Kosterlitz-Thouless Transition in the Smectic Vortex State of a Layered Superconductor

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We investigate the relation between the vortex structure and the dissipation mechanism in strongly layered superconductors when a large magnetic field is applied parallel to the layers. At high enough temperatures the vortex lattice undergoes a transition to a smectic state characterized by a vanishing interlayer shear. The elastic properties of this state lead to novel conditions for Kosterlitz-Thouless-type behavior, resulting in an algebraic current-voltage characteristic down to vanishing current densities.

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The interplay between the vortex structure and the pronounced intrinsic pinning in strongly layered superconductors such as $Bi_2Sr_2CaCu_2O_8$ (BiSCCO) has recently attracted a lot of interest.[1-3] A large critical current density is obtained for the situation where both the magnetic field and the current flow are directed along the Cu-O planes.[4] With the Lorentz force parallel to the c axis, dissipation is initiated by an activation process where a finite segment of a vortex (nucleus) jumps across the superconducting layer. Such activated creep phenomena depend in a nontrivial way on the underlying vortex structure.

In this Letter we investigate the relation between the vortex structure and the dissipation mechanism in strongly layered superconductors. We show that the expanding nucleus produces a string of dislocations within the vortex lattice, which in turn leads to the confinement of the nucleus. For high enough temperatures the vortex lattice is expected to melt[5-7] and the layered structure of the material supports a smectic vortex state which is characterized by a vanishing interlayer shear. As a consequence the confining string "melts," giving way to a logarithmic interaction between the pancake vortices. This logarithmic interaction is preserved even at large distances as it is due to the elastic properties of the smectic state and leads to a novel Kosterlitz-Thouless-type behavior.[8,9]

Intrinsic pinning in layered superconductors has been studied both experimentally and theoretically. A particularly puzzling result is the independence of the resistance upon the angle ϑ between the magnetic field $\mathbf{H}$ and the probing current density $\mathbf{j}$ ($\mathbf{j},\mathbf{H}\perp\mathbf{c}$) as reported by Woo *et al.*[10] and by Iye, Nakamura, and Tamegai.[11] Kes *et al.*[12] have suggested that a finite field component perpendicular to the layers might explain these findings, whereas in our approach the field is mainly directed parallel to the superconducting layers. We will give some comments on this phenomenon below. For moderate magnetic fields and $\mathbf{j}\perp\mathbf{H}$, Chakravarty, Ivlev, and Ovchinnikov[13] have shown that the single-vortex nucleus is confined at low current densities in the case of a vortex lattice and the activation of a vortex bundle leads to a diverging activation energy at small current densities. Without the concept of a liquid vortex state it seems to be impossible to explain the activated flux-flow measurements which show no divergent dependence of the activation energy on the current density.[2] Here we study the situation of high fields where the magnetic flux is concentrated along the layers and where a new type of Kosterlitz-Thouless (KT) behavior is found. Also, we explicitly discuss the general case of an arbitrary angle between the current flow and the magnetic field. Kosterlitz-Thouless behavior in layered high-T_c superconductors has been reported by various authors, predominantly in moderate[14] or zero[15] magnetic fields.

In the following we first discuss the nucleation process for the case of a vortex lattice. The problem is formulated within dislocation theory[5] which allows us to treat the angular dependence in a straightforward way. We then discuss the transition to the smectic vortex state which is characterized by a vanishing interlayer shear. For the smectic state, the elastic part of the activation energy changes dramatically and a logarithmic dependence on the size of the nucleus is found at all length scales. We determine this energy using simple arguments based on continuum elastic theory and also present a more careful derivation using the London approach. We find that the current-voltage characteristic is given by a power law down to low current densities.

Consider the situation where a large magnetic field $\mathbf{H}$ is applied parallel to the Cu-O planes (see Fig. 1, we choose a coordinate system with $\mathbf{z}\parallel\mathbf{c}$ axis, $\mathbf{y}\parallel\mathbf{H}$). For a magnetic field $H=H_d=\Phi_0/2\sqrt{3}\Gamma d^2$ the size of the unit cell along the c axis equals the interlayer distance d. Here, $\Gamma=\lambda_c^2/\lambda_{ab}^2$ is the anisotropy ratio which is ~ 3000 in BiSCCO (Ref. 16) and $\lambda_c,\lambda_{ab}\simeq 0.14$ μm denote the London penetration depths for fields directed along the Cu-O planes and parallel to the c axis. When H is increased beyond H_d the vortex cores start to concentrate along each interlayer plane. For BiSCCO, $H_d\simeq 5$ T ($d\simeq 15$ Å) and the intralayer distance between the cores is $l=\Phi_0/Hd$ ($l\simeq 2800$ Å for $H=H_d$). Upon application

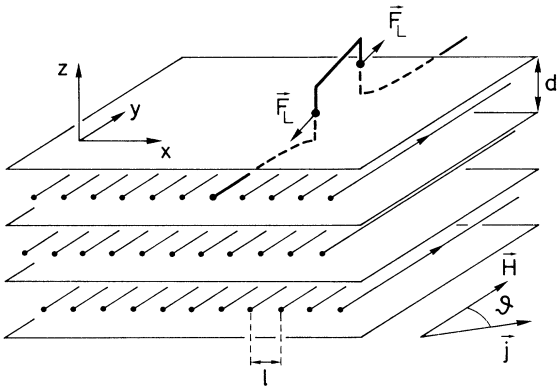

FIG. 1. Vortex structure for large magnetic fields $H \geq H_d$ in a strongly layered superconductor. At high enough temperatures the vortex lattice melts and the interlayer shear modulus vanishes, resulting in a smectic vortex state as illustrated (drawing not to scale).

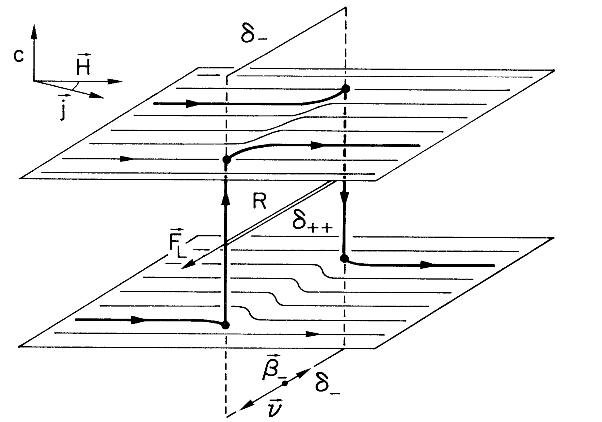

FIG. 2. Nucleation process in the vortex lattice. The moving pancake vortices (vertical segments of the loop) generate a string of three dislocations $\boldsymbol{\nu}_-$, $\boldsymbol{\nu}_{++}$, and $\boldsymbol{\nu}_-$ along their path with the total topological charge zero (drawing not to scale).

of a current **j** along the Cu-O planes the Lorentz force tends to push the vortices up. Because of the strong intrinsic pinning along the c axis, the motion has to proceed via a nucleation process as shown in Fig. 1. A finite segment of the vortex is activated across the layer.[17] The nucleus is bounded by two pancake vortices[18] parallel to the c axis. The Lorentz force acting on the pancake vortices drives them into opposite directions such that the nucleus expands and the vortex moves up by one layer.

Let us consider the energies involved in creating the nucleus and in the expansion. We have to account for three contributions; the magnetic energy E_{mag}, the work done by the Lorentz force E_L, and the elastic energy E_{el}. (i) Consider first the magnetic energy between the two pancake vortices separated by a distance R: For a strictly two-dimensional system (no interlayer coupling, $\lambda_c = \infty$) we obtain the magnetic energy $E_{\text{mag}}(R) = 2e_0 d \times \ln(R/\xi_{ab})$, where $e_0 = (\Phi_0/4\pi\lambda_{ab})^2$ and ξ_{ab} denotes the correlation length in the plane, $\xi_{ab} \simeq 38$ Å in BiSCCO. (We neglect the core condensation energy $E_c = e_0 d/2$ since our calculation is only logarithmically correct.) As we turn on the Josephson interaction between the layers ($\lambda_c < \infty$), the logarithmic interaction is cut off[13] at $R \simeq d\sqrt{\Gamma}$. For $R > d\sqrt{\Gamma}$ the magnetic interaction increases like $1/R$,

$$E_{\text{mag}}(R) = 2e_0 d[\ln(d/\xi_c) - d\sqrt{\Gamma}/4R], \quad R < \lambda_c .$$

(ii) The second contribution is due to the driving Lorentz force F_L (see Fig. 1) which adds an energy $E_L(R) = -j\Phi_0 dR/c$ to the total energy of the nucleus. (iii) The third contribution is the elastic energy produced by the distortion of the vortex structure between the Cu-O layers. Whereas the contributions E_{mag} and E_L are of a general nature, this third part depends crucially on the vortex structure. We start with a discussion of the vortex lattice. Depending on the angle ϑ between the magnetic field **H** and the current density **j**, the Lorentz force drives the pancake vortices to move past the vortex lines in the planes. As a pancake vortex crosses one of the vortex lines, a reswitching process rearranges the vortex lattice such that a string of dislocations is produced along the path of the moving vortex. The process is illustrated in Fig. 2. The string consists of three dislocations δ_-, δ_{++}, and δ_- with Burgers vectors $\boldsymbol{\beta}_-$, $\boldsymbol{\beta}_{++}$, and $\boldsymbol{\beta}_-$, $-2\boldsymbol{\beta}_- = \boldsymbol{\beta}_{++} - (2l,0,0)$, and dislocation axes $\boldsymbol{\nu}_- = \boldsymbol{\nu}_{++} = \boldsymbol{\nu}$ pointing along the path of the moving pancake vortex. For an angle $\vartheta = 0°$ (90°) the dislocations are pure screw (edge) whereas their character is mixed for all angles in between. Let us estimate the elastic energy of the string: For $\vartheta = 0°$ (90°) the defect involves both shear along the z axis and tilt (compression) along the y (x) axis. The shear energy density is $\varepsilon_s = c_{66}(\partial u/\partial z)^2/2 \simeq c_{66}(l/d)^2/2$, with u denoting the displacement field along x and c_{66} the interlayer shear modulus.[19] Similar expressions hold for the tilt (ε_t) and compression (ε_c) energy densities. For the equilibrium configuration the relevant elastic energies are equal ($\varepsilon_s = \varepsilon_t$ for $\vartheta = 0°$, $\varepsilon_s = \varepsilon_c$ for $\vartheta = 90°$) and we obtain the length scales for tilt and compression, $t = d(c_{44}/c_{66})^{1/2}$ and $c = d(c_{11}/c_{66})^{1/2}$. The string contributes an elastic energy $E_{\text{el}}(R,\vartheta) = dR(c\varepsilon_c \sin^2\vartheta + t\varepsilon_t \cos^2\vartheta)$ to the activation energy of the nucleus. Using known expressions for the (nonlocal) elastic moduli

$$c_{11}(\mathbf{k}) \simeq c_{44}(\mathbf{k}) = (H^2/4\pi)(1 + \lambda_c^2 k_x^2 + \lambda_{ab}^2 k_z^2)^{-1},$$

$$c_{66} = \Phi_0 H/(8\pi\lambda_{ab})^2 \Gamma^{3/2},$$

we obtain the estimate $E_{\text{el}}(R) \simeq e_0 R/\sqrt{\Gamma}$ independent of the angle ϑ. Here we have used the Brillouin-zone cutoffs $k_x \simeq 2\pi/l$ and $k_z \simeq \pi/2d$. In our expression for the elastic moduli we have neglected corrections due to finite reciprocal-lattice vectors which become relevant for wave vectors near the Brillouin-zone edge and thereby underestimate somewhat the magnitude of $E_{el}(R)$.

Summing up all three contributions we obtain the energy of the nucleus $E_n = E_{mag} + E_L + E_{el}$. The linear increase of the elastic energy with distance R has to compete with the energy gain due to the Lorentz force, leading to the confinement of the nucleus for current densities smaller than the critical value $j_1 = ce_0/\Phi_0 d\sqrt{\Gamma}$. For BiSCCO we find a value of $j_1 \sim 8\times10^6(1-T/T_c)$ $\mathrm{A\,cm^{-2}}$.

In the above calculation we have neglected the barrier for crossing and reswitching of the vortices. Consider the case of a pure screw dislocation ($\vartheta = 0°$). A simple estimate for the reswitching barrier is given by the maximal elastic energy of the vortex before reswitching, which is $E_r = e_0 l/\sqrt{\Gamma}$. However, this is exactly the energy needed to produce the segment l of the string which is already taken into account by $E_{el}(R)$. Within the precision of our calculation we thus find no angular dependence of the nucleation energy.

We have to conclude that for current densities $j < j_1$ there is no dissipation due to single-vortex activation across the layers as long as the underlying vortex structure is a lattice. However, for large enough temperatures the vortex lattice is expected to melt. The resulting smectic state (see Fig. 1) is characterized by a vanishing interlayer shear modulus $c_{66} = 0$. When the magnetic field is increased beyond $3H_d$, the shear modulus of the lattice is exponentially suppressed by a factor of order $p^2\exp(-p)$, $p = \pi H/\sqrt{3}H_d$, and the melting of the lattice becomes even more favorable.[20] The vanishing shear in the smectic state leads to the melting of the confining string. The vortices relax to the configuration shown in Fig. 3 which is characterized by two pairs of edge dislocations. Note that the type of dislocations studied here represent magnetic monopoles if considered as objects in isolated planes.

We have to recalculate the elastic energy contribution to the nucleus for the smectic vortex state. A simple estimate can be found from dislocation theory, where the energy of an edge dislocation pair separated by a distance R is given by $(dl^2/2\pi)c_{44}\ln(R/l)$, showing the well-known logarithmic dependence on distance R. For quantitative results we recalculate the elastic energy within the framework of the London model. The free energy of a system of vortices placed at positions $\mathbf{R}_k$ is

$$F = \frac{\Phi_0^2}{8\pi}\int \frac{d^3q}{(2\pi)^3}\frac{|v_x|^2+|v_y|^2+|v_z|^2\gamma(q)}{1+\lambda_c^2(q_x^2+q_y^2)+\lambda_{ab}^2 q_z^2},$$

with

$$\mathbf{v}(\mathbf{q}) = \sum_k \int d\mathbf{R}_k \exp(-i\mathbf{q}\cdot\mathbf{R}_k)$$

and $\gamma(q) = (1+\lambda_c^2 q^2)/(1+\lambda_{ab}^2 q^2)$. The term proportional to $|v_z|^2$ produces the magnetic energy of the two pancake vortices discussed above. Here we are interested in the elastic energy between the planes which is provided by the terms $|v_x|^2+|v_y|^2$. We integrate over q_z and obtain

$$F - E_{mag} = \frac{\Phi_0^2}{16\pi\lambda_{ab}} \sum_{\substack{nm\\NM}} \int d\vec{R}_N^n\, d\vec{R}_M^m\, Q_{N-M}(\vec{R}_N^n - \vec{R}_M^m),$$

with

$$Q_N(R) = \int \frac{d^2q}{(2\pi)^2}\frac{\exp[i\vec{q}\cdot\vec{R} - d|N|(1+\lambda_c^2q^2)^{1/2}/\lambda_{ab}]}{(1+\lambda_c^2q^2)^{1/2}}.$$

We denote by $\vec{R}_N^n$ the position of the nth vortex in the layer N. $\vec{R}_N^n$ is a planar vector with components $\vec{R}_N^n = (nl + u_N(nl,y), y)$. For a slowly varying displacement field $u_N(nl,y)$ we can substitute the summations over n and m by an integration and obtain for the quadratic elastic energy the result

$$E_{el} = \frac{B^2d^2}{16\pi\lambda_{ab}}\sum_{NM}\int d^2r_1\, d^2r_2 \nabla u_N(\vec{r}_1)\nabla u_M(\vec{r}_2) \times Q_{N-M}(\vec{r}_1 - \vec{r}_2).$$

The above expression does not depend on displacement gradients along the c axis $(u_N - u_M)$ due to the absence of interlayer shear.

The displacement field $u_N(\vec{r})$ can be split in two terms, $u_N = u_N^r + u_N^s$, where u_N^r is the fluctuating regular part of the field which provides the liquid properties. The singular part u_N^s describes the four edge dislocations shown in Fig. 3, $u_N^s(\vec{r}) = [s(\vec{r}) - s(\vec{r}-\vec{R})](\delta_{N,1} - \delta_{N,0})$ with $s(\vec{r}) = (l/2\pi)\arctan(y/x)$. Because of the condition $\nabla^2 u_N^s = 0$ the contributions of the regular and singular parts decouple. For large distances $R \gg d\sqrt{\Gamma}$ only small wave vectors $q \ll 1/d\sqrt{\Gamma}$ are relevant and the interaction between the vortices becomes local. The elastic energy shows the expected logarithmic behavior, $E_{el}(R) = 2e_0 d$

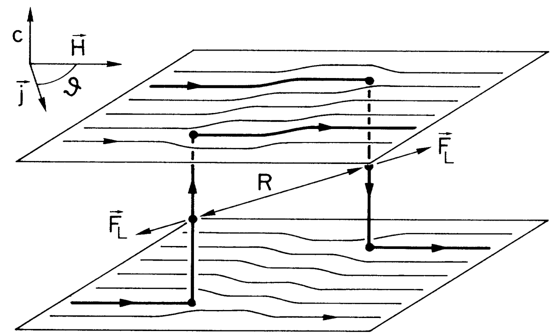

FIG. 3. Nucleation process in the vortex liquid. The vanishing interlayer shear has led to the melting of the elastic string present in the vortex lattice. After relaxation, a pair of edge dislocations (magnetic monopoles) appears in each plane with a logarithmic interaction between the two monopoles constituting each pair.

$\times \ln(R/d\sqrt{\Gamma})$. For small distances a linear dependence is obtained, $E_{el}(R) \simeq e_0 R/\sqrt{\Gamma}$, $R < d\sqrt{\Gamma}$. Comparison with (i) above shows that the logarithmic interaction smoothly changes its origin from magnetic to elastic as we cross $R = d\sqrt{\Gamma}$ and *persists to arbitrarily large distances.*

We substitute the elastic energy for the liquid into the expression for the activation energy $E_n(R)$ and minimize with respect to R in order to obtain the critical size of the nucleus $R_c = 2e_0 c/j\Phi_0$ and the activation energy $U \simeq 2e_0 d \ln(2ce_0/j\Phi_0\xi_{ab})$. Typical values for BiSCCO are $R_c \simeq 80$ μm and $U \simeq 130$ meV (a temperature $T_c - 5$ K and a current density $j = 10^3$ A cm^{-2} have been chosen). The logarithmic dependence on the current density produces the KT-type algebraic current-voltage characteristic $V \sim j^{a(T)}$ with a temperature-dependent exponent $a(T) = 2e_0 d/k_B T$. This simple expression for $a(T)$ is valid if $a \gg 1$ and a jump $\Delta a = 2$ is expected to occur at the KT transition temperature, similar to the one observed in the experiment of Ref. 15.

In the liquid state, the deconfining critical current density j_1 introduced above becomes the crossover current density separating the two regimes where the logarithmic interaction is of magnetic ($j > j_1$) and of elastic ($j < j_1$) origin, respectively. Note that for the zero- or moderate-field cases the interpretation of the algebraic current-voltage characteristic in terms of KT behavior is limited by the condition $j \gtrsim j_1$.[5] In the high-field case discussed above, however, KT behavior is expected over the whole range of current densities down to very low currents.

Regarding the angular dependence of the nucleation process we have to take into account the reswitching barrier which can be rewritten as $E_r = 2\sqrt{3} e_0 d H_d/H$. In order to have a smooth diffusion process for generating the nucleus, the reswitching barrier E_r has to be smaller than the thermal energy $k_B T$. This is possible only for sufficiently high magnetic fields; for BiSCCO, $H \gtrsim 15$ T. Thus we find that for large enough fields the reconnection ripple in the potential-energy landscape is small and within our exponential accuracy there is no dependence of the dissipation on the angle between **j** and **H**.

In summary, we have shown that the vanishing interlayer shear, characteristic of the smectic vortex state, leads to the deconfinement of the single-vortex nucleus. The relaxed excitation is characterized by two pairs of edge dislocations which produce the logarithmic interaction between the two pancake vortices at large distances $R > d\sqrt{\Gamma}$. The elastic properties of the smectic state provide the necessary conditions for the realization of KT-type behavior at all length scales. For high enough magnetic fields our model is able to explain (within exponential accuracy) the independence of the dissipation upon the angle between the magnetic field and the applied current.

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[1]M. Tinkham, Phys. Rev. Lett. **61**, 1658 (1988).

[2]T. T. M. Palstra, B. Batlogg, R. B. van Dover, L. F. Schneemeyer, and J. V. Waszczak, Appl. Phys. Lett. **54**, 763 (1989); Phys. Rev. B **41**, 6621 (1990).

[3]Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).

[4]M. Tachiki and S. Takahashi, Solid State Commun. **70**, 291 (1989); B. I. Ivlev and N. B. Kopnin, J. Low Temp. Phys. **77**, 413 (1988); Phys. Rev. Lett. **64**, 1828 (1990).

[5]M. C. Marchetti and D. R. Nelson, Phys. Rev. B **41**, 1910 (1990); D. R. Nelson and S. Seung, Phys. Rev. B **39**, 9153 (1989).

[6]E. H. Brandt, Phys. Rev. Lett. **63**, 1106 (1989).

[7]A. Houghton, R. A. Pelcovits, and A. Sudbø, Phys. Rev. B **40**, 6763 (1989).

[8]V. L. Berezinsky, Zh. Eksp. Teor. Fiz. **61**, 1144 (1971) [Sov. Phys. JETP **34**, 610 (1972)].

[9]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).

[10]K. C. Woo, K. E. Gray, R. T. Kampwirth, J. H. Kang, S. J. Stein, R. East, and D. M. McKay, Phys. Rev. Lett. **63**, 1877 (1989).

[11]Y. Iye, S. Nakamura, and T. Tamegai, Physica (Amsterdam) **159C**, 433 (1989).

[12]P. H. Kes, J. Aarts, V. M. Vinokur, and C. J. van der Beek, Phys. Rev. Lett. **64**, 1063 (1990).

[13]S. Chakravarty, B. I. Ivlev, and Yu. N. Ovchinnikov, Phys. Rev. Lett. **64**, 3187 (1990); Phys. Rev. B **42**, 2143 (1990).

[14]D. H. Kim, A. M. Goldman, J. H. Kang, and R. T. Kampwirth, Phys. Rev. B **40**, 8834 (1989).

[15]S. N. Artemenko, I. G. Gorlova, and Yu. I. Latyshev, Phys. Lett. A **138**, 428 (1989).

[16]D. E. Farrell, S. Bonham, J. Foster, Y. C. Chang, P. Z. Jiang, K. G. Vandervoort, D. J. Lam, and V. G. Kogan, Phys. Rev. Lett. **63**, 782 (1989).

[17]For a description of vortices in layered superconductors, see S. Doniach, in *High Temperature Superconductivity,* Proceedings of the Los Alamos Symposium 1989, edited by K. S. Bedell *et al.* (Addison-Wesley, Reading, MA, 1990), p. 410.

[18]J. Clem, Bull. Am. Phys. Soc. **35**, 260 (1990).

[19]This corresponds to the easy shear mode with modulus $c_{66}^{(e)}$ in the notation of V. G. Kogan and L. J. Campbell, Phys. Rev. Lett. **62**, 1552 (1989).

[20]B. I. Ivlev, N. B. Kopnin, and V. L. Pokrovsky, J. Low Temp. Phys. **80**, 187 (1990).