4}(\bar{\mu}-\bar{m}_Z)^2]^{1/2}\}^{-1/2}\,, \tag{11c}$$

and $x_\gamma=\tilde{m}_\gamma^2/\tilde{M}_L^2$, $x_\pm=\tilde{M}_\pm^2/\tilde{M}_L^2$, $\tan\phi_\pm=\tilde{\mu}_0/\tilde{M}_\pm$, and $\tilde{M}_\pm=|\lambda_\pm|$. Here $\theta_+=1$ and $\theta_-=-1$ for $\lambda_-<0$. For the numerical evaluation of Eqs. (11) we let $\phi_1=\phi_2=\phi$, $\mu=\tilde{m}_e=m_{3/2}$, and $A=1$ and use the SU(5) relation $\tilde{m}_1=\frac{5}{3}\tilde{m}_2\tan^2\theta_W$. Results are exhibited in Table I. The evaluation of the individual pole contributions $(\tilde{\gamma},\tilde{Z}_\pm)$ of Eqs. (11) shows that while the $\tilde{Z}_\pm$-exchange contributions are relatively small for small gaugino masses, they can contribute significantly (up to $\approx\frac{1}{3}$ in magnitude of the total) for large gaugino masses, i.e., $\tilde{m}_1\sim m_{\tilde{e}}$.

The result of Table I shows that the experimental bound on $d_e$ of Eqs. (1) already constrains $CP$ violation in the electroweak sector of the supersymmetric theory. Thus, for example, Table I shows that maximal $CP$ violation in the electroweak sector is ruled out except for selectron masses $\gtrsim 200$ GeV or photino masses $\gtrsim 750$ GeV. Further from Table I, one finds that for a normal size spectrum of the photino and of the selectron, the $CP$-violating phase is required by the experimental limit on $d_e$ to be significantly smaller, i.e., $\sim 10^{-2}$ of the maximum value. This limit will become even more stringent as the experimental limits on $d_e$ improve.

In conclusion, we note that the method discussed here to deduce the effective Lagrangian Eq. (6) can be extended directly to the chargino sector and the resulting effective supergravity Lagrangian can be used for the investigation of a full array of $CP$-violating phenomena induced via the supersymmetric electroweak sector below the scale of $SU(2)_L\times U_Y(1)$ breaking.

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[11]The result of Eq. (6) reduces to the standard result (Refs. 8 and 10) when *CP*-violating phases are set to zero, i.e., $\tilde{\mu}_0=0$ and $\beta_i=0$.

[12]Our conclusion on the allowed region of maximal *CP* violation is not sensitive to the approximation used here. Further, the approximation is not unreasonable in view of the fact that $\tilde{m}_{\gamma Z}\approx -0.36\tilde{m}_\gamma$ and the current data from the CERN $e^+e^-$ collider LEP is consistent with $\tan\alpha \lesssim 0.77$. [See J. Lopez and D. V. Nanopoulos, Mod. Phys. Lett. A **5**, 1259–1264 (1990).]