Tamayo and Klein Reply: Both Bray[1] and Moseley, Gibbs, and Jan[2] have used renormalization-group techniques to investigate the change in the exponent characterizing critical slowing down in model B with the implementation of an arbitrary-range Kawasaki exchange rather than the usual nearest-neighbor one. Both calculations appear to find results which contradict the findings reported in our Letter.[3]

Several comments are relevant.

(1) The result we obtained in Ref. 3 is that the relaxation time $\tau \sim \xi^{z_k}/L^2$, where ξ is the correlation length, z_k is the exponent associated with critical slowing down for model B with a local conservation law, and L is the mean exchange distance for the Kawasaki algorithm with longer-range exchange.

The result $z=z_k-2=2-\eta$ quoted in the Comment by Bray is the special case where $L=\xi$ which is the case for which we have numerical results. Our numerical results were obtained from a finite-size scaling analysis in which the maximum exchange distance of up-down pairs was the system size.

(2) The "equivalent simulations" by Moseley, Gibbs, and Jan quoted in Bray's Comment were in fact an implementation of the Monte Carlo renormalization group.[2]

(3) We have reanalyzed our data and reconfirmed the results in Ref. 3. That is not to say that we have ruled out any subtle effects such as the possibility that much longer runs would change the results. However, at this point we stand by the statement that the best fit to the data is $z=2-\eta$ in the finite-size scaling runs reported in Ref. 3.

(4) Finally, we offer a possible explanation of the discrepancy between our results and the results of Bray (and Moseley, Gibbs, and Jan). This explanation is implicit in Ref. 3 but perhaps was not stated as clearly as one would have liked.

It will be helpful to obtain a simplified scaling argument for part of the results in Ref. 1. The renormalization-group treatments in Refs. 1 and 2 assume the thermodynamic limit. Specifically, the system size is large compared to the correlation length. We now consider the relaxation in a subsystem the size of the correlation length which is the quantity of interest. Clearly, in such a subsystem the magnetization is not strictly conserved. However, we can make an estimate of the change in the amount of magnetization flowing into the system during a decorrelation time.

We know that the rate of change of the magnetization per unit volume in a region of size ξ^d near a critical point is

$$\partial\phi/\partial t \sim 1/\xi^{z+\beta/\nu}, \tag{1}$$

where the ξ^{-z} comes from the vanishing of the current due to critical slowing down and the $\xi^{-\beta/\nu}$ term comes from the vanishing of the magnetization per unit volume as the critical point is approached. In order to obtain the rate of change of the magnetization we must multiply by the volume that can leave the system in one time unit. For the Kawasaki algorithm with nearest-neighbor exchange this is the surface area ξ^{d-1} multiplied by a unit width. Therefore the rate at which the magnetization changes ρ is proportional to

$$\rho \sim \xi^{d-1}/\xi^{z+\beta/\nu}. \tag{2}$$

In order to obtain the change in the magnetization during a decorrelation time we multiply Eq. (2) by ξ^z. We then divide by $\xi^{d-\beta/\nu}$ to get the ratio $\Delta\Phi$ of the change in magnetization to the mean value of the magnetization:

$$\Delta\Phi \sim \xi^z\rho/\xi^{d-\beta/\nu}=\xi^{-1}. \tag{3}$$

Equation (3) implies that for nearest-neighbor Kawasaki exchange the fluctuations in the magnetization can be ignored as the critical point is approached. This result is still valid if the Kawasaki exchange is further than nearest neighbor but finite.

For Kawasaki exchange of the range of the correlation length, corresponding to $\sigma=0$ in Ref. 1, the volume of magnetization that can leave the system in one time unit is proportional to ξ^d since the "surface" has a thickness ξ. This makes $\Delta\Phi \sim 1$.

The ratio of the equilibrium fluctuations $\xi^d\chi_T$, where χ_T is the isothermal susceptibility, to the magnetization squared $(\xi^d\phi)^2$ is also of order 1. These results imply that nearest-neighbor Kawasaki exchange preserves the order parameter or magnetization conservation but exchange of the order of the correlation length introduces changes in the magnetization of the same order as the equilibrium fluctuations. Hence Kawasaki exchange of the order of ξ should result in the dynamics of model A.

This argument is in essential agreement with Refs. 1 and 2 but it also makes it clear that long-range Kawasaki exchange coupled with the thermodynamic limit implicit in the renormalization group results in the loss of the conservation law, global as well as local. It is this loss of the conservation that is the difference between the results of Refs. 1 and 2 and our work reported in Ref. 3. The finite-size scaling with either arbitrary Kawasaki exchange or the Creutz demon preserves the global conservation.

In effect we have decoupled the conservation law from the diffusion dynamics. This is what we have argued is *not* done in Refs. 1 and 2.

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[1]A. Bray, preceding Comment, Phys. Rev. Lett. **66**, 2048 (1991).

[2]L. Moseley, P. W. Gibbs, and N. Jan (to be published); Ref. 4 in preceding Comment.

[3]P. Tamayo and W. Klein, Phys. Rev. Lett. **63**, 2757 (1989).