u- and d-Quark Masses in Nambu's BCS Model

Bipin R. Desai
Department of Physics, University of California, Riverside, California 92521
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It is suggested that $m_d > m_u$ is a natural consequence of Nambu's four-fermion interaction model (with the BCS mechanism) if a small admixture of the charged-current interaction is included.

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The u and d quarks share some unique properties: They are nearly degenerate and d, despite its smaller electric charge compared to u's, is heavier than u, contrary to any simple electromagnetic arguments, and contrary to the situation that exists in other quark families. The standard model has no explanation for this because the fermion–Higgs-boson Yukawa couplings which give rise to the masses are arbitrary.

Recently, Nambu has proposed a four-fermion interaction model based on the BCS mechanism in which the dynamics is determined essentially by fermion bubble diagrams.[1-3] A remarkable consequence of this model is that, in the bubble approximation, the t-quark-to-Higgs-boson mass ratio is 1:2. The Higgs-boson self-coupling is thereby related to the fermion–Higgs-boson Yukawa coupling. This approach has been extended in a recent paper by Bardeen, Hill, and Lindner.[4] And, in a closely related work, Kaus and Meshkov[5] have discussed the quark-lepton mass hierarchies.

Much of the above-mentioned effort has been devoted to the t quark and related subjects. We point out, however, that there are some interesting consequences of the Nambu model for u and d quarks. In particular, d being heavier than u appears as a natural consequence of this model. This observation is based on writing the four-fermion interaction in the form

$$\mathcal{L}_I = G_{\rm NC} J_{\rm NC}^\dagger J_{\rm NC} + G_{\rm CC} J_{\rm CC}^\dagger J_{\rm CC} , \tag{1}$$

where the neutral current $J_{\rm NC}$ is given by

$$J_{\rm NC} = \sum_i \bar{q}_L^i q_R^i = \bar{u}_L u_R + \bar{d}_L d_R + \bar{c}_L c_R + \bar{s}_L s_R + \bar{t}_L t_R + \bar{b}_L b_R . \tag{2}$$

The coefficients in the above equation are all taken to be unity even though we realize that there is no *a priori* reason for doing so. Our assumption seems a natural one based on simplicity.

The charged current $J_{\rm CC}$ is given by

$$J_{\rm CC} = \bar{u}_L d_R' + \bar{c}_L s_R' + \bar{t}_L b_R' . \tag{3}$$

The primes above refer to the gauge eigenstates which are related to the unprimed quark eigenstates through Kobayashi-Maskawa (KM) angles [because of symmetry, primes will be irrelevant in (2)]. We note that the use of gauge eigenstates in (3) is an assumption on our part based on the traditional treatment of the fermion states.

The contributions of the bubble diagrams to the fermion masses are divergent. If, following Nambu,[1,2] we introduce a cutoff Λ such that

$$G\Lambda^2 = 1 , \tag{4}$$

where $G_{\rm NC} = G\lambda_{\rm NC}$, $G_{\rm CC} = G\lambda_{\rm CC}$, then the masses of the quarks are given by

$$m_i = \lambda_{\rm NC} m_i + \lambda_{\rm CC} \sum_j F_{ij} m_j , \tag{5}$$

where F_{ij} is a matrix whose elements are squares of the KM matrix elements.

Our model is an extension of the original Nambu model[1,4,5] recognizing the possible presence, among other things, of the $J_{\rm CC}^\dagger J_{\rm CC}$ interaction (as well as the $\bar{c}c\bar{c}c$ term, for example). We note below that the presence of a small $J_{\rm CC}^\dagger J_{\rm CC}$ term gives $m_d > m_u$.

To illustrate, we take, for simplicity, the cosines involved in the KM matrix to be unity, keeping only the terms that involve $s_i = \sin\theta_i$. We then obtain

$$m_d = \lambda_{\rm NC} m_d + \lambda_{\rm CC}[m_u + s_1^2 m_c + (s_1 s_2)^2 m_t] , \tag{6a}$$

$$m_u = \lambda_{\rm NC} m_u + \lambda_{\rm CC}[m_d + s_1^2 m_s + (s_1 s_3)^2 m_b] , \tag{6b}$$

$$m_s = \lambda_{\rm NC} m_s + \lambda_{\rm CC}[m_c + s_1^2 m_c + (s_2 + s_3)^2 m_t] , \tag{6c}$$

$$m_c = \lambda_{\rm NC} m_c + \lambda_{\rm CC}[m_s + s_1^2 m_d + (s_3 + s_2)^2 m_b] , \tag{6d}$$

$$m_b = \lambda_{\rm NC} m_b + \lambda_{\rm CC}[m_t + (s_3 + s_2)^2 m_c + (s_1 s_3)^2 m_u] , \tag{6e}$$

$$m_t = \lambda_{\rm NC} m_t + \lambda_{\rm CC}[m_b + (s_2 + s_3)^2 m_s + (s_1 s_2)^2 m_d] . \tag{6f}$$

We note first of all that if we take $m_d \approx m_u = 0$ as the input masses on the right-hand side of Eqs. (6a) and (6b) then, because the s_i's are small, the important contributions to the u and d masses come almost entirely from their nearest neighbors c and s. Substituting the values of s_i, and taking the current masses to be $m_s = 200$ MeV and $m_c = 1.3$ GeV (Ref. 6), one finds

$$m_d \simeq \lambda_{\rm CC} \times (65 \text{ MeV}), \quad m_u \simeq \lambda_{\rm CC} \times (10 \text{ MeV}) , \tag{7}$$

and therefore $m_d > m_u$.

If, in order to obtain quantitative estimates of m_u and

m_d, we take (6a) and (6b) seriously, then eliminating λ_{NC} and λ_{CC} we find

$$\frac{m_d}{m_u}=\frac{m_u+65\ \text{MeV}}{M_d+10\ \text{MeV}}\,. \tag{8}$$

If we take, for example,

$$m_u \simeq 5\ \text{MeV} \tag{9a}$$

we find, from the quadratic equation (8) for m_d, that the only positive solution is

$$m_d \simeq 14\ \text{MeV}\,. \tag{9b}$$

These values are to be compared with the most recent determination of the current masses, $m_u=5.6\pm1.1$ MeV and $m_d=9.9\pm1.1$ MeV.[6] They are quite satisfactory considering the approximations involved.

An examination of Eqs. (6a) and (6b) shows that consistent conditions for the parameters λ_{NC} and λ_{CC} are given by

$$\lambda_{NC}\approx 1-\epsilon,\quad \lambda_{CC}\approx\epsilon^2\,. \tag{10}$$

Equations 6(c)–(6f) for $\epsilon \ll 1$ then simply correspond to the self-interaction equation (the gap equation) of the original Nambu picture for the heavy quarks with only the λ_{NC} interaction, i.e.,

$$m_q\approx\lambda_{NC}m_q,\quad \lambda_{NC}=G_{NC}\Lambda^2\approx 1\,. \tag{11}$$

It is an interesting question to ask if one could obtain the correct masses of all the quarks in terms of λ_{NC} and λ_{CC} by taking together Eqs. (6a)–(6f). This will be the subject of a separate paper but it is important to point out here that there is a fine-tuning problem in the coupling constant G, as noted by Bardeen, Hill, and Lindner,[4] specifically where the t-quark equation is concerned.

It needs to be pointed out how significantly different the above results for m_u and m_d are compared to naive electromagnetic arguments which would predict the mass difference Δm ($=m_d-m_u$) to be given by

$$\Delta m\approx(\alpha_d-\alpha_u)m=-\tfrac{1}{3}\,\alpha m\,,$$

where α is the fine-structure constant and m is a typical quark mass ($\approx$ MeV). This not only has the wrong sign but has, in contrast to (9a) and (9b), a magnitude of the order of one-hundredth of the expected value.[7]

The purpose of this Letter was to suggest an explanation of what has always been a mystery in the past: why d with a smaller electric charge is heavier than u. We find that Nambu's BCS model explains this rather simply on the basis of the charged-current interaction.

Finally, it is very important to determine whether the interaction (1) arises from a gauge theory. In this connection we note that the Higgs sector will be more complicated in our case than what Nambu[1,2] and Bardeen, Hill, and Lindner[4] had considered earlier where the Higgs boson was a $t\bar{t}$ bound state. Even though having a huge mass gives t a special status, once one resorts to the dynamical picture for the Higgs boson then $b\bar{b}$, $c\bar{c}$, etc., should also contribute to the Higgs boson. This subject is currently being investigated by us.[8]

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[6]See Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988); C. A. Dominguez and E. de Rafael, Ann. Phys. (N.Y.) **174**, 372 (1987); J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982). The values quoted are the running masses evaluated at 1 GeV^2.

[7]For a calculation to obtain the correct d-u mass difference within the context of supersymmetry see B. R. Desai and G. Xu, Phys. Rev. D **41**, 2214 (1990).

[8]It is interesting to note that if one takes charge-($+\frac{2}{3}$) coupling different from charge-($-\frac{1}{3}$) coupling in the "NC" contribution, i.e., if the λ_{NC} in (6a), (6c), and (6e) is assumed different from the λ_{NC} in (6b), (6d), and (6f), then the qualitative result (7) still remains true. So $m_d > m_u$ is qualitatively current though quantitative estimates now get messier.