Comment on "Topological Glass Transition in Entangled Flux State"

In a recent Letter, Obukhov and Rubinstein[1] predict extremely slow relaxation of a thermally entangled flux-line lattice in high-T_c superconductors. The relaxation times for thermally assisted disentangling are found to be very large, $\propto \exp\{\exp[(\Lambda/d)^2]\}$ for single flux lines and $\propto \exp[(\Lambda/d)^6]$ for collective motion of many flux lines, where $d \approx (\phi_0/B)^{1/2}$ is the flux-line spacing and $\Lambda \approx (2\pi k_B TL/\epsilon_3)^{1/2}$ the fluctuation amplitude of the ends of a flux line of length L (slab width) and effective line tension ϵ_3 in a "flux liquid." This very stable (even in the absence of pinning) entangled glass or liquid state is a consequence of topological constraints imposed on the wandering flux lines and depends crucially on the *assumption* of a relatively large energy barrier for crossing or cutting of flux lines, for which the authors[1] adopt Nelson's[2] estimate

$$U_{\rm cut} \approx 2lH_{c1}\phi_0 = l\phi_0^2 \ln\kappa/2\pi\mu_0\lambda^2 \approx 50k_BT\,, \qquad (1)$$

for $\mu_0 H_{c1} \approx 80$ G (at $T=77$ K in Bi-Sr-Ca-Cu-O) and "cutting length" $l = 10$ Å (interlayer spacing, flux lines perpendicular to the Cu-O layers).

We would like to note that the cutting energy of vortices may actually be much lower, such that thermally activated vortex cutting may become an effective mode of disentanglement. In particular, the factor $\ln\kappa$ in (1) comes from the magnetic-field energy at *large* distances $>\lambda$, which is not involved in cutting. Note also that in isotropic London superconductors (with coherence length $\xi \rightarrow 0$) the magnetic interaction between vortices *vanishes* if these run *perpendicular* to each other, as is the case in all crossing points in the figures of Ref. 1. Such vortices do not see other and thus cross easily.

The estimate (1) is based on the magnetic interaction of vortices which are *straight and parallel* over a length of several penetration depths λ. Though this scalar two-dimensional (2D) vortex interaction $\propto K_0(r_2/\lambda)$ (r_2 is the vortex distance, K_0 is the McDonald function diverging for $r_2 \rightarrow 0$), used in Ref. 2 and in several following papers, does not diverge when formally applied to cutting of nonparallel vortices, the correct 3D London interaction between *curved* vortices is quite different, namely, *vectorial* and *nonlocal* along the flux lines:[3,4]

$$U = \frac{\phi_0^2}{8\pi\mu_0\lambda^2}\sum_{i,j}\int\int \frac{\exp(-r_3/\lambda)}{r_3}\, d\mathbf{r}_i \cdot d\mathbf{r}_j\,, \qquad (2)$$

where $r_3 = |\mathbf{r}_i - \mathbf{r}_j|$ is the 3D distance between vortex line elements $d\mathbf{r}_i$ (i is the vortex index). This 3D interaction becomes even softer when the finite vortex core and the material anisotropy are accounted for and when the flux density B increases.[4,5] Furthermore, as also mentioned in Ref. 1, in *layered* superconductors cutting of flux lines (chains of point vortices in different layers) becomes easy when the coupling between layers is weak.[6]

For a pair of *stiff straight* vortices tilted by an angle α with respect to each other the interaction can be calculated analytically from (2)[7,8]

$$U(a,\alpha) = (\phi_0^2 \cot\alpha/2\mu_0\lambda)\exp(-a/\lambda) \qquad (3)$$

(a is the shortest vortex distance). A more realistic cutting energy may be calculated from (2) *numerically* by choosing appropriate boundary conditions and allowing the vortices to curve spatially in order to minimize their total energy before they intersect: This enhances the cutting angle α and reduces the cutting energy due to a trade-off between increasing self-energy and decreasing mutual interaction of the vortices. A first such computation was performed by Wagenleithner,[9] who (for his boundary conditions) found the vortex-pair configuration to be unstable when $a < 2\lambda$. We find similar instabilities of entangled vortices in a flux-line *lattice* and in anisotropic superconductors. We thus suggest that the statistical mechanics of the vortex lattice should employ the correct 3D interaction (2) between vortices in order to avoid results which possibly do not apply to real vortex lattices.

A further remark to Ref. 1 is that in *real* superconductors each flux line is pinned by *many* pins, e.g., by oxygen vacancies. It is thus little relevant whether the vortex lattice is entangled or has internal viscosity since it would not flow between pins even if it were "liquid" in the absence of pins. And even if there were only a few strong pins, these would exert a very *weak* total pinning force since only the (small) vortex cores are pinned, and large linear or planar pins cannot pin curved flux lines sufficiently. Finally, an ideally stiff vortex lattice could not be pinned at all by random pins since all pinning forces cancel when the lattice cannot adjust to the pins; by the same token, internal viscosity will *reduce* the pinning of the vortex lattice.

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E. H. Brandt and A. Sudbø
AT&T Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

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