Excitation of Large-Amplitude Ion-Wave Wake Fields

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Plasma wake fields in the ion-wave regime have been excited by injecting ion bunches with a variety of shapes. An excited-wave amplitude ($\delta n/n_0$) of up to 17% has been observed for a bunch falloff time less than the wake-field period. The large-amplitude wake wave damps out, along with oscillations of the wave envelope. A simple linear theoretical model is developed, which can explain most of the observed phenomena, except for the large-amplitude oscillations.

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Plasma-based high-energy particle acceleration has been intensively investigated for the last ten years, and some of the new ideas for particle accelerators have successfully been demonstrated.[1-11] The plasma wake-field accelerator is one such new scheme. Investigations of the plasma wake field began with the pioneering work of Chen *et al.*,[8] and the first experimental evidence was reported by Rozenzweig and co-workers[9,10] followed by Nakanishi *et al.*[11] In these investigations, superrelativistic electron bunches of short pulse duration propagating through a high-density plasma were employed. The excited wake field has an acceleration or deceleration gradient with typical values of about 150 keV/m in Ref. 10 and 10–12 MeV/m in Ref. 11. Specifically, Rozenzweig and co-workers have observed the wave shapes excited by the driving bunch, including nonlinear effects such as higher harmonics in the frequency spectrum of the wake plasma wave. The excited wave typically has a period of about 40–50 psec, and therefore the effects of pulse shaping of the driving bunch have not yet been clarified. One reason may be the difficulty of precisely controlling the bunch shape for such a short time duration. In the work by Nakanishi *et al.*, a train of six bunches with energy of about 250 MeV travels through the plasma and the wake field excited by each bunch interacts with the others to make a resonance structure where the whole wake field adds up to "decelerate" all of the bunches in the train.

In the present study, we have succeeded in exciting a wake field in the ion-wave regime with a long pulse duration (typically 50 μsec) by employing a variety of driving bunch shapes; specifically the falloff time of the driving bunch can be made short compared with the wave oscillation period. The latter is one of the key requirements for exciting a large-amplitude wake field, although the phenomena are observed in the nonrelativistic regime. The necessity of pulse shaping has been suggested by Bane, Chen, and Wilson,[12] but no experimental investigations have, to our knowledge, been reported. The excited wake ion wave has a maximum amplitude $\delta n/n_0$ up to $\simeq 17\%$. Such a large-amplitude wave has rarely been observed so far in the ion-wave regime except for the ion-wave soliton which has a maximum amplitude of 20%.[13,14] Therefore, the resultant wave is also useful for studying a variety of nonlinear wave phenomena including strong wave-particle interactions. In the beat-wave experiments, however, Umstadter *et al.*[3] have reported the observation of a large-amplitude electron plasma wave with a maximum amplitude of 16%, which is vital for the success of beat-wave accelerators. Rozenzweig *et al.*[10] have also observed a nonlinear plasma wake field with an estimated amplitude of $\gtrsim 30\%$ and discussed particle-trapping effects, although the nonlinear behavior in the ultrarelativistic case has previously been treated theoretically.[15,16]

The ion-regime experiments are conducted in a double-plasma machine which consists of a cylindrical stainless-steel chamber of 80 cm length by 60 cm diameter covered with many multidipole magnets as illustrated in Fig. 1.[14] Typical plasma parameters are an argon plasma density n_0 of $(2\text{–}5)\times 10^8\ \text{cm}^{-3}$, electron temperature $T_e \simeq 2\text{–}3$ eV, ion temperature $T_i \lesssim T_e/10$, and argon neutral gas pressure $p_0 \simeq (2\text{–}3)\times 10^{-4}$ Torr (vacuum base pressure $\lesssim 1\times 10^{-6}$ Torr). The separation grid between the two plasmas consists of stainless-steel mesh with 300 lines/in. The signals, including an ion bunch for exciting the wake field and the excited wake wave, are received by a tiny cylindrical probe (0.5 mm diameter by 1 mm length) which is biased up to 10 V with

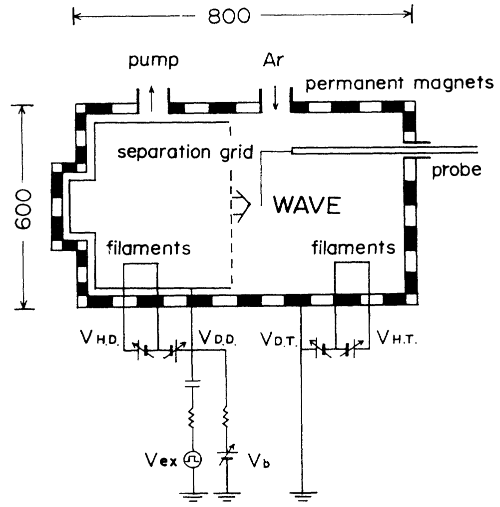

FIG. 1. Schematic description of the experimental apparatus.

respect to ground potential in order to detect the electron saturation current. The plasma potential is about 2–3 V positive from the ground potential.

Typical wave forms observed on the oscilloscope screen are shown in Fig. 2. Here, the top trace displays the launched ion bunch (in 5 V/div) and the bottom trace shows the excited wake field (in 10 mV/div). As clearly seen from the figure, following ion-bunch termination, the wake field is excited with a nearly sinusoidal wave form and with weak temporal damping. More precisely, the wave form is not exactly sinusoidal but even triangle shaped. A ramp-shaped bunch is found to excite a wave with the largest amplitude. In Fig. 2(c), a half-cycle sinusoidal bunch wave form is employed, and only the present narrow wave with width of about 8 μsec can excite the wake field; otherwise, no recognizable wake field can be excited. The maximum amplitude $\delta n/n_0$ of the wake field shown in Fig. 2(a) is $\simeq 13\%$ (26% peak to peak), where δn is the wave amplitude of the density fluctuations and n_0 is the steady-state electron density. In the initial stages of the excited wake field, we see a strong spike, the nature of which is not fully understood yet. However, it may be related to the excitation of an ion-wave soliton.[14] Since a ramp-shaped ion bunch can excite a wake field for a wide range of parameters, hereafter, only this shape of driving bunch was employed.

In order to obtain more insight into the nature of the wake field, we have performed several experiments. Figure 3(a) displays the wave amplitude excited by the ion bunch as a function of the falloff time ΔT. Here, a rise-time constant of 53 μsec and a maximum amplitude of 7 V are kept constant throughout the experiments. When the falloff time constant ΔT is increased, the wave amplitude is found to decrease sharply followed by several bounces. Here, the solid circles stand for the third peak counted from the beginning of the wake field and the

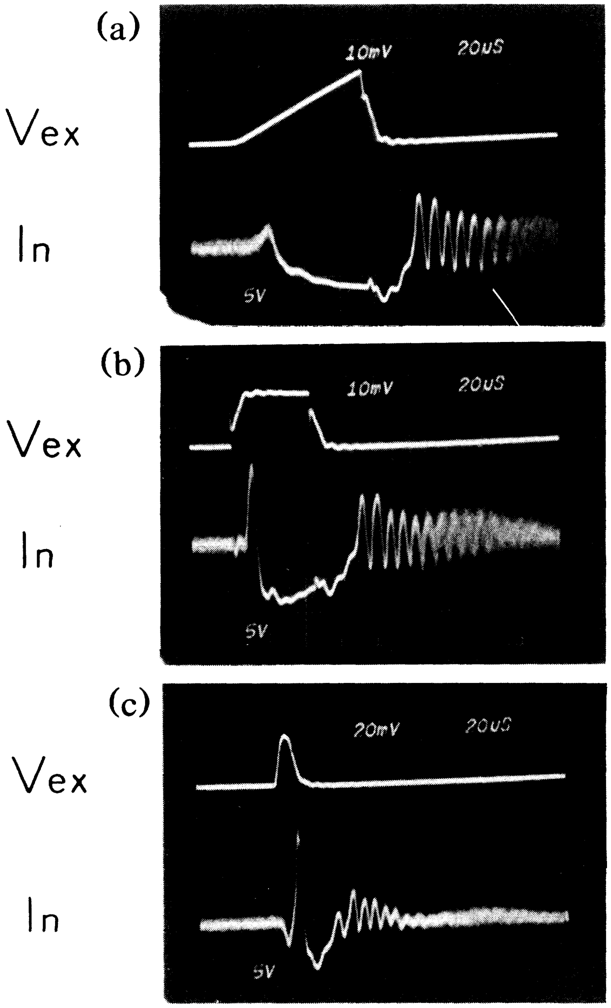

FIG. 2. Typical examples of wave forms excited by a variety of ion-bunch shapes. A ramp-shaped ion bunch is used in (a), a square shape in (b), and a sinusoidal shape in (c). In (c), only this shape can excite the wake field for the present parameters.

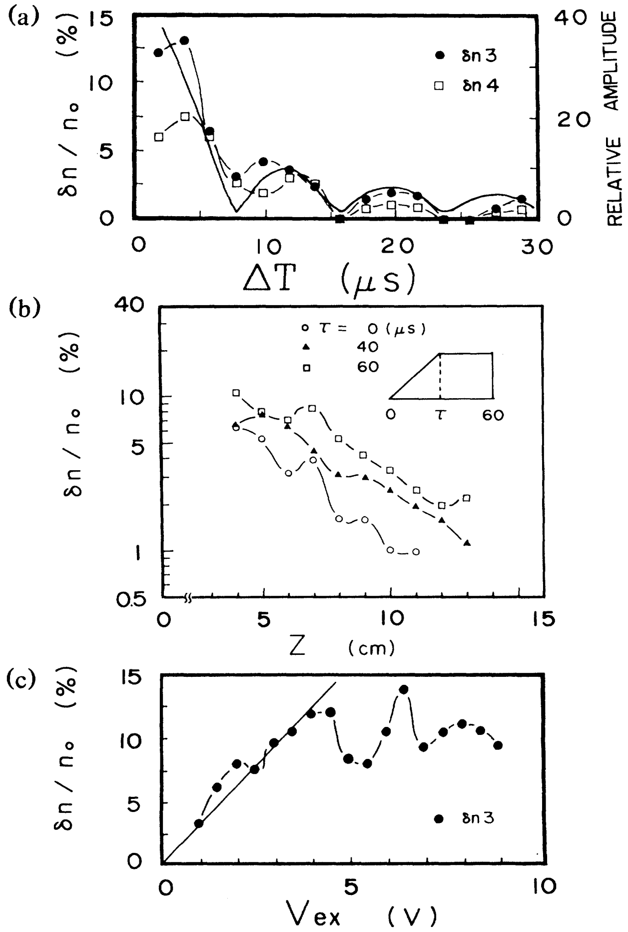

FIG. 3. Characteristics of the wake field observed in the ion-wave regime. (a) Wave amplitude as a function of the falloff time constant ΔT; (b) damping characteristics of the wave as a parameter of the rise-time constant τ; and (c) wave amplitude as a function of the excitation voltage.

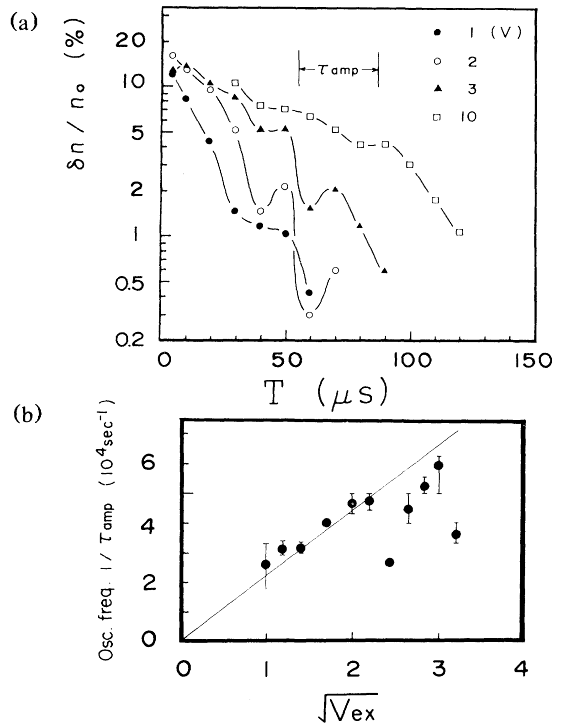

FIG. 4. Nonlinear wave phenomena. (a) Damping characteristics as a function of the exciting voltages; (b) inverse of the oscillating period of the amplitude oscillation vs the square root of the exciting voltages.

open squares for the fourth peak. The solid line shows the theoretical result which will be discussed latter.

When the rise-time constant becomes shorter, with a constant falloff time of 2 μsec and an excitation amplitude of 4 V, the spatial damping of the wave becomes stronger as shown in Fig. 3(b). The shape of the driving bunch used in this experiment is schematically shown in the inset. The open squares correspond to a ramp shape with a linear rise time of 60 μsec, the solid triangles to a rise-time constant of 40 μsec with a plateau of 20 μsec, and the open circles to a square wave with sharp rise time of less than 2 μsec.

The amplitude of the wake field increases almost linearly with the driving-bunch excitation voltage as shown in Fig. 3(c). Here, the third peak of the wake field is plotted. As clearly seen, the wake field increases almost linearly up to about $\delta n/n_0=12.5\%$ at $V_{\mathrm{ex}}=4$ V, although some bumps are seen at $V_{\mathrm{ex}}\simeq 2$ V. Several runs show that the maximum saturation amplitude $\delta n/n_0$ obtained so far is $\simeq 17\%$ for the present plasma parameters.

When the amplitude change of the wake field is plotted as a function of time, amplitude oscillations are observed as seen in Fig. 4(a), i.e., the envelope of the wake field oscillates while damping. As the amplitude of the wake field increases, the oscillation period of the wave envelope decreases. In Fig. 4(b), the inverse of the oscillation period τ_{amp} is plotted as a function of the square root of the maximum excitation voltage of the ion bunch. Up to about 4 V, τ_{amp}^{-1} increases linearly, and then gradually saturates. This phenomenon is the nonlinear Landau damping which has been previously studied,[17,18] but the maximum amplitude of the present experiments is significantly larger than that of the earlier investigations. When the wave amplitude becomes larger, the average damping rate of the wake field becomes smaller, as also seen in Fig. 4(a). Further precise investigations on these phenomena are now under way.

The dispersion relation of an ion acoustic wave can be written as $\omega^2=\omega_{pi}^2(\lambda_D k)^2\{1+(\lambda_D k)^2\}^{-1}$, where ω_{pi} is the ion plasma frequency, λ_D is the Debye length, and otherwise standard notation is employed. The wake field in the present situation may be expressed as[19]

$$E(y)=-4\pi R_0\int_0^y d\eta\cos[k_p(y-\eta)]\rho_b(\eta)\,. \tag{1}$$

Here, a correction factor R_0, which arises from the shape of the ion bunch, is given as

$$R_0=\begin{cases}1 & (k_pa\gg 1),\\ -(k_pa)^2\ln(k_pa) & (k_pa\ll 1),\end{cases} \tag{2}$$

and the wave number of the wake field k_p is

$$k_p=\omega/(V-V_d)\,, \tag{3}$$

where a and V are, respectively, the radial size and the velocity of the ion bunch, and V_d is the drift velocity of the background plasma (in the present case $V_d=0$). If we employ a triangular-shaped ion bunch as shown in Fig. 5, the charge density may be written as

$$\rho_b(\eta)=\rho_0 f(\eta)$$

and

$$f(\eta)=\begin{cases}k_p\eta & (0\le y\le b),\\ \dfrac{b'-\eta}{b'-b}k_pb & (b\le y\le b').\end{cases} \tag{4}$$

When the ion bunch propagates through the plasma, the fluctuating potential of the wake field can be obtained by inserting Eq. (4) into Eq. (1), yielding

$$\phi(y)=\begin{cases}\dfrac{4\pi\rho_0R_0}{k_p}\left[\dfrac{1}{k_p}\sin(k_py)+\dfrac{1}{k_p}\left[1+\dfrac{b}{b'-b}\right]\sin\{k_p(y-b)\}-\dfrac{1}{k_p}\dfrac{b}{b'-b}\sin\{k_p(y-b')\}\right] & (b\neq b') \quad (5\mathrm{a})\\ \dfrac{4\pi\rho_0R_0}{k_p}\left[\dfrac{1}{k_p}\sin(k_py)+\dfrac{1}{k_p}\sin\{k_p(y-b)\}+b\cos\{k_p(y-b)\}\right] & (b=b')\,. \quad (5\mathrm{b})\end{cases}$$

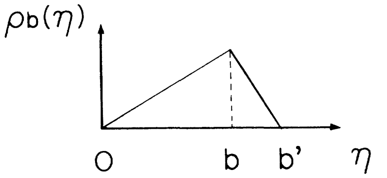

FIG. 5. Characteristic shape of the ion bunch used in the theoretical model.

Numerical results obtained from Eq. (5a) are shown by the solid line in Fig. 3(a), showing fairly good agreement with the experiments. Here, $R_0=1$ (one-dimensional case), $V_{\text{bunch}} \simeq 4.3\times10^5$ cm/sec, and $V_{\text{wake}} \simeq 1.8\times10^5$ cm/sec are employed. If we also assume that the density of the ion bunch is proportional to the exciting voltage V_{ex}, the wave amplitude increases with V_{ex} as shown in Fig. 3(c) by the solid line, again showing a linear dependence in the small-amplitude regime. When the pulse width is changed with the excitation voltage held constant in Eq. (5a), the amplitude of the excited wake field increases with the pulse width, accompanied by a small-amplitude fluctuation of the period of about 10 μsec. The experimental results (not shown here), however, have almost constant amplitude, but with a small fluctuation in amplitude with a period of $\simeq 11$ μsec, showing fairly good agreement. The present theoretical interpretations are based on the linear theory, but the observed wake fields must be in a nonlinear regime, and we need more elaborate theoretical work for a full understanding of the present phenomena such as shown in Ref. 15. Furthermore, in the previous studies performed by the Argonne group, a nonlinear nature in both longitudinal and transverse wake fields has been observed in the strong relativistic regime. In the present study, however, the situation is completely different: The phenomena are one dimensional and in a low-frequency nonrelativistic regime. However, a similar nature of the phenomena has been observed.

In summary, we have excited the wake field in the ion-wave regime by an ion bunch with a variety of shapes. The experimental results show that a ramp-shaped ion bunch with a sharp falloff time can excite a large-amplitude wake field with $\delta n/n_0$ up to $\simeq 17\%$. The amplitude of the wake field decreases with an increase of the falloff time constant of the incident ion bunch, along with oscillations of the amplitude. The theoretical model introduced here can explain most of the experimental results in the linear regime. Large-amplitude waves damp out, along with oscillations of the envelope of the wake waves. The inverse of the period of the envelope oscillation is proportional to the square root of the excitation voltages of the wave.

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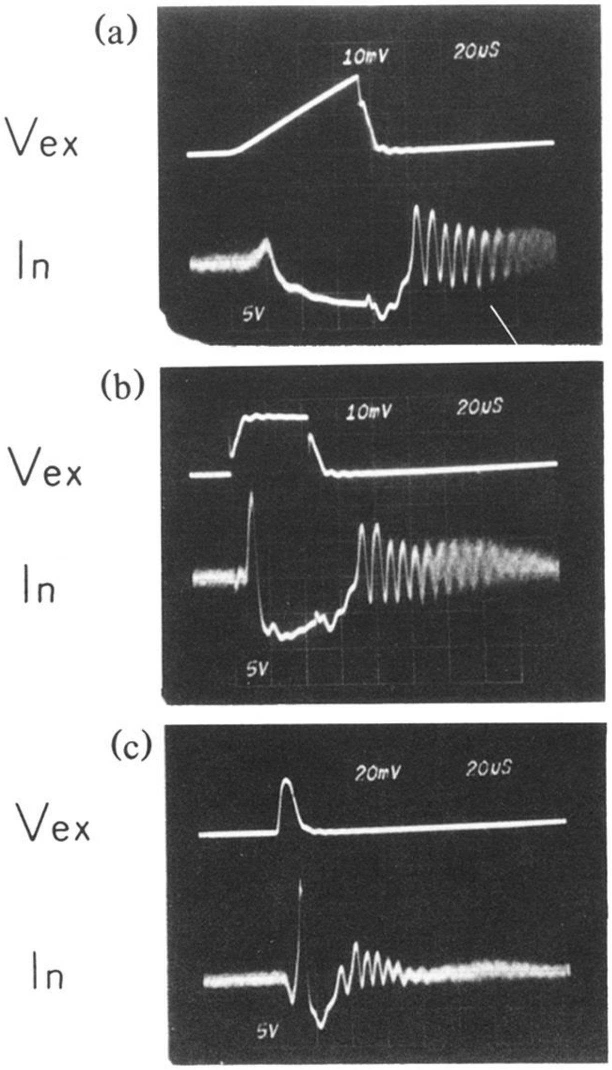

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