Scaling Theory of Marginal Fermi Liquids

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The infrared singularities of the recently introduced marginal-Fermi-liquid theory are treated within a perturbative renormalization-group approach. Marginal Fermi liquids are stable in a limited range of the parameters; otherwise non-Fermi-liquid behavior is obtained. Experimental ramifications on the conductivity and spin relaxation are discussed.

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The understanding of the copper-oxide-based high-temperature superconductors will be complete when besides explaining the superconducting mechanism, the anomalous normal-state properties are accounted for as well. In the absence of a complete microscopic description phenomenological approaches may provide much sought after additional insight. A recent proposition is the marginal-Fermi-liquid (MFL) picture of Varma *et al.*,[1] which accounts for numerous experimental data by starting from a central hypothesis, namely, the assumption of a special form for the imaginary part of the density correlator $\chi(\mathbf{q},\omega)$. This assumption leads to a quasiparticle renormalization factor Z which vanishes at the Fermi surface, but only in the weak form of an inverse logarithm, and hence the "marginal" notion. Other treatises, starting with rather different physical pictures, share the idea that the uniqueness of the high-T_c compounds is essentially described by a $Z=0$ quantum liquid. In particular, this feature is present in the "Luttinger-liquid" approach,[2] drawing an analogy with the one-dimensional Fermi gas,[3] which exhibits the decoupling of spin and charge excitations. Strongly suppressed Z factors accompany the appearance of a pseudogap within the framework of the spin-bag mechanism of high-T_c superconductivity.[4] Also, they appear in the nested[5] and in the almost localized Fermi-liquid theories.[6] In what follows we adopt the basic assumptions of the marginal-Fermi-liquid picture. These lead to the appearance of infrared singularities in the theory. We develop a scaling approach to treat these singularities and explore the resulting phase diagram as well as some experimental consequences.

The basic hypothesis of the marginal-Fermi-liquid theory is that microscopic processes of a less specified origin yield an unconventional contribution to the imaginary part of the density—and similarly the spin—correlator, which takes the form $\mathrm{Im}\chi(\mathbf{q},\omega)\sim N(0)f(\mathbf{q})\times\mathrm{sgn}\omega$, where $N(0)$ is the density of states and $f(\mathbf{q})$ is a smooth function of the momentum. (We will work at zero temperature in most of the paper.) Furthermore, the leading effect of these same microscopic interactions on the single-particle Green's function is assumed to come from the process where the intermediate state includes a low-energy particle-hole pair, described by the above correlator (the "anomalon"), coupled to the electron by a residual interaction g. This residual interaction represents all other diagrams, typically describing the electron scattering off the constituents of the anomalon; thus g is a four-point vertex.[7] In this two-stage picture it is assumed that the dominant effect of the microscopic strong interaction is represented by the anomalon, and thus the low-energy residual interaction g is in fact weak and not even necessarily repulsive. We accept these assumptions as the formal definitions of the MFL theory. Our point is that infrared singularities are present in the theory, and necessitate a renormalization-group treatment. Indeed, the application of the Kramers-Kronig relation shows that the real part of the density correlator is logarithmically divergent in the infrared: $\mathrm{Re}\chi(\mathbf{q},\omega)\sim(2/\pi)N(0)f(\mathbf{q})\ln(\omega_c/\omega)$, where ω_c is an ultraviolet cutoff. Insertion of the anomalon into the one- and two-particle Green's functions thus leads to logarithmic corrections in those quantities as well.

Before proceeding we emphasize that including these terms by no means amounts to double counting. Indeed, the very existence of the residual interaction g allows for higher-order processes, such as the random-phase-approximation (RPA) diagrams and possibly further parquet terms, thus rendering the above correlator the irreducible one instead of the full function. And, since these diagrams contain higher powers of $\ln(\max[\omega,T])$, they cannot be discarded at low energies. Therefore, the correlator of Ref. 1 is a good approximation to the response function at higher temperatures and energies, and we expect the renormalization effects arising from the residual interactions to become important only at lower T,ω.

For the sake of generality we assume that the bare interaction depends on the (magnitude of the) momentum transfer: $g=g(q)$. Upon expanding the vertex in spherical harmonics we keep only two partial waves: $g(2k_F\times\cos\theta)\approx g_0^0+g_2^0\cos2\theta$, where the external momenta are taken close to the Fermi surface. This approximation can be motivated, e.g., by a short screening length in a Thomas-Fermi-type expression, and we expect that at least the basic tendencies resulting from the momentum dependence will be represented. We also mention that in a related work it has been demonstrated that even if the scaling is started with many channels, typically two of them become exponentially dominant over the others.[8] The four-point vertex at the momentum transfer $q=p_3-p_1$ is written as $\Gamma(q,q',K)_{12;34}=g(q)\delta_{13}\delta_{24}-g(q')\delta_{14}\delta_{23}$, where $q'=p_4-p_1$ is the exchange momentum, K is the sum of the incoming momenta, and the spin indices are explicitly displayed. The scattering geometry in two dimensions forces one of the above three external momenta to be zero. We choose $K=0$, which leads to the convenient parametrization $q\approx 2k_F\cos\theta$ and $q'\approx 2k_B\sin\theta$. The external frequencies are chosen as in the case of the 1D Fermi gas, with energy transfer ω.[9]

The first step of the scaling procedure is the computation of the leading logarithmic terms for the vertex and for the Green's functions. Fortunately, in the present formulation only vertex contributions appear in leading order in g: from the direct and exchange particle-hole channels, and from the Cooper channel (Fig. 1). The momentum dependence of the correlator is expanded up to $l=2$ for similar reasons as for the vertex itself, and we write $\chi(q,\omega)=[\chi_0+\chi_2\cos2\theta]\ln(\omega_c/\omega)$. The angle dependence is present if for no other reason than because of the f-sum rule, which requires that $\mathrm{Re}\chi(q,\omega)\sim q^2/\omega^2$ for $v_Fq\ll\omega$, so the correlator cannot be logarithmic everywhere.[10] This in turn unavoidably induces an angle dependence for the vertex as well, generating g_2 even if its bare value is zero. When the angle-dependent couplings are inserted into the correlators, different angular moments of the *integrand* appear. Once again we keep only the two leading terms: χ_{il} denotes the quantity obtained by performing the last angular integral for χ_i with a $\cos(l\phi)$ weight factor ($l=0,2$), where ϕ is the angle between p_1 and the integrational variable p. In performing the momentum integrals we assume in the usual spirit of the Fermi-liquid theory that the dominant contributions emerge from regions not too far from the Fermi surface. We include the natural assumption that the Cooper channel is also logarithmic at $K=0$. Since this channel is isotropic, and all its higher angular momenta are vanishing, it contributes only a $\chi_{00}^C\equiv\chi_C\ln(\omega_c/\omega)$ term. Finally, the contributions to the g_l's are identified

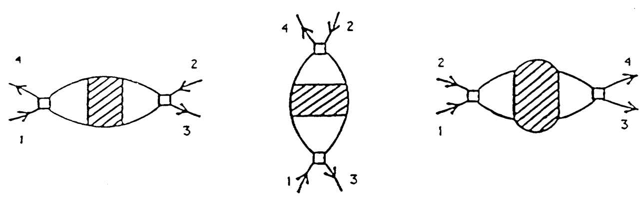

FIG. 1. The basic parquet diagrams with four-point vertices used to construct Eq. (1).

by comparing the coefficients of the $\cos(l\theta)$ terms on the two sides of the vertex equation.

Having obtained the perturbative expressions for the couplings, the scaling equations are derived by reducing the ultraviolet cutoff frequency ω_c, in complete analogy with the techniques applied to the one-dimensional Fermi gas.[9] This approach is different from the standard quantum renormalization procedure[11] because no infrared singularities are associated with the momentum dependence, and so we could average over the external momenta on the Fermi surface. The following scaling equations are obtained:

$$\begin{aligned}\partial g_0(\xi)/\partial\xi &= [\chi_C-\chi_{00}]g_0^2+[-2\chi_{02}+\chi_{20}-\chi_{22}]g_0g_2\\&\quad+[\chi_{00}-\tfrac{1}{2}\chi_{02}-\tfrac{1}{4}\chi_{20}-\tfrac{1}{2}\chi_{22}]g_2^2\,,\\ \partial g_2(\xi)/\partial\xi &= \chi_{20}g_0^2+2\chi_{00}g_0g_2\\&\quad+[-\tfrac{1}{2}\chi_C+\tfrac{1}{2}\chi_{00}-\chi_{02}+\chi_{20}-\tfrac{1}{2}\chi_{22}]g_2^2\,,\end{aligned}\tag{1}$$

where $\xi=\ln(\omega_c'/\omega_c)$, and the $g_l(\xi)$ denote the renormalized couplings. A negative right-hand side means decreasing couplings with decreasing temperatures. The scaling equations sum up the parquet diagrams. These contain higher powers of $\ln(\omega_c/\omega)$ as can be seen by explicit calculation, using the simplest assumptions about the integrands in the correlator.

The fixed-point structure of these equations is the following. Each derivative vanishes along two straight lines crossing the origin. The requirement that these pairs of rays coincide yields a criterion for the χ_{il}'s. *If* this ("MFL") condition is met, then we have an attractive and an unstable fixed line, as shown in Fig. 2(a). The generic situation is, however, that the condition is not fulfilled, in which case a typical flow diagram looks like that in Fig. 2(b). (Depending on the five parameters, naturally other flow trajectories can occur as well.) We note, however, that χ_C is typically negative, and in the fully nested one-dimensional case, $\chi_C=\chi(00)$. Thus in the present situation, without large nesting, we expect $|\chi_{00}|<|\chi_C|$, and so (for $|g_0|\gg|g_2|$) the *magnitude* of an attractive g_0 increases with decreasing temperature. Therefore, Fig. 2(b) shows a typical situation.

We determine the Fermi-liquid characteristics from the quasiparticle renormalization factor:

$$Z\sim\{[g_0^0g_0(\omega)+\tfrac{3}{4}g_2^0g_2(\omega)]\ln(\omega_c/\omega)\}^{-1}\,,\tag{2}$$

where $g_l(\omega)\equiv g_l(\ln(\omega_c/\omega))$, and the above form is necessary to avoid double counting. Three distinct behaviors can be identified with the help of Z: (i) Both coupling constants scale to zero, typically as a power law. Z then rapidly increases for small frequencies and since it is bounded by 1 from above, the Fermi-liquid behavior is restored. (ii) Both couplings remain bounded, but at least one of them scales to a nonzero fixed point. In this case Z vanishes, as an inverse logarithm, describing the marginal-Fermi-liquid phase. (iii) At least one

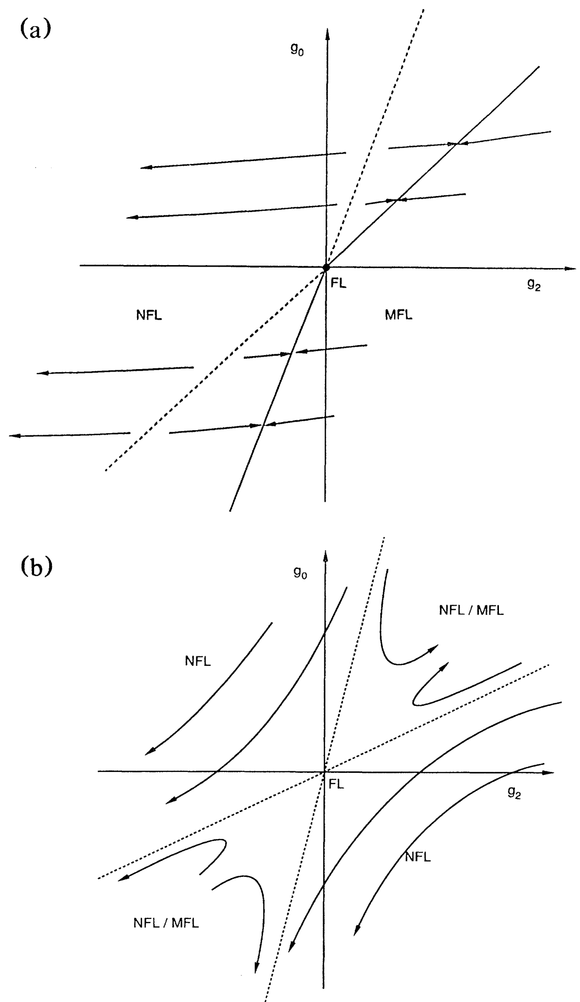

FIG. 2. (a) The phase diagram when the MFL condition is satisfied. FL denotes the Fermi-liquid fixed point, MFL the marginal-Fermi liquid region, and NFL the non-Fermi-liquid region. (b) The phase diagram when the MFL condition is not met. The NFL/MFL label indicates the eventually non-Fermi-liquid behavior with a large MFL crossover.

of the couplings scales to large values, typically as an inverse power law. Z now vanishes, as a power law at low frequencies, clearly destroying the Fermi-liquid behavior.

Figure 2(a) shows the situation, when the MFL condition is satisfied. The (lower-) right half plane—bounded by the dashed-line separatrix—scales to the stable fixed line, shown as solid. Using the above characterization we identify this as the marginal-Fermi-liquid phase. This fixed line is analogous to the Tomonaga-Luttinger line in one dimension. The difference is that there the Fermi liquid is destroyed even *on* the line by the forward scattering,[3,9] but in the present case, according to the basic assumptions, it is only driven marginal. On the other hand, the (upper-) left half plane gives rise to non-Fermi-liquid behavior. As mentioned, the MFL condition holds only for special values of the parameters, and thus a typical situation is represented in Fig. 2(b), where the marginal-Fermi-liquid fixed line becomes unstable towards strong coupling.

The physical mechanism driving the non-Fermi-liquid behavior is that in low-dimensional interacting systems the scattered particle induces a phase shift to *all* other particles, essentially changing the whole Hilbert space, and, as a reaction, experiences a strong interaction itself.[2]

Regarding the issue of the separation of the spin and charge degrees of freedom[2] an approximate criterion may be the decoupling of the corresponding vertices. This leads to angle-dependent conditions, and we conclude that within this approach spin-charge separation is not a robust feature in the weak-coupling regime.

We now turn to the measurable characteristics of the above phases. Following Ref. 1, the dominant term for the dc resistivity is obtained by inserting the anomalon into the current correlator, yielding

$$\rho(T) \sim [g_0(T)^2 + \tfrac{3}{4} g_2(T)^2]T\,, \tag{3}$$

where the temperature dependence of the effective couplings is obtained by integrating the scaling equations to $\omega = T$. When the above MFL condition is far from being satisfied, then the magnitude of the couplings rapidly increases and one should see an insulatorlike rise in the resistivity. On the other hand, the temperature dependence of $\rho(T)$ in the high-T_c superconductors is typically linear, in some cases with an upturn at lower temperatures.[12] Thus *if* these materials are described by the MFL theory, then this suggests that the MFL condition is only weakly violated. In this case the $\delta g_0 = 0$ and the $\delta g_2 = 0$ rays are close to each other, and the couplings flow for a long temperature interval in this region with their values changing only moderately, and only at an exponentially low crossover temperature do they turn towards strong coupling. The crossover temperature is higher if the bare interactions are stronger. This leads to an upward bending of the resistivity at low temperatures. Of course, if such a behavior is observed experimentally it can be attributed to other effects, like sample imperfections, as well. (Traditional scenarios utilizing, e.g., normal Fermi-liquid formulas are less successful in explaining other properties, such as the magnetic susceptibity.) If, however, the above scaling is the dominant reason for the bending, then compounds with larger bare coupling strength should bend upwards at higher tempertures. The couplings can be extracted, e.g., from the slope of the resistivity, if the ratio n/m^* is known. This correlation between slope and bending is clearly absent if defects cause the upturn. We mention that in the case of the yttrium compounds, when samples with O_7 and $O_{6.63}$ are compared, such a correlation seems to be present, with a pronounced upturn for $O_{6.63}$.[12]

Another prominent observable is the spin-relaxation time in the NMR experiments. At temperatures high above T_c, $1/T_1$ can be fitted rather satisfyingly with the form $1/T_1 = aT + b$.[1,13] The MFL theory accounts for the first, Korringa-type term by the bare spin correlator, and for the second one by the anomalous spin-response function. The first perturbative correction to $1/T_1$ is of the order of g, as the leading term of the RPA-type expansion for $b(g)$ reads $b(g)/b \approx 1 + g_0(T)N(0)\ln(\omega_c/T)$. This gives rise to a pronounced *downturn* in $1/T_1$ for attractive interactions, as the temperature is lowered. This downward deviation from the earlier formula is a general feature of the experiments, often starting at $(3\text{-}4)T_c$.[13] Superconducting fluctuations affect the spin-relaxation time in the same way,[14] but in general one would expect them to be dominant only closer to the transition, leaving room for the above corrections at higher temperatures. We do not attempt a quantitative fit to the NMR data, since there are indications that the spin structure factor has a strong q dependence.[15] We note, however, that in the materials with stronger upturn or bending of the resistivity, like Y-Ba-Cu-O with $O_{6.63}$ and the lanthanum compounds, in general the downward turn in $1/T_1$ is more pronounced.[13] In our picture this correlation emerges naturally, since both effects derive from a common cause: the renormalization of the couplings g_i. We also mention that in Ref. 14, when couplings extracted from high-energy regions were used to fit low-energy data, some discrepancies remained. The present renormalization of the couplings, which *does* take into account the vertex corrections, is a possible explanation for that. Finally, the singlet pairing vertex can be determined as well. Its s-wave component equals the renormalized value of $g_0(T)$, and its singularity determines T_c. To summarize, for attractive interactions spin fluctuations are suppressed, while, on the other hand, charge and superconducting fluctuations are enhanced, in close analogy to the one-dimensional case.

In conclusion, we investigated the infrared singularities in the recently introduced marginal-Fermi-liquid picture. A multiplicative renormalization-group technique was used to sum up the most divergent diagrams. We constructed the phase diagram on the basis of the scaling equations. The marginal-Fermi-liquid phase is stable in a large parameter region, when a special condition is satisfied. Away from, but close to this condition, it becomes a large crossover regime, extending to low temperatures. While the renormalization effects might not be necessary for the analysis of some experiments, in the crossover regime they can provide a natural explanation for the temperature dependence of the resistivity and the spin-relaxation rate in NMR in high-T_c compounds.

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