Comment on "Critical Dynamics and Global Conservation Laws"

In a recent Letter, Tamayo and Klein[1] suggested that the dynamic critical exponent z for a system with only global conservation of the order parameter should be simply related to the exponent z_B obtained with local conservation (model B), namely, $z=z_B-2$. In particular, the exact result[2] $z_B=4-\eta$ implies $z=2-\eta$, i.e., $z<z_A$, where z_A is the dynamic exponent for a system with no conservation laws (model A). This prediction is surprising since intuitively one expects that the constraint imposed on the dynamics by the conservation law can only hinder relaxation, so one should have $z \geq z_A$.

In this Comment I consider a class of models which interpolate between local and global conservation laws.[3] In the notation of Ref. 1, the equation of motion for the order parameter, written in terms of its Fourier components, is

$$\frac{\partial \psi_{\mathbf{k}}(t)}{\partial t} = -\Gamma_0(\mathbf{k}) \frac{\delta F}{\delta \psi_{-\mathbf{k}}} + \theta_{\mathbf{k}}(t) , \tag{1}$$

where $F[\psi]$ is the Ginzburg-Landau-Wilson free energy functional and $\theta_{\mathbf{k}}(t)$ is a Gaussian white noise with $\langle\theta\rangle=0$ and

$$\langle \theta_{\mathbf{k}}(t) \theta_{-\mathbf{k}'}(t') \rangle = 2\Gamma_0(\mathbf{k}) \delta_{\mathbf{k},\mathbf{k}'} \delta(t-t') .$$

For model A, $\Gamma_0(\mathbf{k})$ is a constant, while for a local conservation law (model B), $\Gamma_0(\mathbf{k})=\lambda_0 \mathbf{k}^2$. I consider the class of models defined by $\Gamma_0(\mathbf{k})=\lambda_0|\mathbf{k}|^\sigma$, with $\sigma \geq 0$. Using conventional renormalization-group (RG) arguments I find that $z=2+\sigma-\eta$ provided $\sigma>\sigma_c=z_A-2+\eta$. Otherwise, $z=z_A$. This change of behavior at a critical value of σ is a consequence of an interchange of stability of the conserved and nonconserved fixed points; i.e., *the conservation law is irrelevant for* $\sigma<\sigma_c$. Thus $z=\max(2+\sigma-\eta, z_A)$. In particular, $z \geq z_A$ in accord with intuitive reasoning.

The RG calculation is straightforward, and follows the treatment of model B given in Ref. 2. The first step is to divide through Eq. (1) by $\Gamma_0(\mathbf{k})$. Anticipating corrections to the equation of motion due to coarse graining, and dropping the subscripts, I write

$$1/\Gamma(\mathbf{k}) = 1/\lambda|\mathbf{k}|^\sigma + 1/\Gamma . \tag{2}$$

This model is in the same universality class as the original because $\Gamma(\mathbf{k}) \to \lambda|\mathbf{k}|^\sigma$ for $|\mathbf{k}| \to 0$. The second term on the right-hand side of (2) is generated by momentum-independent contributions to the response-function self-energy.[2] The RG procedure consists of eliminating modes with momenta in the range $\Lambda/b<|\mathbf{k}|<\Lambda$, where Λ is an ultraviolet momentum cutoff, and then rescaling momenta, times, and fields via $\mathbf{k}=\mathbf{k}'/b$, $t=b^z t'$, and $\psi_{\mathbf{k}'/b}(b^z t')=b^{(2-\eta)/2}\psi'_{\mathbf{k}'}(t')$. The rescaling of λ is trivial, because coarse graining does not lead to any contributions nonanalytic in $\mathbf{k}$. Hence λ is changed only trivially, by the change of scale.[2] This gives

$$(1/\lambda)' = b^{2+\sigma-\eta-z}(1/\lambda) . \tag{3}$$

Thus, *provided* $1/\lambda$ *is nonzero at the fixed point* (i.e., provided the conservation law is *relevant*) we have trivially $z=2+\sigma-\eta$. Note that this result holds for the "subdiffusive" case $\sigma>2$ as well as the "superdiffusive" case $\sigma<2$. The case $\sigma=2$ reproduces the conventional model-B result $z_B=4-\eta$.

To test whether the conservation law is indeed relevant, we treat the first term on the right-hand side of (2) as a small perturbation to the model-A equation of motion. At the model-A fixed point, $1/\lambda$ still renormalizes as in (3), but with $z=z_A$: $(1/\lambda)'=b^{2+\sigma-\eta-z_A}(1/\lambda)$. Hence the conservation law is asymptotically irrelevant for $\sigma<\sigma_c=z_A-2+\eta$, and model-$A$ critical behavior is recovered. Since $z_A>2-\eta$ quite generally, $\sigma_c>0$. Technically, by investigating the stability of the model-A fixed point against "weak conservation," we have proved the irrelevance of the conservation law for $\sigma<\sigma_c$ only when $1/\lambda$ is sufficiently small. I see no reason, however, to expect a critical value of $1/\lambda$ above which the conservation law becomes relevant. Investigating directly the stability of the conserved fixed point is a much harder problem.

The case of infinite-ranged spin exchange considered in Ref. 1 is described by the limit $\sigma \to 0+$, corresponding to the global constraint $\psi_{\mathbf{k}=0}(t)=0$. This case, therefore, belongs to the class of systems with $\sigma<\sigma_c$, for which $z=z_A$.

The numerical results presented in Ref. 1 were interpreted as being consistent with $z=2-\eta$. I cannot explain these data. However, equivalent simulations with infinite-range spin exchange performed by Moseley, Gibbs, and Jan[4] are entirely consistent with $z=z_A$.

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[4] L. L. Moseley, P. W. Gibbs, and N. Jan (to be published).

