

Experimental Observation of Landau Levels in Nonperiodic (Fibonacci) Superlattices

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Absorption spectra of GaAs/GaAlAs Fibonacci superlattices (FSL) with magnetic fields parallel to the layers show self-similarity at field values scaled by τ^2 [$\tau = (1 + \sqrt{5})/2$]. The one-dimensional nonperiodicity of the FSL affects the carrier motion perpendicular to the layers very differently from that parallel to the layers: In B_{\parallel} , where carriers have to cross several barriers to complete a cyclotron orbit, clear Landau levels are observed, whereas in B_{\perp} the merging of the magnetic levels superimposed on the irregular eigenvalue spectrum of the FSL does not yield equally spaced spectra.

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Systems with nonperiodic quasi-one-dimensional (1D) potentials, such as Fibonacci superlattices (FSL), are accessible analogs of noncrystalline solids, which over the years have become a subject of increasing importance.¹⁻³ FSL are artificially grown layered structures with varying layer thickness or composition. These purposely disordered quasi-1D systems, besides providing a "controllable" kind of nonperiodicity, are self-similar, meaning that their properties are similar at different length scales. Experiments probing directly the geometry of the FSL, e.g., x-ray diffraction⁴ or Raman scattering by acoustic phonons,⁵ have brought out this self-similarity. Moreover, the electronic states of nonperiodic SL, and in particular FSL, have been investigated theoretically^{6,7} and experimentally⁸⁻¹⁰ at zero magnetic field, showing the existence of regions of localized states and delocalized states.

In this paper we study the effects of nonperiodicity and self-similarity on the energy-level structure when a magnetic field is applied parallel to the layers. In this field configuration, carriers move in cyclotron orbits of radius $(\hbar/eB)^{1/2}$ in the direction of variation of the nonperiodic potential. Therefore, a magnetic field can be considered as a tool to study vertical motion in nonperiodic SL, and the length scale on which the carriers "see" the nonperiodicity can be varied by the field. A condition for the observation of such effects is that the barriers are thin and low enough in order to allow an efficient tunneling of the carriers through the barriers; i.e., we consider a weakly disordered system.

It has been shown theoretically¹¹ that, because of the same scaling behavior of the Schrödinger equation of motion of carriers in a magnetic field and of the Fibonacci potential, the density of states (DOS) of a FSL as a function of a field parallel to the layers may be self-similar at different field values, and here we demonstrate the effect experimentally. This type of property is entirely new, and different from the previously studied sys-

tems,^{4,5,12} since it is independent of the exact geometry of the structure and it is brought about by the variation of an external parameter.

We report measurements of the luminescence intensity of the near-band-edge emission as a function of the excitation energy (luminescence excitation spectroscopy), in magnetic fields both parallel and perpendicular to the FSL layers, and compare the results with similar experiments in periodic superlattices (PSL). We have verified that the shape of the excitation spectra taken at different positions in the luminescence line was the same and that the measurements reflect the shape of the DOS. The exciting radiation, with wavelengths between 700 and 800 nm, was generated by a Kr⁺-pumped CR599 dye laser using LD700 as a dye. The experiments were performed at 1.8 K and in fields up to 22 T produced by a 10-MW polyhelix magnet.

The two investigated FSL consisted of molecular-beam-epitaxy-grown sequences w_n of the elementary layers a (1.12-nm Al_xGa_{1-x}As barrier, with $x = 0.2$ for one and 0.4 for the other sample) and b (1.69-nm GaAs well) generated according to the Fibonacci series in the following way. Let w_n be the concatenation of w_{n-2} and w_{n-1} if n is odd, and w_{n-1} and w_{n-2} if n is even. Starting with $w_1 = a$ and $w_2 = b$, we find subsequently $w_3 = ab$, $w_4 = abb$, $w_5 = ababb$, $w_6 = ababbabb$, etc. For increasing n , the ratio of the lengths of w_n and w_{n-1} approaches $\tau = (1 + \sqrt{5})/2$, the golden mean. The self-similarity of the resulting nonperiodic structure follows from the transformations

$$abb \rightarrow b', \quad ab \rightarrow a', \quad (1a)$$

$$ab \rightarrow b'', \quad b \rightarrow a''. \quad (1b)$$

Equation (1a) transforms w_n into w_{n-2} corresponding to a scaling of the length in structure by a factor τ^2 (see Wang and Maan¹¹); (1b) transforms w_n into the reverse of w_{n-1} (i.e., w_{n-1} read from left to right), and the scal-

ing factor becomes τ .

The FSL studied were of generation w_{13} (233 layers a and b). For comparison, we also measured a 3.5-nm/1.1-nm and a 3.8-nm/1.1-nm GaAs/Al_{0.4}Ga_{0.6}As periodic SL.

In Figs. 1(a) and 1(b) we show the luminescence excitation spectra of a FSL in magnetic fields parallel (B_{\parallel}) and perpendicular (B_{\perp}) to the SL layers and in Figs. 1(c) and 1(d) the analogous results for the PSL. The first striking feature is the much stronger anisotropy between the B_{\parallel} and B_{\perp} spectra of the FSL compared to that of the PSL. While in the PSL in both field configurations, clearly equidistant, linearly field-dependent transitions (Landau levels) can be distinguished, such transitions can only be identified in the B_{\parallel} spectra of the FSL and not in the B_{\perp} spectra. This latter result would *a priori* not be expected, since in B_{\parallel} the carriers have to tunnel through a nonperiodic array of barriers, while they can complete their cyclotron orbit within each GaAs well in B_{\perp} . A further important difference between PSL and FSL is that with B_{\parallel} [Figs. 1(a) and 1(c)] in the PSL the Landau levels become more pronounced, in a regular fashion, as the field increases, while the intensity of the peaks in the FSL varies irregularly with increasing field strength. Moreover, the width of the levels in the PSL is seen to be independent of the magnetic field [see inset of Fig. 1(c)], whereas the levels in the FSL clearly broaden with field [see inset of Fig. 1(a)].

These results are a direct consequence of the shape of the DOS of a system with a 1D potential in a magnetic field. When the field is *perpendicular* to the SL layers, the carriers orbit in the plane of the layers, and the energy spectrum consists of Landau-level fans originating from each of the eigenvalues of the confining 1D poten-

tial. Contrary to the PSL, the eigenvalues of the FSL are grouped at irregularly spaced bunches and the resulting level structure does not resemble a simple Landau-level fan. In particular, Landau levels with higher Landau-level indices of lower eigenvalues merge at higher fields with those with lower indices of higher eigenvalues, and thus the resulting DOS shows no individual levels. For instance, the peak visible in the B_{\perp} spectrum at 20 T is the result of such a convolution. In the *parallel* field instead, the carriers move in the direction of the Fibonacci potential on a length scale which can be varied with the field. The energy spectrum in this case results from the Schrödinger equation describing the carrier motion in the FSL growth direction (z) in a magnetic field parallel to the layers (e.g., the x direction). When we express all lengths in units of the cyclotron radius l and all energies and potentials in units of $\hbar\omega_c = \hbar eB/m^*$, the Schrödinger equation reads

$$\left[- \left(\frac{d^2}{d(\zeta - \zeta_0)^2} \right) + (\zeta - \zeta_0)^2 + v_{SL}(\zeta) \right] \psi(\zeta - \zeta_0) = E\psi(\zeta - \zeta_0), \quad (2)$$

where $l\zeta_0 = \hbar k_y/eB$ (the position of the “center” of the cyclotron orbit in the growth direction). The wave functions in the x and y directions are plane waves with wave vectors k_x and k_y , respectively.

In the absence of a potential v_{SL} , Eq. (2) is the well-known harmonic-oscillator equation with degenerate eigenvalues $E_N = N + \frac{1}{2}$ (N is the Landau-level quantum number) for all orbit centers ζ_0 . In the presence of $v_{SL}(\zeta)$, Eq. (2) leads in general to a ζ_0 -dependent dispersion. When $d_{\text{bar}} < \hbar/(2mV_0)^{1/2}$ (d_{bar} and V_0 are the width and the height of the barrier) and when the aver-

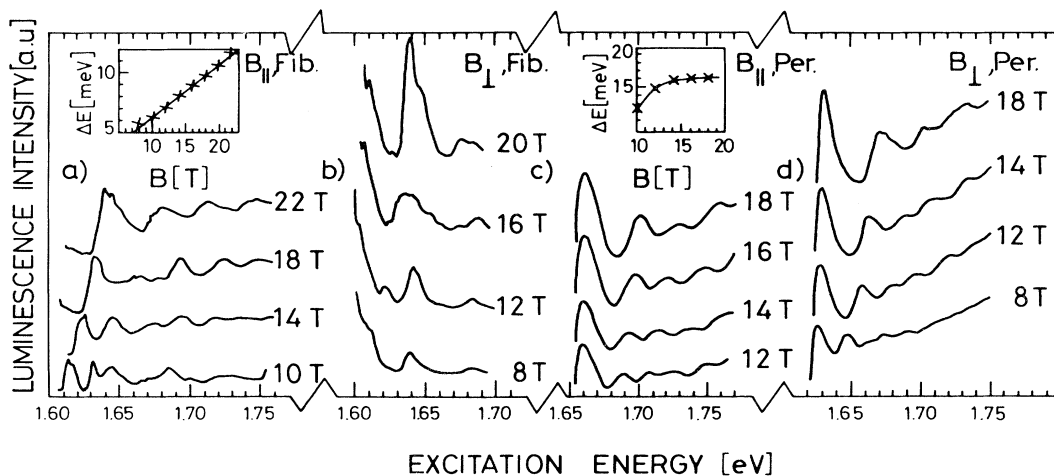


FIG. 1. Luminescence excitation spectra at different values of the magnetic field of the Fibonacci SL with 20%-Al barriers (a) in a field parallel to the layers and (b) in a perpendicular field; and of periodic SL (c) in a parallel field (3.8-nm-GaAs/1.1-nm-Al_{0.4}Ga_{0.6}As) and (d) in a perpendicular field (3.5-nm-GaAs/1.1-nm-Al_{0.4}Ga_{0.6}As). (a), (c) insets: The width of magnetic levels as a function of parallel field in, respectively, the Fibonacci and the periodic SL.

age periodicity of the potential is small compared to $[(2N+1)/(N+1)]^{1/2}$ (the "period" of ϕ_N), $v_{\text{SL}}(\zeta)$ can be considered as a perturbation and alters only slightly the unperturbed harmonic-oscillator wave functions ϕ_N . In first-order perturbation the eigenvalues of Eq. (2) can then be written as¹¹

$$(N + \frac{1}{2}) + \langle \phi_N(\zeta - \zeta_0) | v_{\text{SL}} | \phi_N(\zeta - \zeta_0) \rangle \\ = (N + \frac{1}{2}) + \langle v_{\text{SL}}(\zeta_0) \rangle. \quad (3)$$

In this approximation for a periodic potential, $\langle v_{\text{SL}} \rangle$ is independent of the origin of the cyclotron orbit ζ_0 , and of the extent of the wave function ϕ_N . The width of the Landau levels in periodic SL is thus independent of the magnetic field as was confirmed experimentally^{13,14} and as can also be seen in Fig. 1(c).

In the case of a Fibonacci potential it has been shown¹¹ that the dispersion $\langle v_{\text{SL}}(\zeta_0) \rangle$, and therefore also the DOS, is self-similar for values of the magnetic field related by τ^{2n} ($n = \dots -2, -1, 0, 1, 2, \dots$), provided that all energies are measured in units of the cyclotron energy. This result was obtained on the basis of an exact solution of (2), but can more easily be understood in terms of the perturbation approach (3). At a given field, Eq. (3) calculated for all values of ζ_0 with respect to the Fibonacci potential gives the orbit-center dispersion and

the shape of the DOS at that field. At a τ^4 -times-lower magnetic field, i.e., a τ^2 -longer magnetic length, (3) can be evaluated for the actual potential, but also for a suitably averaged potential, both yielding the same orbit-center dispersion. In this average, a barrier followed by a wide well (*abb*) is replaced by a new well *b'* with a width equal to *abb* and a potential equal to the average over *abb*, and similarly the actual *ab* is replaced by a new barrier *a'* with a width like *ab* and a potential equal to the average over *ab*. This suitable averaging corresponds to the transformation (1a) for which the Fibonacci sequence is self-similar and implies a scaling in the length by τ^2 . Decreasing the field by a factor of τ^4 thus merely corresponds to a scaling of both the magnetic length and the length scale of the potential with τ^2 . As a consequence, the DOS of a FSL in a parallel field is the same at fields related by a factor τ^4 .¹¹ Scaling with τ in length and τ^2 in field using transformation (1b) is similar but the argument is somewhat more complicated.¹¹

To demonstrate this self-similarity in the spectra at different parallel magnetic fields we calculate $\Sigma(B_0, B)$, the sum of the squares of the differences between a spectrum at a given field B_0 and spectra at other values of the magnetic field (with the energy scale normalized to the cyclotron energy), scaled in a way as to allow for comparison between different samples and field geometries:

$$\Sigma(B_0, B) = \left[\int \left(\frac{I_{B_0}(E)}{\langle I_{B_0} \rangle} - \frac{I_B(E)}{\langle I_B \rangle} \right)^2 dE / \int \left(\frac{I_{B_0}(E)}{\langle I_{B_0} \rangle} + \frac{I_B(E)}{\langle I_B \rangle} \right)^2 dE \right]^{1/2}, \quad (4)$$

where $I_B(E)$ is the luminescence intensity at excitation energy E normalized to the cyclotron energy and at a field B , and $\langle I_B \rangle$ is the average intensity of the spectrum. The closer $\Sigma(B_0, B)$ is to zero, the more similar the spectra at B_0 and B are. For instance, for a simple Landau-level spectrum with equidistant peaks of the same shape, (4) would give zero for all fields. In Fig. 2(b), we show $\Sigma(B_0, B)$ for both FSL in B_{\parallel} and compare with similar results for the PSL [Fig. 2(a)]. For the FSL a clear minimum at a value of the magnetic field for which $B/B_0 = 0.43 \pm 0.04$ is observed, while for the PSL $\Sigma(B_0, B)$ increases smoothly. In B_{\perp} , for the PSL, $\Sigma(B_0, B)$ behaves similarly, but in this configuration the FSL shows a very rapid increase for lower values of B/B_0 [Fig. 2(c)], demonstrating that the spectra do not resemble each other. Since only $\Sigma(B_0, B)$ for FSL in B_{\parallel} shows a minimum at a value 0.43 ± 0.04 and since this value is very close to the expected self-similarity ratio, $1/\tau^2 = 0.38$, we can conclude that the B_{\parallel} spectra of the FSL show indeed the predicted self-similar behavior. An implication of this self-similarity is that the width of the peaks increases linearly with the field as we have indeed observed (Fig. 1).

In this analysis we considered the shape of the spectra only in terms of the broadening due to the nonperiodic potential, but, of course, several other aspects like the

field-independent intrinsic broadening of the peaks due to scattering, the nonparabolicity of the conduction band, the valence-band structure, and excitonic effects also contribute partially to the shape of the peaks. In particular, the finite broadening of the peaks which makes structure only become clearly visible at 6 T makes the spectra between high and low fields begin to be dissimilar, as can be seen from the fact that also $\Sigma(B_0, B)$ of the PSL in B_{\parallel} shows a decrease. We attribute the difference between the observed value of 0.43 and the theoretically expected value of 0.38 ($1/\tau^2$) and the fact that $\Sigma(B_0, B)$ does not reach zero at this value to these effects.

It is important to stress that this self-similarity does not simply reflect any particular ratio between the magnetic length at some field with the layer thicknesses involved. Instead, for any field B_0 one can find other field values ($\tau^2 B_0$, B_0/τ^2 , B_0/τ^4 , etc.) which will show in principle self-similar spectra, which implies that the spectra are not fractal in the sense that one observes a spectrum in a spectrum, etc., as was the case with the other experiments.^{4,5} Another new phenomenon is that the self-similarity is a consequence of the variation of an external parameter.

In summary, we have shown that luminescence excita-

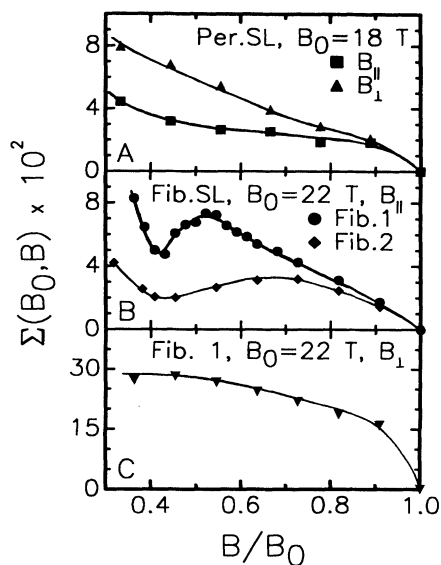


FIG. 2. Similarity $\Sigma(B_0, B)$ of the spectrum, normalized to the cyclotron energy, at a field B_0 , with the normalized spectra at other values of the magnetic field: (a) for a 3.5-nm-GaAs/1.1-nm- $\text{Al}_{0.4}\text{Ga}_{0.6}\text{As}$ periodic SL in parallel and perpendicular fields, with $B_0 = 18$ T; (b) for the Fibonacci SL (Fib. 1, 20%-Al barriers, and Fib. 2, 40%-Al barriers) in parallel fields, with $B_0 = 22$ T; and (c) for a Fibonacci SL (Fib. 1) in a perpendicular field, with $B_0 = 22$ T (note the scale change). The lines are a guide to the eye.

tion spectra of nonperiodic FSL in magnetic fields perpendicular and parallel to the SL layers are strongly anisotropic. The parallel-field spectra of the nonperiodic SL show clear Landau levels, which, in contrast to those in periodic SL, broaden as a function of field, vary non-uniformly in intensity, and show a self-similar behavior as a function of the parallel magnetic field. This is a

consequence of the identical scaling behavior of the magnetic Hamiltonian and the Fibonacci potential.

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