Nonlocality of a Single Photon

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We propose an experiment which demonstrates the nonlocal properties of a single-photon field via phase-sensitive measurements. This is the first proposal which demonstrates nonlocality and a violation of Bell's inequality with a single photon rather than a correlated photon pair.

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In this paper we wish to propose an experiment which will demonstrate in a most striking way the nonlocal properties of a single photon. In essence the effect of the single photon is felt at two spatially separated detectors. This manifests itself as an enhancement of the two-photon coincidence count rate when homodyne measurements are performed on the one-photon field at two different positions.

The nonlocal property of the photon field is evident in the Young's interference experiment performed by Grangier, Roger, and Aspect[1] where only a single photon is incident on the two slits. The interference fringes observed are explained quantum mechanically by the interference of the two paths the photon may take. However, the observed interference fringes may be explained by a classical field theory which is nonlocal. In the experiment we propose, the effects predicted may not be duplicated by any classical theory.

Discussions concerning the nonlocality of quantum mechanics were initiated by Einstein, Podolsky, and Rosen[2] (EPR) and later formulated in a rigorous fashion by Bell[3,4] via his famous inequalities.

Bell's inequality has traditionally been used to demonstrate the failure of local causality in quantum mechanics, since it places bounds on the degree of correlation which can exist between measurements made at two spatially separated detectors if local causality was valid. In quantum mechanics, a measurement affects the entire system being measured, so that the result of a measurement at one detector depends not only on the local parameters at that detector, but may be coupled via a quantum correlation in the system to the parameters at the other.

In the many configurations which have been proposed to demonstrate violations of Bell's inequality, a common feature is that a pair of particles is involved, generated by some interaction which imposes a high degree of correlation between them. The system consists of the pair of particles, which maintain the correlation between themselves as they separate. These are subsequently responsible for the violation of Bell's inequality. Indeed, in the experiments in optics which have demonstrated a violation of Bell's inequality, correlated pairs of photons have been generated either by a two-photon atomic cascade[5,6] or by nondegenerate parametric amplification.[7-9] Related experiments to observe nonlocal effects in two-photon interference also involve a correlated pair generated by nondegenerate parametric amplification.[10]

In this paper we describe a configuration for demonstrating the nonlocal nature of quantum-mechanical states which does not rely on having two correlated particles. Indeed, a special case of the general result shows that the field generated by a single photon can have a pronounced nonlocal effect on two homodyne detectors giving rise to EPR correlations and a violation of Bell's inequalities. We contrast the result with those expected from a naive particle theory and a classical wave theory.

We consider a pair of homodyne detectors (indexed by subscript k), each of which consists of a 50-50 beam splitter, a coherent local oscillator with amplitude $\alpha_k = \alpha\exp(i\theta_k)$, and two photodetectors in the output ports. The inputs to these homodyne detectors are themselves derived from a third 50-50 beam splitter, as shown in Fig. 1. Other schemes involving homodyne detection have been proposed to investigate the EPR paradox[11] and violations of Bell's inequalities.[12]

Referring to Fig. 1, we see that homodyne detector k may be regarded as making a measurement of mode b_k, with a local parameter θ_k. This local parameter is

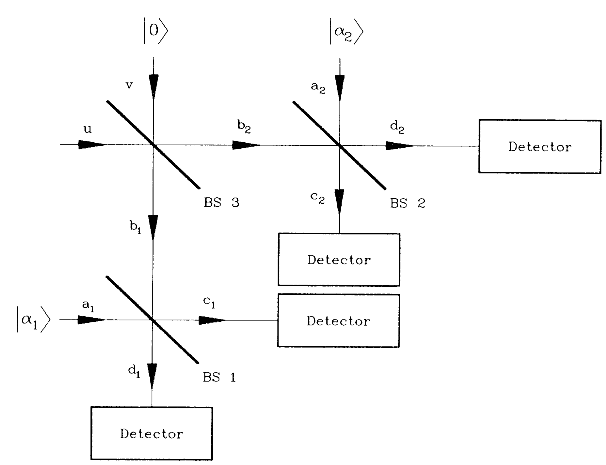

FIG. 1. Proposed experimental configuration.

analogous to the angle of the analyzer used in the conventional two-particle EPR experiment. We wish to determine the probabilities with which the individual photodetectors respond, and the coincidence probabilities for pairs of photodetectors, one in each homodyne detector.

The transformation between the mode operators shown in Fig. 1 is given by

$$\begin{pmatrix} \hat{c}_k \\ \hat{d}_k \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{b}_k \end{pmatrix}, \quad \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} \hat{v} \\ \hat{u} \end{pmatrix}. \tag{1}$$

Thus the modes input to the detectors may be expressed in terms of the input-mode operators by

$$\begin{pmatrix} \hat{c}_1 \\ \hat{d}_1 \\ \hat{c}_2 \\ \hat{d}_2 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & i/2 & 0 & -1/2 \\ i/\sqrt{2} & 1/2 & 0 & i/2 \\ 0 & -1/2 & 1/\sqrt{2} & i/2 \\ 0 & i/2 & i/\sqrt{2} & 1/2 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{v} \\ \hat{a}_2 \\ \hat{u} \end{pmatrix}. \tag{2}$$

This enables us to calculate the coincidence probabilities between the detectors directly in terms of the input fields.

We begin first by considering vacuum inputs to the modes $\hat{u}$ and $\hat{v}$. The local oscillators are assumed to be in coherent states $|\alpha e^{i\theta_1}\rangle$, $|\alpha e^{i\theta_2}\rangle$. The intensities at all detectors are found to be equal

$$\langle I_{c_1}\rangle = \langle I_{c_2}\rangle = \langle I_{d_1}\rangle = \langle I_{d_2}\rangle = \tfrac{1}{2}\alpha^2 . \tag{3}$$

The two-photon coincidence rates due to rare chance coincidences between the local oscillators are also equal between the pairs of detectors

$$\langle I_{c_1} I_{c_2}\rangle = \langle I_{d_1} I_{d_2}\rangle = \langle I_{c_1} I_{d_2}\rangle = \langle I_{d_1} I_{c_2}\rangle = \tfrac{1}{4}\alpha^4 . \tag{4}$$

We now consider the input of a single photon in mode $\hat{u}$ while the mode $\hat{v}$ is the vacuum. The state of the two-mode field $\hat{b}_1$ and $\hat{b}_2$ after the first beam splitter is then an entangled state[13,14] of a one-photon state and the vacuum

$$|\psi\rangle = (1/\sqrt{2})(i|1\rangle_{\hat{b}_1}|0\rangle_{\hat{b}_2} + |0\rangle_{\hat{b}_1}|1\rangle_{\hat{b}_2}) , \tag{5}$$

which is precisely the same state as one gets (except for a phase factor) for a one-photon state incident on the two slits in Young's interference experiment.

The photon count probabilities at the individual detectors are now

$$\langle I_{c_1}\rangle = \langle I_{c_2}\rangle = \langle I_{d_1}\rangle = \langle I_{d_2}\rangle = \tfrac{1}{2}\alpha^2 + \tfrac{1}{4} . \tag{6}$$

Thus the intensities at each detector are increased by $\frac{1}{4}$, being the probability that the one-photon input is detected by any given detector. The coincidence count probabilities between the pairs of detectors differ, now depending on which pair is considered. We find

$$\langle I_{c_1} I_{c_2}\rangle = \langle I_{d_1} I_{d_2}\rangle = \tfrac{1}{4}\{\alpha^4 + \alpha^2[1 + \sin(\theta_1 - \theta_2)]\} , \tag{7}$$

$$\langle I_{c_1} I_{d_2}\rangle = \langle I_{d_1} I_{c_2}\rangle = \tfrac{1}{4}\{\alpha^4 + \alpha^2[1 - \sin(\theta_1 - \theta_2)]\} . \tag{8}$$

The coincidence probabilities depend on the phase difference between the local oscillators $\theta_1 - \theta_2$; if this is set to $-\pi/2$, we get the minimum possible coincidence probability of $\frac{1}{4}\alpha^4$ between detector pairs $(\hat{c}_1, \hat{c}_2)$ and $(\hat{d}_1, \hat{d}_2)$ and the maximum coincidence probability of $\frac{1}{4}\alpha^4 + \frac{1}{2}\alpha^2$ between pairs $(\hat{c}_1, \hat{d}_2)$ and $(\hat{d}_1, \hat{c}_2)$. We shall be most interested in the situation where α is small compared to 1.

Let us first try to interpret these results from a naive particle viewpoint. The great enhancement of the singles count probability over that with vacuum inputs is easily understood by the above argument. On the other hand, a coincidence between two detectors is expected to be a rare event since there is only one incident photon, and a coincidence can only occur if an additional photon is generated by the (weak) local oscillator of the homodyne detector which the photon does *not* reach. Since these two photons are detected at two spatially separated detectors and have apparently arisen from independent sources, we would not expect any correlation between the paths of these photons within each homodyne detector. Nevertheless, the quantum-mechanical analysis shows such a correlation is present. In fact, this correlation is so great that for the choice of phases given above, no additional coincidences (above the vacuum level) occur for particular detector pairs, whereas there is a relatively large coincidence probability (proportional to the local oscillator intensity) for the other pairs.

Nonlocal intensity correlations and their dependence on the local oscillator phases are not unexpected from a classical wave description of light. A classical analog to the single-photon input is a wave of low amplitude and unspecified phase. We may formally obtain the results for the classical wave theory from the quantum-mechanical calculation by substituting the wave amplitude $\beta e^{\pm i\phi}$ for $\hat{b}$ and $\hat{b}^\dagger$, respectively, and averaging over the random phase ϕ. It is easy to check that the predicted average intensities and intensity correlations are given by

$$\langle I_{\hat{c}_1}\rangle = \langle I_{\hat{c}_2}\rangle = \langle I_{\hat{d}_1}\rangle = \langle I_{\hat{d}_2}\rangle = \tfrac{1}{2}\alpha^2 + \tfrac{1}{4}\beta^2 , \tag{9}$$

$$\langle I_{\hat{c}_1} I_{\hat{c}_2}\rangle = \langle I_{\hat{d}_1} I_{\hat{d}_2}\rangle = \tfrac{1}{4}\{\alpha^4 + \alpha^2\beta^2[1 + \sin(\theta_1 - \theta_2)] + \tfrac{1}{4}\beta^4\} , \tag{10}$$

$$\langle I_{\hat{c}_1} I_{\hat{d}_2}\rangle = \langle I_{\hat{d}_1} I_{\hat{c}_2}\rangle = \tfrac{1}{4}\{\alpha^4 + \alpha^2\beta^2[1 - \sin(\theta_1 - \theta_2)] + \tfrac{1}{4}\beta^4\} . \tag{11}$$

If we consider the coincidence probabilities as a function of $\theta_1 - \theta_2$, we see that they vary between $\frac{1}{4}(\alpha^4 + \frac{1}{4}\beta^4)$ and $\frac{1}{4}(\alpha^4 + 2\alpha^2\beta^2 + \frac{1}{4}\beta^4)$. This corresponds to a "visi-

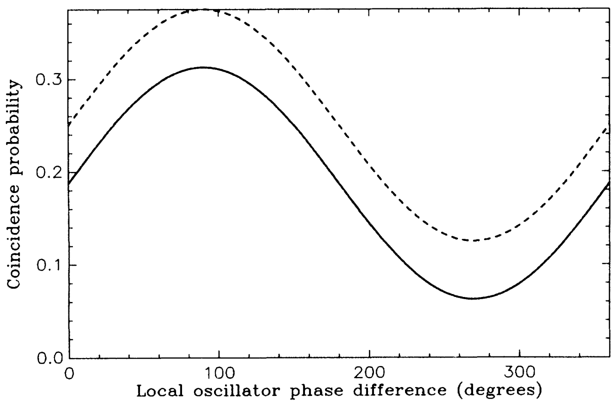

FIG. 2. Coincidence probabilities for quantum-mechanical model (solid line) and classical wave model (dashed line).

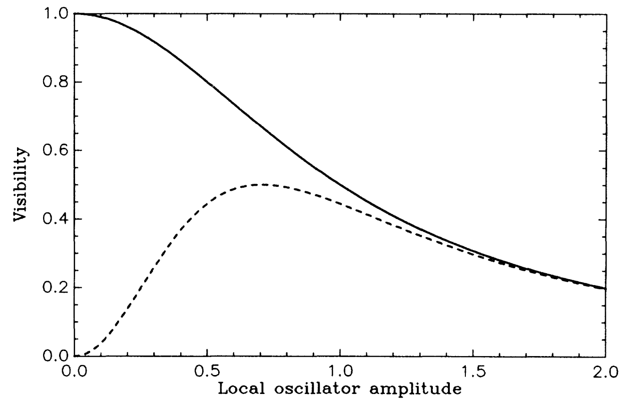

FIG. 3. Variation of visibility with local oscillator amplitude for quantum-mechanical model (solid line) and classical wave model (dashed line).

bility" of

$$\mathcal{V}=\rho/(\rho^2+\rho+\tfrac{1}{4}), \tag{12}$$

where $\rho=(\alpha/\beta)^2$. The visibility attains a maximum value of $\frac{1}{2}$ when $\rho=\frac{1}{2}$. By contrast, the visibility as calculated from the quantum-mechanical result is

$$\mathcal{V}=1/(\alpha^2+1). \tag{13}$$

This can be made arbitrarily close to unity by choosing a sufficiently small value of α. Figure 2 shows the coincidence probabilities $\langle I_{\hat{c}_1}I_{\hat{c}_2}\rangle=\langle I_{\hat{d}_1}I_{\hat{d}_2}\rangle$ as a function of the local oscillator phase difference for the quantum-mechanical and classical results with $\beta=1$ and $\alpha=1/\sqrt{2}$. This gives the same singles count probability of $\frac{1}{2}$ in each detector, and the local oscillator amplitudes are optimized for maximum visibility in the classical result. However, the quantum-mechanical visibility is considerably larger than that expected classically. This is clearly seen in Fig. 3 where the visibility $\mathcal{V}$ is plotted as a function of the coherent local oscillator amplitude α for the quantum-mechanical single-photon state and the classical wave model with $\beta=1$.

We thus see that by measuring the coincidence probability in a pair of detectors, it is possible to distinguish between the classical and quantum-mechanical models. If the detector efficiencies are less than unity, coincidences will be missed, but the ratio of minimum to maximum coincidence rates as the relative phase of the local oscillators is varied is unaffected, provided that spurious coincidences due to the dark count rates of the detectors are removed. Even without compensating for the dark count rates, it is possible to test the quantum-mechanical prediction that for the correct choice of local oscillator phases, there is no increase in the coincidence count rate in one of the detector pairs when the one-photon input is applied.

Preparation of a single-photon state may be achieved experimentally by using the signal beam of a parametric amplifier while monitoring photons in the idler beam.[15] Hong and Mandel[16] describe an experiment in which a nearly pure single-photon state was produced using this method. If the pump for this parametric amplifier is derived by frequency doubling a coherent beam, this provides a convenient source for the local oscillators required in this experiment.

In order to rigorously rule out classical explanations for the quantum-mechanical result, it is necessary to show that Bell's inequality may be violated. The use of phase-sensitive detectors for showing violations of Bell's inequalities have previously been discussed,[12,17] and only a summary of the results relating to this experiment will be given. An intensity correlation coefficient is used which involves all four photodetectors

$$E(\theta_1,\theta_2)=\frac{\langle(I_{\hat{d}_1}-I_{\hat{c}_1})(I_{\hat{d}_2}-I_{\hat{c}_2})\rangle}{\langle(I_{\hat{d}_1}+I_{\hat{c}_1})(I_{\hat{d}_2}+I_{\hat{c}_2})\rangle}. \tag{14}$$

Evaluating this in terms of the statistics of the input mode $\hat{u}$ where $\hat{v}$ is the vacuum yields

$$E(\theta_1,\theta_2)=-\frac{\alpha^2\{\langle\hat{u}^\dagger\hat{u}\rangle\sin(\theta_2-\theta_1)+|\langle\hat{u}^2\rangle|\sin(\theta_2+\theta_1-\xi)\}}{\alpha^4+\langle\hat{u}^\dagger\hat{u}\rangle\alpha^2+\frac{1}{4}\langle\hat{u}^{\dagger 2}\hat{u}^2\rangle}, \tag{15}$$

where $\langle\hat{u}^2\rangle=R\exp(i\xi)$. When a single-photon input is used for $\hat{u}$, this reduces to

$$E(\theta_1,\theta_2)=[1/(\alpha^2+1)]\sin(\theta_1-\theta_2). \tag{16}$$

If the coefficient of $\sin(\theta_1 - \theta_2)$ is greater than $1/\sqrt{2}$, it is well known that this functional form for the correlation allows a violation of Bell's inequalities. This is clearly possible if α is made sufficiently small. It has been shown[17] that such a violation of Bell's inequalities is not possible if $\hat{u}$ is in a coherent state, no matter how small the input amplitude may be.

In conclusion, some of the most striking features of nonlocality in quantum mechanics may be demonstrated using phase-sensitive measurements on the field produced by a single photon. These effects may not be explained classically using a particle, wave, or hidden-variable theory involving local causality.

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