Experimental Observation of Period-Doubling Suppression in the Strain Dynamics of a Magnetostrictive Ribbon

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(Received 8 October 1990)

Suppression and shift of period-doubling bifurcation due to a periodic perturbation is experimentally observed in the strain dynamics of a magnetically driven magnetostrictive ribbon. First experimental verification of the predicted scaling law relating the shift of the bifurcation point to the perturbation amplitude is made. The predicted switchlike nonlinearities in the perturbing signal around the bifurcation point are also observed.

PACS numbers: 75.80.+q, 05.45.+b

The effects of perturbations on dynamical instabilities of a nonlinear system have attracted considerable interest.[1-6] Specifically, Bryant and Wiesenfeld[6] have found that a system exhibiting period-doubling (PD) bifurcation at $f_0/2$ (f_0 is the fundamental frequency) when perturbed by a signal whose frequency f_1 is near the period-doubled frequency $f_0/2$ exhibits several universal properties, including (i) a suppression and subsequent shift of the bifurcation point as the perturbation amplitude is increased, (ii) a scaling law relating the shift of the bifurcation point to the perturbation amplitude, (iii) parametric amplification of the perturbation signal including nonlinear effects such as gain saturation and discontinuous response at a critical perturbing amplitude, and (iv) the emergence of a closely spaced set of peaks in the frequency response spectrum. Bryant and Wiesenfeld,[6] motivated by the center manifold theorem, argued that the crucial dynamics of a system exhibiting supercritical PD bifurcation and subjected to a perturbing signal are captured by the following normal form:[6]

$$\dot{x} = \mu x - x^3 + \varepsilon \cos(\delta t) , \qquad (1)$$

where $x(t)$ is the slowly varying envelope function for the full (vector) dynamical variable $\mathbf{X} \in R^N$, μ is the bifurcation parameter, ε is the perturbation amplitude (assumed positive), and $\delta = f_1 - f_0/2$ is the detuning of the perturbation frequency. The universal properties related to the PD suppression have not been thoroughly verified experimentally[7] and its practical ramifications have yet to be explored. To the best of our knowledge we report the first experimental verification of all the universal properties related to the PD bifurcation suppression and shift.[6] The experiment was accomplished by observing the strain response of a magnetically driven amorphous ferromagnetic ribbon.

The experiment involved measuring the dynamic strain response of a magnetically driven $Fe_{78}B_{13}S_9$ amorphous magnetostrictive ribbon (Metglas 2605S-2) using a fiber-optic Mach-Zehnder interferometer (Fig. 1). In such ribbons, the strain e is related to the applied magnetic field H by $e = CH^2$, where C is the effective magnetostriction parameter.[8] A small portion (< 5 mm) of an as-cast ribbon (50 mm × 12 mm × 25 μm) was bonded to the optical fiber comprising one arm of the interferometer. The phase shift of light propagating in the fiber attached to the unannealed ribbon is a direct measure of strain in the ribbon. Details of the fiber-optic interferometer are well documented elsewhere.[9] The magnetostrictive ribbon was positioned vertically, clamped at the top end and kept free at the other end (Fig. 1). The interferometer was contained in a solenoid which was driven by a two-channel frequency synthesizer (HP3326A), providing a longitudinal magnetic field $H = H_{dc} + h_0 \cos(2\pi f_0 t)$, where H_{dc} is the applied dc field and h_0 is the amplitude of the sinusoidally varying field. The strain response exhibits a rich PD cascade as the magnetic field is increased.[10] The perturbing signal at f_1 was added with the second channel of the synthesizer. The power-spectral-density output of the strain

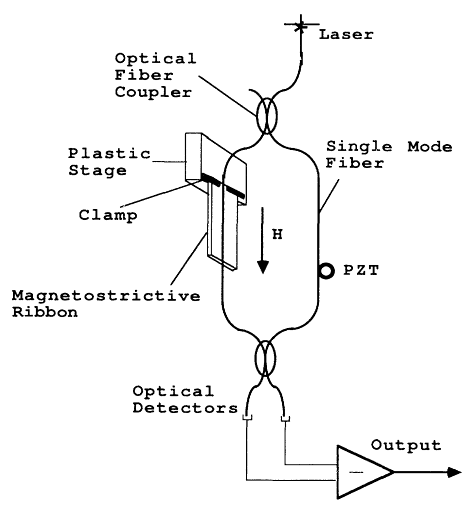

FIG. 1. Schematic of a fiber-optic Mach-Zehnder interferometer used to measure the strain dynamics of a magnetostrictive ribbon. PZT, piezoelectric device (linear device) used for calibration purposes.

response was measured with a dynamic signal analyzer (HP3562A). The amplitudes of the applied magnetic fields were obtained by measuring the voltage drop across a 1-Ω resistor in series with the solenoid. Previous experiments exploiting the ΔE effect (reversible changes in the Young's modulus as a function of applied field) in magnetostrictive ribbons to observe quasiperiodic motion and intermittent and sustained chaos at low frequencies ($f < 1$ Hz) have been reported.[11] However, our experiment does not rely on the ΔE effect (the ΔE effect is negligible in unannealed Metglas ribbons) and measures the dynamic ($f = 6.8$ kHz) magnetostrictive response of the ribbon directly with the fiber-optic Mach-Zehnder interferometer.

In this experiment, the drive amplitude h_0 is the bifurcation parameter [equivalent to μ in Eq. (1)] and h_1 is the perturbing-signal amplitude [equivalent to ε in Eq. (1)]. For an unperturbed system ($h_1 = 0$), clean PD bifurcation in the strain response of the unannealed ribbon was observed for the following drive parameters: $h_0 = 0.593$ Oe and $f_0 = 6.8$ kHz at a fixed bias field $H_{dc} = 4$ Oe. We shall refer to the unperturbed bifurcation point as h_{0c} and measure the shift in the bifurcation point with respect to h_{0c}. A perturbing signal [$h_1 \times \cos(2\pi f_1 t)$] oscillating at $f_1 = 3.3998$ kHz ($\delta = 0.2$ Hz) was added in order to observe its effects on the PD bifurcation. For small h_1 no significant effects on the PD bifurcation were observed. Upon increasing h_1, the PD bifurcation was abruptly suppressed at $h_1 = h_{1B}$. Figure 2(a) shows the switchlike response of the amplitude of the PD bifurcation as a function of the perturbing-signal amplitude h_1. The bifurcation point where the symmetrical oscillator becomes unstable has now shifted to a new bifurcation point which is above the bifurcation point of the unperturbed system ($h_{0,\text{shifted}} > h_{0c}$). The theory[6] also predicts that upon suppression of the PD bifurcation, the frequency components at $\omega_0/2 \pm m\delta$, where m is an odd integer, undergo a dramatic gain at the critical perturbation amplitude h_{1B}. Figure 2(b) shows the switchlike nonlinear gain in the strain response of the perturbing signal ($m = -1$) at $h_1 = h_{1B}$. For $h_1 > h_{1B}$ gain saturation is observed.

The theory of Bryant and Wiesenfeld[6] also predicts that the presence of the perturbing signal shifts the bifurcation point. Above the shifted bifurcation point, the solution $e_0(t)$ undergoes a symmetry-breaking bifurcation thereby producing a discontinuous change in the attracting solution for $e_0(t)$ at the bifurcation point. This discontinuity is expected in the amplitudes of all the spectral components. The $\omega_0/2$ component is expected to jump from zero to a finite value, while the ω_1 component is supposed to drop by 81.7% from its peak amplitude.

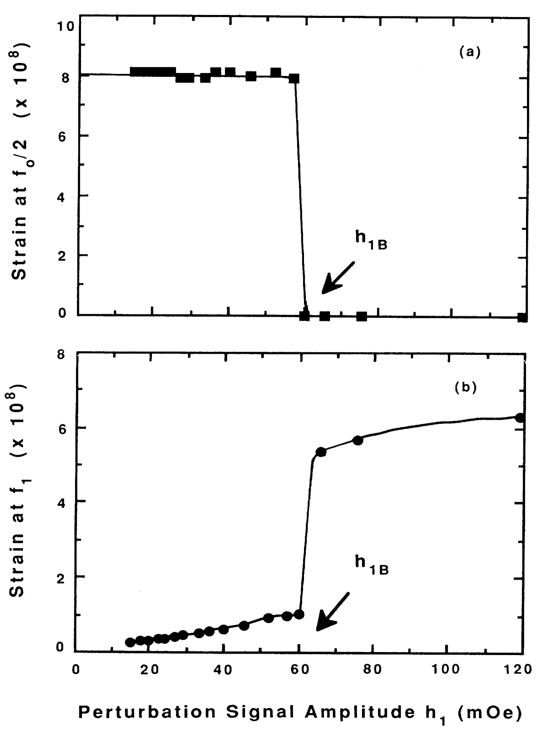

FIG. 2. (a) Suppression of strain at $f_0/2$ as a function of perturbation signal amplitude. (b) Amplification of the perturbing signal. The strain response at $f_0/2$ is suppressed at $h_1 = h_{1B}$, while the perturbing signal is amplified at $h_1 = h_{1B}$.

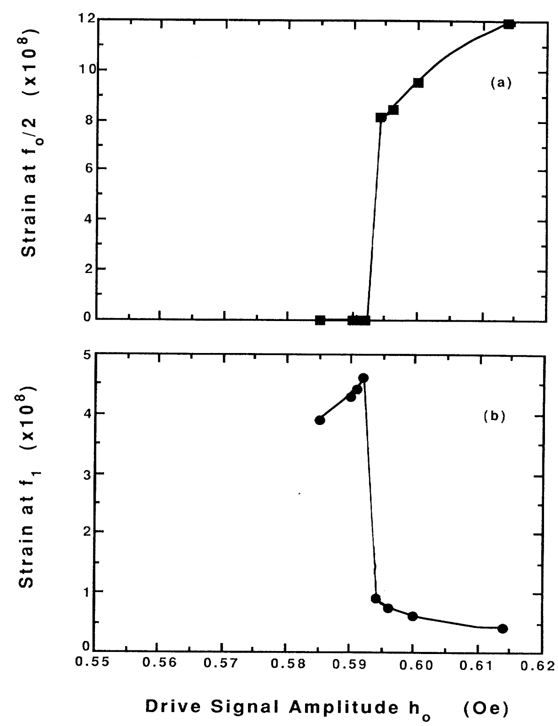

FIG. 3. (a) Emergence of period-doubling instability above the shifted bifurcation point ($h_{0,\text{shifted}} > h_{0c}$). (b) A switchlike discontinuity in the perturbing-signal amplitude above the shifted bifurcation point.

Experimental observations confirm these theoretical predictions. A slight increase in the drive amplitude h_0 reintroduces the PD bifurcation (as observed in the power spectra) thereby indicating that the bifurcation point in parameter space has indeed shifted to a higher value of h_0 (i.e., $h_{0,\text{shifted}} > h_{0c}$) due to the presence of the perturbing signal. Figure 3(a) shows the $\omega_0/2$ component jumping from zero to a finite value as the bifurcation parameter is increased above the shifted bifurcation point. The behavior of the perturbing signal amplitude near the shifted bifurcation point is shown in Fig. 3(b). The amplitude of the ω_1 component increases prior to reaching the shifted bifurcation point, after which its amplitude drops by 82%, in close agreement with the predicted 81.7% drop.[6]

The scaling law relating the shift of the bifurcation point to the perturbation amplitude has been found to be given by Ref. 6, $\mu_B \propto \varepsilon^{2/3}$ (where μ_B is the shifted bifurcation point) which becomes $(h_0 - h_{0c}) \propto (h_1)^{2/3}$ for the experiment described here. The shift power law was verified by plotting measured values of the bifurcation point $[\log(h_0 - h_{0c})]$ as a function of the perturbation amplitude ($\log h_1$) for $\delta = 0.2$ Hz (Fig. 4). The solid line is drawn through the lowest amplitude point with a slope of 0.653 ± 0.018, in good agreement with the predicted $\frac{2}{3}$ power law.

Fourier analysis performed by Bryant and Wiesenfeld[6] also revealed that for every odd multiple n of $\omega_0/2$ there are a group of closely spaced peaks at frequencies

$$\omega_{nm} = n(\omega_0/2) + m\delta\,, \tag{2}$$

where n is odd and m can be even or odd. Above the bifurcation point there is a pair of complementary asymmetrical attractors, corresponding to two possible phases of the period-doubled response. The asymmetry results in a complete spectrum with all m values allowed. Fig-

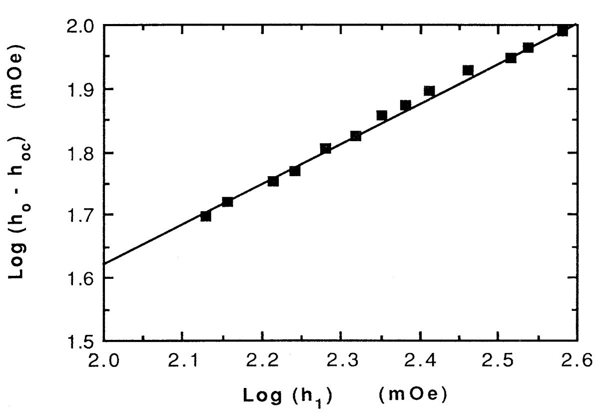

FIG. 4. Scaling law relating the shift of the bifurcation point $(h_0 - h_{0c})$ to the perturbation-signal amplitude (h_1). Slope of the log-log plot is 0.653 ± 0.018 in close agreement with the predicted $\frac{2}{3}$ power law.

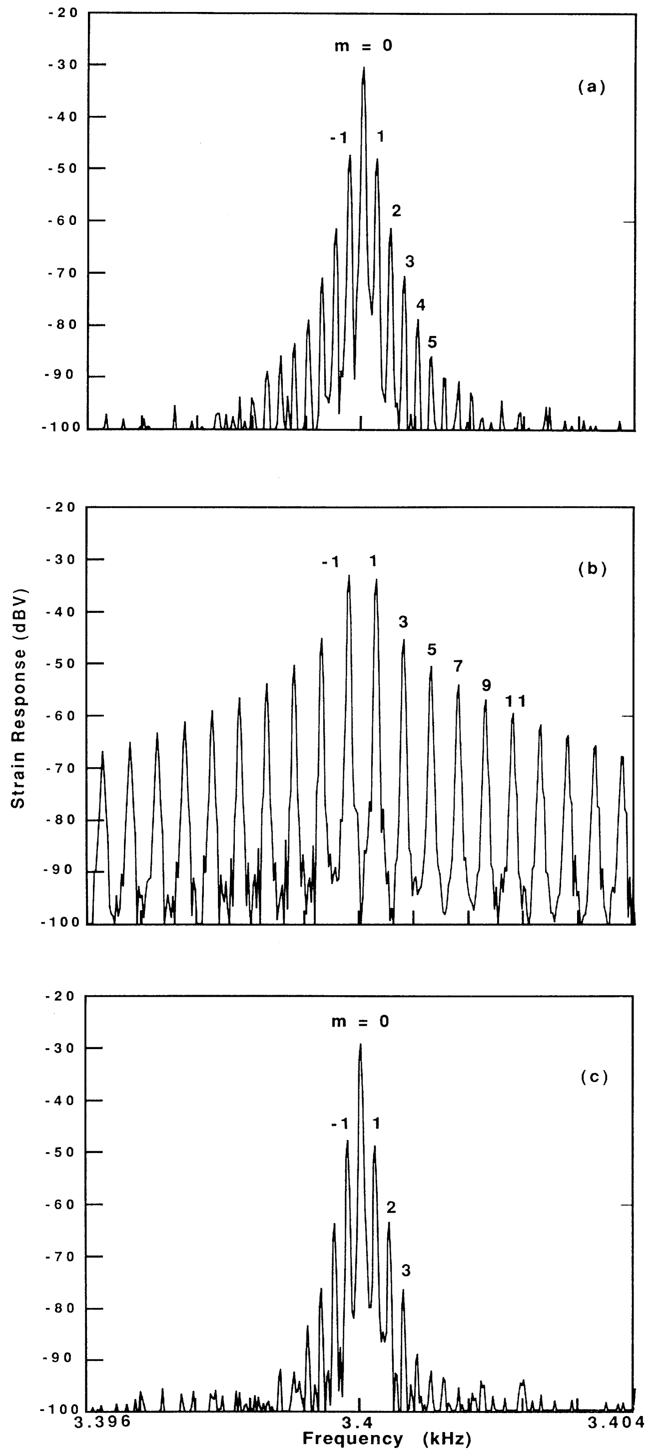

FIG. 5. Power spectra of the strain response around $f_0/2$ ($=3.4$ kHz). The spectra show $\omega_{nm} = n(\omega_0/2) \pm m\delta$ for $n = 1$ and $-20 < m < 20$, where $\delta = 0.2$ Hz. (a) $h_1 < h_{1B}$ for $h_0 = h_{0c}$, even and odd m values are allowed. (b) $h_1 = h_{1B}$ for $h_0 = h_{0c}$, only odd m are allowed. Notice the gain in all the odd-m components. (c) $h_{0,\text{shifted}} > h_{0c}$ for $h_1 = h_{1B}$, all m values allowed with $m =$odd components losing strength.

ure 5(a) shows the Fourier spectra of the strain response for $h_1 < h_{1B}$. The central peak in the spectrum [Fig. 5(a)] is $\omega_0/2$ [$n=1$ in Eq. (2)], while both the even and odd values of m are allowed. As the perturbation amplitude increases to $h_1 = h_{1B}$, $\omega_0/2$ is suppressed and only the peaks corresponding to the odd values of m are allowed [Fig. 5(b)]. It is also clear from Fig. 5(b) that peaks corresponding to odd values of m undergo a nonlinear gain upon the suppression of $\omega_0/2$ as depicted in Fig. 2(b) for the $m=-1$ case. Increasing the amplitude of the signal drive above the bifurcation point produces the Fourier spectra of Fig. 5(c). Both the odd and even values of m are allowed again but the strength of the m=odd peaks decreases in a switchlike manner, while m=even increases discontinuously as predicted by the theory. Figure 3 depicts the case for $m=0$ and $m=1$ as the signal amplitude is varied from below the bifurcation point to above the bifurcation point.

In conclusion, suppression of PD and nonlinear parametric effects were observed in the strain dynamics of a periodically perturbed magnetostrictive ribbon. The scaling law $[(h_0 - h_{0c}) \propto (h_{1B})^{2/3}]$ relating the shift of the bifurcation point to the perturbation-signal amplitude was experimentally verified for the first time. Switchlike nonlinearities (amplification) in the perturbing-signal amplitude near the bifurcation point were also observed. We hope to apply these results to observe parametric gain in a fiber-optic magnetometer operating in the four-photon mode and also extend this work to the study of suppression and shift of Hopf bifurcations.

We acknowledge the partial support for this work provided by the Office of Naval Research. We also acknowledge useful discussions with Dr. Kurt Wiesenfeld on the subject of PD suppression and shift.

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[1]J. Heldstab, H. Thomas, T. Geisel, and G. Radons, Z. Phys. B **50**, 141 (1983).

[2]K. Wiesenfeld, J. Stat. Phys. **38**, 1071 (1985); Phys. Rev. A **32**, 1744 (1985).

[3]C. Jeffries and K. Wiesenfeld, Phys. Rev. A **31**, 1077 (1985).

[4]K. Wiesenfeld and B. McNamara, Phys. Rev. Lett. **55**, 13 (1985); Phys. Rev. A **33**, 629 (1986).

[5]B. Derighetti, M. Ravani, R. Stoop, P. F. Meier, E. Brun, and R. Badii, Phys. Rev. Lett. **55**, 1746 (1985).

[6]P. Bryant and K. Wiesenfeld, Phys. Rev. A **33**, 2525 (1986).

[7]Period-doubling suppression has been observed in bouncing-ball experiments; however, other related universal properties were not explored in these experiments. See, K. Wiesenfeld and N. B. Truffilaro, Physica (Amsterdam) **26D**, 321 (1987); P. Pieranski, Phys. Rev. A **37**, 1782 (1988).

[8]J. D. Livingston, Phys. Status Solidi (a) **70**, 591 (1982).

[9]A. Dandridge, J. H. Cole, W. K. Burns, T. G. Gialorrenzi, and J. A. Bucaro, in *Optoelectronic Technology and Lightwave Communications Systems,* edited by C. Lin (Van Nostrand Reinhold, New York, 1989), Chap. 21.

[10]S. T. Vohra, F. Bucholtz, D. M. Dagenais, and K. P. Koo, in Proceedings of the Thirty-Fifth Magnetism and Magnetic Materials Conference, San Diego, California, 1990 [J. Appl. Phys. (to be published)].

[11]W. L. Ditto, S. Rauseo, R. Cawley, C. Grebogi, G. H. Hsu, E. Kostelich, E. Ott, H. T. Savage, R. Segnan, M. L. Spano, and J. A. Yorke, Phys. Rev. Lett. **63**, 923 (1989); W. L. Ditto, M. L. Spano, H. T. Savage, S. N. Rauseo, J. Heagy, and E. Ott, Phys. Rev. Lett. **65**, 533 (1990).