New Test of Quantum Mechanics: Is Planck's Constant Unique?

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We discuss the possibility that different realms of physics are described by distinct quantization constants. From the consistency of existing data, we infer limits on the differences between hypothetically distinct quantization constants for different elementary particles. Since the existence of multiple Planck constants implies violations of space-time symmetries, these limits may be viewed as precise tests of fundamental conservation laws, including the conservation of linear momentum and energy.

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The concept of a universal quantization constant has been central to modern physics since its introduction by Planck in 1900.[1] As accustomed as we have become to thinking of Planck's constant, $\hbar = h/2\pi$, as the unique quantum in terms of which both angular momentum and action are measured, it cannot be logically excluded that different realms of physics are in fact described by distinct quantization constants.[2] To appreciate that such a possibility is meaningful phenomenologically, it is useful to recall the classic experiments of Beth[3] and Holbourn[4] which directly measured the angular momentum of the photon in macroscopic units. By passing circularly polarized light of known intensity through a quartz retardation plate suspended from a torsion fiber, Beth was able to determine the angular momentum transmitted to the plate by a single photon. His result was consistent with the theoretical expectation, $J = \hbar$. A similar result was obtained by Holbourn, who found $J/\hbar = 1.05 \pm 0.15$. It is significant that the Beth-Holbourn experiments can, in principle, be adapted to measure the intrinsic angular momentum of any particle. An ensemble of such experiments, each employing a different elementary particle, may be viewed as a direct and unambiguous test of the conventional assumption that the angular momenta of all particles are quantized in terms of the same $\hbar$. As we discuss below, the issues that are raised by such a hypothetical multiplicity of Beth-Holbourn experiments are accessible in a metrologically more robust fashion by other means.

We begin by demonstrating that by appropriately combining the results of various high-precision experiments, we can identify and extract distinct Planck constants from existing data. After discussing the consistency of the results we obtain, we show that within the framework of nonrelativistic quantum mechanics, the introduction of multiple Planck constants leads to violations of space-time symmetry laws, such as energy and linear momentum conservation. Thus, a test of the uniqueness of Planck's constant may be viewed as a test of quantum mechanics and/or the aforementioned conservation laws.

Our analysis involves an examination of a subset of the existing precision measurements employed in the familiar least-squares adjustment[5] of the fundamental constants. It should be noted that the extremely high accuracies which have been obtained in many of these measurements are possible because they involve experimentally accessible *combinations* of fundamental constants. Attainment of such accuracies would be extraordinarily difficult, if not impossible, if measurements were restricted to direct determinations of the individual constants themselves. For example, while it is possible to measure $\hbar$ by an experiment of the Beth-Holbourn-type, and e by the Millikan "oil drop" method, it would be difficult in either case to achieve a precision well below the 10^{-3} level. By contrast, the current experimental limits on the combinations $2e/h$ (Josephson effect), and h/e^2 (quantized Hall effect), are quoted at the level of several parts in 10^8.

Directly or indirectly, Planck's constant enters into many of these experimentally accessible combinations of constants. Often its presence can be ascribed to the direct application of a quantization principle for a particular elementary particle. For example, the Josephson frequency-to-voltage relation, $2e/h = \nu/V$, arises when one measures an electrical energy $qV = 2eV$ which is related to the frequency ν of an oscillating *classical* electric field. A quantum-mechanical description of the electron is necessary for the derivation of this relation; however, the electromagnetic field enters only classically. Since the essential quantization principle applies to the electron, we identify the h in the Josephson relation as h_e, the quantization constant for the electron.

In the following we distinguish four hypothetically distinct values of Planck's constant, h_e, h_p, h_γ, and h_n, which we associate with the electron, proton, photon, and neutron, respectively. We shall examine several of the relevant precision measurements with the aim of isolating the essential quantization principle(s) inherent in each. From this reinterpreted set of data, it is possible to extract ratios of these distinct h's. In principle one can carry out a complete least-squares adjustment of the fundamental constants, with the inclusion of each of these subscripted Planck constants. Such a procedure will, however, produce only a minimal improvement in sensitivity. Except where more accurate results exist, we will use the experimental quantities quoted by Cohen and Taylor[5] in their latest least-squares adjustment of the fundamental constants. Included in that adjustment are several electrical measurements which involve the use of "as-maintained" (as opposed to SI) electrical standards. Currently the most accurate complete set of such as-maintained measurements are those recently carried out at the National Institute of Standards and Technology (NIST). These results, summarized by Cage *et al.*,[6] will be denoted by the subscript NIST. We also note the existence of an elegant and important result, a recent measurement involving neutrons by Krüger, Nistler, and Weirauch.[7]

A convenient point of departure for our analysis is the consideration of several expressions for the fine-structure constant $\alpha = e^2/4\pi\epsilon_0 \hbar c$, which depend on different combinations of measured quantities. We note that the uniqueness of e has been determined to very high accuracy.[8] We also note that all of the measurements under consideration concern nonrelativistic phenomena. Thus c enters only via its role as the definition of the SI length scale, and similarly ϵ_0 and μ_0 enter as defined quantities. We therefore attribute any differences in these metrologically distinct α's to differences in the h's which appear in them. Although the interpretation of some of these quantities requires the use of higher-order corrections which may involve additional "distinct" h's (or relativistic corrections), our discussion will be limited to the lowest-order appearances of h in each expression.

The first expression for α involves the quantized Hall resistance R_H, which is conventionally written as $R_H = h/e^2$. The condition which relates R_H and h results from the quantization of orbital angular momentum for a two-dimensional electron gas in a magnetic field.[9] Therefore, we identify the h in R_H as the "electronic" Planck's constant h_e, and denote the value of α obtained in this way as α_1, where[6]

$$\alpha_1 = \frac{\mu_0 c}{2R_H} = \frac{e^2}{4\pi\epsilon_0 c}\frac{1}{\hbar_e}. \tag{1}$$

Accurate measurements of R_H are not made directly in SI units, but require the use of as-maintained units.[6] In such units Eq. (1) assumes the form

$$\alpha_1 = \mu_0 c (2 r_{K\text{-NIST}} \Omega_{\text{NIST}})^{-1}, \tag{2}$$

where Ω_{NIST} is the as-maintained ohm, and $r_{K\text{-NIST}}$ is the dimensionless number which expresses R_H in terms of Ω_{NIST}.

The second expression for α makes use of the relation[6]

$$\alpha_2 = \left[\frac{(\mu_p'/\mu_B) R_H E_J}{2\mu_0 R_\infty \gamma_p'}\right]^{-1/3}, \tag{3}$$

where R_∞ is the Rydberg constant, γ_p' is the (low-field) proton gyromagnetic ratio in a spherical sample of water, μ_p'/μ_B is the ratio of the proton magnetic moment (in a spherical sample of water) to the Bohr magneton, and $E_J = 2e/h$ is the Josephson frequency-to-voltage ratio. As in the case of α_1, the expression in Eq. (3) must be recast in terms of as-maintained electrical quantities in order to achieve an interesting level of precision. This gives[6]

$$\alpha_2 = \left[\frac{(\mu_p'/\mu_B) r_{K\text{-NIST}} \nu_{J\text{-NIST}}}{2\mu_0 R_\infty \gamma'_{p\text{-NIST}}}\right]^{-1/3}, \tag{4}$$

where $\nu_{J\text{-NIST}}$ is the Josephson frequency in terms of as-maintained volts. The quantities in Eq. (3) which *directly* depend on Planck's constant are $R_H = h/e^2$, $E_J = 2e/h$, and $R_\infty = m_e e^4/8\epsilon_0^2 c h^3$, where m_e is the electron mass. We have already identified $h = h_e$ in the expressions for both R_H and E_J.

R_∞ is determined in an experiment where the wavelength of the radiation associated with an electronic transition in atomic hydrogen is measured in absolute units. This procedure may be viewed as equating the difference in the electronic energy between states having different principal quantum numbers n_1 and n_2 to the energy of a photon of wavelength $\lambda = h_\gamma c/E$. To *lowest* order, this electronic energy difference is given by $E = (1/n_1^2 - 1/n_2^2) m_e e^4/8\epsilon_0^2 h_e^2$. We have identified the Planck constant appearing in this expression for the electronic energy with h_e since, in essence, it arises from applying the Bohr quantization condition $\oint p\,dq = nh$ to the electron. The Rydberg constant can thus be written as $R_\infty = m_e e^4/8\epsilon_0^2 c h_e^2 h_\gamma$. (As we discuss below, allowing $h_e \neq h_\gamma$, while at the same time assuming energy conservation, amounts to a test of the Bohr correspondence principle.)

The determination of the combination $(\mu_p'/\mu_B)/\gamma_p'$ relies on a series of measurements where, in effect, the precession frequencies of electrons (in atomic hydrogen) and protons (in water) are compared. The actual detection methods involve sensing the time-varying bulk magnetizations of the samples and involve only classical electromagnetic quantities. We note that the experimental determination of μ_p'/μ_B is actually a determination of the ratio of the respective gyromagnetic ratios. It is only by assuming that $h_e = h_p$ that one may characterize the results of this experiment as a magnetic-moment ratio. For our purposes we note that the combination $(\mu_p'/\mu_B)/\gamma_p'$ does *not* introduce additional distinct quantization constants. We thus conclude that

$$\alpha_2 = \frac{e^2}{4\pi\epsilon_0 c}\frac{1}{(\hbar_e^2 \hbar_\gamma)^{1/3}}. \tag{5}$$

We obtain a third relation for $\alpha=\alpha_3$ by starting from

$$\alpha_3=[(2R_\infty/c)(m_n/m_p)(m_p/m_e)(h/m_n)]^{1/2}, \qquad (6)$$

which expresses α_3 in terms of appropriate ratios of the electron, proton, and neutron masses, m_p, m_e, and m_n. h/m_n has been measured recently in a very elegant determination of the wavelength-to-velocity ratio for a nonrelativistic neutron.[7] We identify this experimental value of h/m_n as h_n/m_n. The only other first-order occurrence of h in Eq. (6) is in R_∞ as previously discussed. Thus

$$\alpha_3=\frac{e^2}{4\pi\epsilon_0 c}\frac{1}{\hbar_e}\left(\frac{\hbar_n}{\hbar_\gamma}\right)^{1/2}. \qquad (7)$$

Combining Eqs. (1), (5), and (7) leads to the relations

$$\hbar_e/\hbar_\gamma=(\alpha_2/\alpha_1)^3, \qquad (8a)$$

$$\hbar_n/\hbar_\gamma=(\alpha_3/\alpha_1)^2. \qquad (8b)$$

While the results of other precision measurements may be combined to form different α's, these are the only two linearly independent ratios of the h's under consideration which can be accurately determined from the above data.

The values of α_1 and α_2 are quoted directly by Cage *et al.*,[6] and α_3 can be obtained from the recently measured value[7] of $h/m_n=3.9560344(16)\times10^{-7}\ \mathrm{m^2\,s^{-1}}$ using Eq. (6). The α's are then given by $\alpha_1^{-1}=137.0359979(32)$, $\alpha_2^{-1}=137.0359840(51)$, and $\alpha_3^{-1}=137.035993(27)$.[7]

Combining these results we obtain

$$\hbar_e/\hbar_\gamma-1=+30(13)\times10^{-8}, \qquad (9a)$$

$$\hbar_n/\hbar_\gamma-1=+7(40)\times10^{-8}, \qquad (9b)$$

where the errors correspond to 1σ. The result in (9a) reflects the level of agreement between α_1 and α_2 noted previously by Cage *et al.*[6]

The existence of a (hypothesized) multiplicity of quantization constants has important implications when one goes beyond the level of the single-particle quantum mechanics thus far invoked. This follows from the realization[10] that the introduction of multiple Planck constants in a system with two or more particles leads to apparent violations of space-time conservation laws. To see how this comes about, consider the interaction between two particles, 1 and 2, having a common mass m, whose coordinates $\mathbf{q}_{1,2}$ and momenta $\mathbf{p}_{1,2}$ satisfy

$$[q_{1x},p_{1x}]=i\hbar_1,\ \text{etc.},$$
$$[q_{2x},p_{2x}]=i\hbar_2,\ \text{etc.}, \qquad (10)$$
$$[q_{1x},q_{2x}]=[p_{1x},p_{2x}]=0,\ \text{etc}.$$

We assume that the one-dimensional Hamiltonian H of the system is given by

$$H=\frac{p_1^2}{2m}+\frac{p_2^2}{2m}+V(\mathbf{q}_1-\mathbf{q}_2), \qquad (11)$$

where V depends only on the separation of 1 and 2, and hence is translationally invariant. If we introduce relative and center-of-mass (c.m.) coordinates in the usual way,

$$\mathbf{u}=\mathbf{q}_1-\mathbf{q}_2,\quad \mathbf{k}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_2),$$
$$\mathbf{R}=\tfrac{1}{2}(\mathbf{q}_1+\mathbf{q}_2),\quad \mathbf{P}=\mathbf{p}_1+\mathbf{p}_2, \qquad (12)$$

then H can be written in the conventional form

$$H=P^2/2M+k^2/2\mu+V(\mathbf{u}),\quad M=2m=4\mu. \qquad (13)$$

It should be emphasized that the relative and c.m. coordinates introduced in Eq. (12) have the same form as in the classical case, which ensures that H will have the proper correspondence limit. Combining Eqs. (10) and (12) the commutation relations satisfied by these coordinates are

$$[u_i,R_j]=[k_i,P_j]=0, \qquad (14a)$$

$$[u_i,k_j]=[R_i,P_j]=\tfrac{1}{2}i(\hbar_1+\hbar_2)\delta_{ij}, \qquad (14b)$$

$$[u_i,P_j]=4[R_i,k_j]=i(\hbar_1-\hbar_2)\delta_{ij}. \qquad (14c)$$

From Eq. (14c)

$$[H,\mathbf{P}]=[V(\mathbf{u}),\mathbf{P}]\neq0, \qquad (15)$$

and hence in this simple model the c.m. momentum $\mathbf{P}$ is not a constant of the motion, if H retains its usual meaning as the generator of time translations.

It is interesting to note that this connection between the introduction of multiple Planck constants and the violation of space-time symmetries can be inferred from the "old quantum theory."[11] Following Bohr, we assume that the energy levels of the electron in the hydrogen atom are given by $E_n=f(n)\hbar_e\omega_n$, where $\hbar_e$ is the previously defined Planck constant for the electron. $f(n)$ is a function of the principal quantum number n, and ω_n is the angular frequency of the electron. Classically, the latter is given by $\omega=8\pi\epsilon_0(2E^3/m_e)^{1/2}/e^2$ for an electron of energy E in a circular orbit, and therefore $\omega_n=\pi e^4 m_e/16\epsilon_0^2\hbar_e^3 f^3(n)$. It follows that for a transition between levels n and m, the change in the electron energy is

$$E_n-E_m=\frac{e^4 m_e}{32\epsilon_0^2\hbar_e^2}\left(\frac{1}{f^2(n)}-\frac{1}{f^2(m)}\right). \qquad (16)$$

This transition gives rise to a photon of frequency ν_{nm}, whose energy E_γ is given by $E_\gamma=h_\gamma\nu_{nm}$, where h_γ is the previously defined photon quantization constant. If conservation of energy is assumed, so that $E_\gamma=E_n-E_m$, then agreement with the Balmer formula is obtained if $f(n)\propto n$. Because of the freedom retained in defining h_e and h_γ, we may follow Bohr and set $f(n)=n/2$, so that

$$\nu_{nm}=\frac{e^4 m_e}{8\epsilon_0^2 h_\gamma h_e^2}\left(\frac{1}{n^2}-\frac{1}{m^2}\right),\quad \omega_n=\frac{\pi e^4 m_e}{2\epsilon_0^2 h_e^3 n^3}. \qquad (17)$$

If Bohr's correspondence principle, $\lim_{n\to\infty}\nu_{n,n-1} = \lim_{n\to\infty}\nu_n$, and energy conservation are invoked, we find from Eq. (17) that $h_\gamma = h_e$. This argument demonstrates in physical terms the connection[12] between energy conservation and the universality of $\hbar$ embodied in Eqs. (10)-(15). It follows from the preceeding discussion that we can retain the correspondence principle, and accommodate $h_\gamma \neq h_e$, if we relax the requirement of energy conservation so that $h\nu_{nm} = E_\gamma = (\hbar_\gamma/h_e)(E_n - E_m)$. This establishes the direct connection between the limits on multiple Planck constants that we have derived, and the test of energy conservation at the quantum level. Considerations similar to the above can be used to test the validity of other space-time symmetries at the quantum level, such as linear and angular momentum conservation, as we will discuss elsewhere.[13]

In summary, we have shown that one can identify physically distinct Planck constants for different elementary particles. We have examined a set of precision measurements in a fashion which explores possible deviations from a universal quantization constant. We have also shown that these limits test the validity of various space-time symmetries at the quantum level.

This analysis can be extended by studying the uniqueness of $\hbar$ in other realms. Some of the questions which may be raised are the following: Does the same quantization constant describe orbital and spin angular momentum, energy and angular momentum, nuclear and electronic spins, bosons and fermions, or particles and antiparticles? We will show elsewhere[13] that in each of these cases the introduction of distinct Planck constants implies specific observable effects whose presence (or absence) can be used to set limits on the constants introduced. Such effects may be expected in atomic fine structure and in the Zeeman effect for fine and hyperfine structure. The possibility that different generations $(e,\mu,\ldots)$ are governed by different Planck constants may be tested by examination of the muonium hyperfine structure and other exotic-atom spectroscopy.

We also observe that the argument from which *CP* violation is deduced in K^0 decays rests on an assumption of angular momentum conservation. Noting that the introduction of multiple quantization constants violates this conservation law, it would be instructive to consider the implications of multiple $\hbar$'s in the neutral kaon system. Finally, and significantly, we note that we have not incorporated into the present analysis the constraints on α and $\hbar$ that are implied by the comparison of theory and experiment for the anomalous magnetic moment of the electron[14] $g-2$. The identification of the specific quantization principles implied in the calculation of $g-2$ is less obvious than in the data we have discussed, and a proper consideration of these questions would require a field-theoretic analysis which is beyond the scope of this paper. These and other questions will be discussed elsewhere.[13]

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