Obukhov and Rubinstein Reply: We have recently demonstrated[1] that topological entanglements freeze large-scale motion of flux lines in high-T_c superconductors. In order for these entanglements to be effective, there has to be a high-energy barrier for the vortex line crossing. It is difficult to accurately calculate this barrier while allowing for the arbitrary shape of flux lines. One may only rely on some order-of-magnitude estimates from oversimplified pictures of vortex line crossing. There are two energies contributing to the barrier: (i) the interaction energy E_i for a pair of vortices, and (ii) the self-energy increase E_s due to flux-line elongation.

The interaction energy for a pair of straight vortices tilted by an angle α with respect to each other was calculated by Brandt, Clem, and Walmsley,[2] $E_i = (\phi_0^2/8\pi\lambda) \times \cot\alpha \exp(-a/\lambda)$, where ϕ_0 is the flux quanta, λ is a penetration depth, and a is an interline separation.

If a section of a vortex line was originally oriented along the field and had length L and later was tilted by an angle β with respect to its original direction, it had to be elongated by $L(1-\cos\beta)/\cos\beta$ in a slab geometry. The energy cost due to this elongation is extensive in line length $E_s = \varepsilon L(1-\cos\beta)/\cos\beta$, where the line tension is $\varepsilon = (\phi_0^2/16\pi^2\lambda^2)\ln(\lambda/\xi)$ and ξ is a correlation length. For long straight lines ($L \gg \lambda$) this self-energy increase E_s is always much larger than the interaction energy E_i.

To lower this elongation energy, let us consider lines that are parallel a distance d apart from each other far from the intersection region (Fig. 1). If distance d is less than the London penetration depth λ, we can estimate the interaction energy change due to crossing $\tilde{E}_i \approx (\phi_0^2/8\pi\lambda^2) d \cot\alpha$. The increase of self-energy due to elongation of the pair of lines is $\tilde{E}_s = 2\varepsilon d \tan(\alpha/4)$. We neglected the additional bending energy at points A, B, C, and D in the expression for the self-energy. At angles α such that $\tan\alpha = \cot\alpha \approx 1$ the two energies $\tilde{E}_i$ and $\tilde{E}_s$ are of the same order of magnitude and we expect this to be a good estimate for the minimum crossing barrier.

Both the discussion above and that of the Comment[3] are limited to the isotropic superconductors. In order to extend these results to the anisotropic case, one needs to rescale distances.[4] The intersection angle in the highly anisotropic picture corresponding to Fig. 1 is very close to π. But the main conclusion that the energy barrier for the flux crossing is of the same order of magnitude as either the extension energy E_s or the interaction energy E_i still holds for the anisotropic superconductors. We estimate this energy by $E_s \propto \epsilon_\parallel d\lambda_\parallel/\lambda_\perp$, where $\varepsilon_\parallel$ and $\lambda_\parallel$ are the line tension and London penetration depth of a vortex oriented along the z direction (normal to CuO_2 planes) and $\lambda_\perp$ is the penetration depth in the plane. In a significant region of phase diagram this estimate is bigger than the minimum one proposed by Nelson,[5] $E_{\mathrm{barrier}} \approx 2\varepsilon_\parallel l$, where the minimal physical length—the interplane spacing l—is used instead of $d\lambda_\parallel/\lambda_\perp$. This minimum estimate was used in our paper.[1]

Another issue raised in the Comment[3] was on the effect of impurities. Single flux lines are easily depinned from weak impurities by thermal fluctuations. There may not be enough strong-pinning centers to hold each individual vortex line. But if the flux lines are entangled with each other, the mobility of the whole network of lines is zero even if only the small fraction of lines is strongly pinned. In recent experiments with proton-irradiated samples[6] it was demonstrated that by increasing the concentration of strong-pinning centers, the critical current increases by an order of magnitude while the critical temperature remains practically uneffected. These experiments support our picture[1,7] of collective pinning of the three-dimensional network of entangled flux lines.

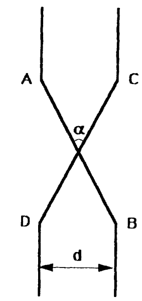

FIG. 1. A configuration of a pair of crossing vortex lines.

In conclusion, we would like to mention that our predictions for H_{c2} based on the estimate of the crossing energy [Eqs. (6) and (7) of Ref. 1] have been confirmed in recent experiments at high fields and low temperatures.[8]

We are indebted to D. R. Nelson for very helpful discussions. This work was supported in part by NSF Grant No. DMR-9058572.

S. P. Obukhov
Landau Institute of Theoretical Physics
Moscow, U.S.S.R.
and University of Florida
Gainesville, Florida 32611

Michael Rubinstein
Eastman Kodak Company
Rochester, New York 14650-2110

Received 27 December 1990
PACS numbers: 74.60.Ec, 74.40.+k, 74.60.Ge

[1]S. P. Obukhov and M. Rubinstein, Phys. Rev. Lett. **65**, 1279 (1990).

[2]E. H. Brandt, J. R. Clem, and D. G. Walmsley, J. Low Temp. Phys. **37**, 43 (1979).

[3]E. H. Brandt and A. Sudbø, preceding Comment, Phys. Rev. Lett. **66**, 2278 (1991).

[4]V. G. Kogan, Phys. Rev. B **24**, 1572 (1981).

[5]D. R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).

[6]L. Civale, A. D. Marwick, M. W. McElfresh, T. K. Worthington, A. P. Malozemoff, F. H. Holtzberg, J. R. Thompson, and P. H. Kess, Phys. Rev. Lett. **65**, 1164 (1990).

[7]S. P. Obukhov, Y. Shapir, and M. Rubinstein (to be published).

[8]G. T. Seidler, T. F. Rosenbaum, D. L. Heinz, A. W. Downey, A. P. Paulikas, and B. W. Veal (to be published).