First Return, Transient Chaos, and Decay in Chaotic Systems

In a recent Letter,[1] Bauer and Bertsch examined the dependence of decay laws on chaoticity. They studied the capture of point particles, bouncing elastically off the walls of a Sinai billiard, by a trap in one of the container walls. For ergodic motion, they find an exponential decay in time of the number of initial particles which remain in the billiard, whereas the nonchaotic system decays according to a power law.

In this Comment, (1) we point out that this problem is very general and has been considered previously in various contexts such as room acoustics,[2-5] transient chaos,[6,7] and coarse-grained properties of classical orbits (first passage within a finite interval).[7,8] (2) It is shown that the proposed classification[1] is not followed by the integrable circular billiard whose decay law is also exponential.[8] (3) For chaotic motion, the exponential decay law is shown to hold only for very small escape rates,[5,8] due to finite time correlations.

(1) Almost a century ago,[2] Sabine studied the decay of sound in concert halls due to the influence of absorption. It is in fact[3,5] equivalent to the escape problem of Ref. 1 within geometrical acoustics. The width Δ of the window for escape must then be identified with $\int ds\, \alpha(s)$, where $\alpha(s)$ is the absorption coefficient at position s of the container perimeter. In the case of Ref. 1, $\alpha(s)=1$ over the width Δ of the window and 0 elsewhere. Then, formula (6) of Ref. 1 for the decay time, $\tau=\pi A_c/p\Delta$, is nothing but the classic expression of the decay time for an acoustic impulse in an auditorium.[2-5] τ is also the mean free time of a particle within an ergodic box of volume V and absorbing surface S ($\tau=4V/cS$), derived more than a century ago by Czuber[9] and Clausius.[10]

Opening a window to allow the escape of particles was also proposed recently[7] as a means to connect the dynamical properties of billiards (internal problem) to the (external) problem of chaotic scattering.[6] The window can be used as the entrance to the billiard for particles coming from outside and the gate of exit from the billiard at their first return to the window. The decay time τ is nothing but the average *dwell time* of the transient chaotic scattering process.

(2) We have recently studied[8] the integrable circular billiard and found numerically that its decay law is also exponential, for sufficiently small window widths Δ, with a decay time $\tau \sim \Delta^{-1}$, up to times of the order of Δ^{-2}. A particle trajectory can be parametrized by the positions α_n of its rebounds on the circle and is equivalent to the iteration of the circle map $\alpha_{n+1}=\alpha_n+\Omega$, where Ω is determined from the initial position and momentum of the particle. Then, the exponential decay law can be tracked back to ergodic properties of the irrational number $\Omega/2\pi$.[8]

(3) We have shown[5,8] that, in the case of the stadium,[11] the decay is more complicated than a pure exponential when Δ becomes larger than 0.3 times the radius R of the circular caps. This stems from the existence of finite-time correlations, present when the average trajectory length is not large. This effect is strong when all particles start at time zero from a single source, as we have studied.[5,8] In contrast, in Ref. 1, the 10^6 particles have their initial positions and momenta (orientations) randomly sampled in phase space. We have carried out[8] a similar simulation for the stadium and observe that the decay law departs from a pure exponential for values of Δ at least twice as large ($\Delta \geq 0.6R$). In this case, the maximum acceptable value of Δ is larger than previously, but must still be sufficiently small so that the escape of a single particle truly corresponds to a Markovian process.

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