Electron Transport in a Mesoscopic Annulus, Driven by a Time-Dependent Magnetic Flux

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Classical and quantum electron transport in an annulus formed by a two-dimensional electron gas in a perpendicular magnetic field is studied. The transport is driven by an enclosed time-dependent magnetic flux, and is described with a Landauer-Büttiker-type formalism. Transport current is generated because the transmission and reflection probabilities of the electrons which are emitted by the reservoirs on the inside and outside of the annulus are modified from their static values. The role of the probes, as well as the effect of scattering between the inside and outside of the annulus, is studied.

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The Landauer-Büttiker formalism for electron transport has been very successful in describing electron transport in mesoscopic conductors.[1] In this description the current through a conductor which connects two electron reservoirs is expressed in terms of (energy-dependent) transmission probabilities $T(E)$ of electrons which are emitted by these reservoirs. An applied voltage V creates a difference $eV=\mu_2-\mu_1$ between the electrochemical potentials of the reservoirs. For a one-dimensional (spin-degenerate) system at zero temperature the current I is then given by

$$I=\frac{2e}{h}\int_{\mu_1}^{\mu_2} T(E)\,dE\,. \tag{1}$$

For small V, Eq. (1) yields a conductance $G=(2e^2/h)\times T(E_F)$, with $T(E_F)$ the transmission probability at the Fermi energy.

The above formalism applies to electron transport in conductors which are subjected to *time-independent* potentials. The electron transport is then due to an imbalance between μ_1 and μ_2, since in equilibrium ($\mu_1=\mu_2$) the current flowing from reservoir 1 to reservoir 2 is exactly canceled by an equal current which flows in the opposite direction. However, electron transport between reservoirs with $\mu_1=\mu_2$ can take place when a *time-dependent* potential (either a scalar potential or a magnetic vector potential) is present.[2]

In this Letter I will study the electron transport (at zero temperature) in an annulus (Corbino disk) with circumference L, which is formed in a two-dimensional electron gas (2DEG) [see Fig. 1(a)]. The 2DEG is subjected to a time-independent perpendicular magnetic field B in such a way that only one (single-spin) Landau level is occupied. In addition, the annulus is threaded by a time-dependent magnetic flux Φ, which increases linearly with time. Electron transport from the outside to the inside of the annulus is induced by the azimuthal electric field $E_\theta=(1/L)d\Phi/dt$. This system has been studied by several authors.[3-6] A special feature is that under certain conditions exactly one electron (per occupied Landau level) is transported for each flux quantum h/e which is added to Φ.

I will present a microscopic description of the electron transport, which takes into account explicitly the coupling between the annulus and the electron reservoirs, as well as the scattering between the inside and outside of the annulus. For this purpose the Landauer-Büttiker formalism, which expresses the current in terms of reflection and transmission probabilities of electrons emitted by reservoirs, is extended to the time-dependent case. The crucial point is that a transport current is generated, because in the presence of a time-dependent Φ the reflection and transmission probabilities are modified from their static values.[7] Because of the straightforward way in which the time-dependent flux can be incorporated, this is an elementary model system for the study of transport induced by time-dependent potentials.

In high magnetic fields electrons with energy E move along equipotential lines, given by $eV(x,y)=E-\frac{1}{2}\hbar\omega_c$. The states which are relevant for the transport are located at the boundaries of the 2DEG where the Landau lev-

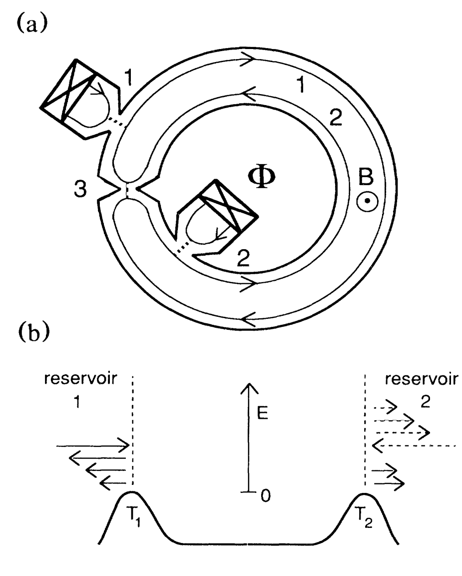

FIG. 1. (a) Schematic layout of the system, illustrating the electron flow in edge channels. (b) Cross section of the annulus, showing the potential barriers in QPCs 1 and 2 (for simplicity, the potential barrier of QPC 3 is not shown). The currents which flow at different energies are shown for electrons injected by reservoir 1 (solid arrows), and for electrons emitted by reservoir 2 (dashed arrows).

el intersects the Fermi energy, and form the edge channels.[8] At these boundaries an electric field $\mathbf{E}$ is present, perpendicular to the boundary. The electrons in edge channels 1 and 2 move in opposite directions with a drift velocity $v_D = |\mathbf{E}|/B$. The system is coupled to 2DEG regions on the outside and inside by means of quantum point contacts (QPCs) 1 and 2. The transport through these QPCs can be described[9] by the (energy-dependent) transmission and reflection probabilities $T_1(E)$ and $T_2(E)$ and $R_1(E)=1-T_1(E)$ and $R_2(E)=1-T_2(E)$. A potential barrier is present in the QPCs. The QPCs reflect all electrons with energies below the barrier, $T_1(E), T_2(E)=0$ for $E<0$, and partially transmit electrons with energies above the barrier, $T_1(E)=T_1$ and $T_2(E)=T_2$ for $E>0$. Note that the zero of the energy scale is chosen to coincide with the top of the barrier of the QPCs. The transmission and reflection probabilities are given by $T(E)=t^2(E)$ and $R(E)=r^2(E)$. The amplitudes r and t appear in the S matrix, which describes the relation between the (complex) amplitudes of the currents which enter and leave the QPCs:

$$S(E)=\begin{pmatrix} it(E) & -r(E) \\ -r(E) & it(E) \end{pmatrix}, \qquad (2)$$

with $r(E)$ and $t(E)$ real and $r^2(E)+t^2(E)=1$. The edge channels on the inner and outer perimeters can be coupled by QPC 3, with $T_3(E)=0$ for $E<E_b$, and $T_3(E)=T_3$ for $E>E_b$. The barrier height[10] E_b for QPC 3 is taken below that of QPCs 1 and 2. When $T_3=1$, the edge channels are decoupled, while when $T_3=0$ the electron flow around the annulus is prohibited, and the system becomes insensitive to Φ. Ideal[8] bulk contacts are connected to the 2DEG regions on the inside and the outside.

For the description of the electron transport, I define the following (energy- and flux-dependent) transmission and reflection probabilities:[11]

$$T_{21}(E,\Phi)=\frac{j_{i2}(\Phi)}{j_{01}(E)},\quad R_{11}(E,\Phi)=\frac{j_{i1}(\Phi)}{j_{01}(E)},$$
$$T_{12}(E,\Phi)=\frac{j_{i1}(\Phi)}{j_{02}(E)},\quad R_{22}(E,\Phi)=\frac{j_{i2}(\Phi)}{j_{02}(E)}, \qquad (3)$$

in which $j_{01}(E)$ and $j_{02}(E)$ indicate the currents (per unit energy) which are emitted into the annulus at energy E by reservoirs 1 and 2, and $j_{i1}(\Phi)$ and $j_{i2}(\Phi)$ indicate the currents which flow back into reservoirs 1 and 2 as a result of the currents $j_{01}(E)$ and $j_{02}(E)$. Note that these currents can have energies different from E. The injected currents are given by $j_{01}(E)=j_{02}(E)=e/h$ for $E<E_F$ and $j_{01}(E)=j_{02}(E)=0$ for $E>E_F$. The currents $I_1(\Phi)$ and $I_2(\Phi)$ which flow into the annulus through QPC 1 and QPC 2 can now be written as $(\mu_1=\mu_2=E_F)$

$$I_1(\Phi)=\frac{e}{h}\int_0^{E_F}[1-R_{11}(E,\Phi)-T_{12}(E,\Phi)]dE, \qquad (4)$$

$$I_2(\Phi)=\frac{e}{h}\int_0^{E_F}[1-R_{22}(E,\Phi)-T_{21}(E,\Phi)]dE. \qquad (5)$$

For static potentials,

$$R_{11}(E,\Phi)+T_{12}(E,\Phi)=R_{22}(E,\Phi)+T_{21}(E,\Phi)=1$$

and $I_1(\Phi)=I_2(\Phi)=0$. As shown below, $I_1(\Phi)$ and $I_2(\Phi)$ can be nonzero in the presence of time-dependent flux because the reflection and transmission probabilities deviate from their static value.

In the classical description of the transport the phase (coherence) of the electrons is not taken into account. The azimuthal electric field E_θ induces an additional drift velocity E_θ/B in the radial direction. Thus electrons which move in edge channel 1 lose an energy $\Delta E=e\times d\Phi/dt$ in one revolution around the annulus, while electrons in edge channel 2 gain an amount ΔE. Figure 1(b) illustrates how transport takes place for the case when $T_3=1$, and $T_1=T_2=T$. A fraction $1-T$ of the electrons injected by reservoir 2 at energy E is reflected at QPC 2. The remaining fraction T is transmitted, and makes one revolution, by which it gains an energy ΔE. Upon reaching QPC 2 again, another fraction T^2 is transmitted back into reservoir 2, and the remainder $T(1-T)$ makes another revolution, and so forth. Eventually the injected current is reflected entirely: $R_{22}(E)=1$ and $T_{12}(E)=0$. [Note that in the classical case the transmission and reflection probabilities do not depend upon Φ, and that current conservation requires that $R_{22}(E)+T_{12}(E)=1$ and $R_{11}(E)+T_{21}(E)=1$.] The situation is different for electrons emitted by reservoir 1. After $N=\mathrm{Int}(E/\Delta E)$ revolutions the energy of the fraction $T(1-T)^N$ has fallen below zero, and as a result this fraction is completely reflected at QPC 1. When its energy has dropped below E_b, it is reflected to the inner perimeter at QPC 3. It will then gain energy and leave the annulus via QPC 2. This implies $T_{21}(E)=T\times(1-T)^N$ and $R_{11}(E)=1-T(1-T)^N$, with $N=\mathrm{Int}(E/\Delta E)$. This shows that the transport current is carried by electrons with incoming energies which are close to energies where the transmission probabilities of QPC 1 and QPC 2 change with energy.[12] $T_{21}(E)$ and $R_{22}(E)$ are shown in Fig. 2(a) for $T=0.3$ and $E_F=18\Delta E$. Evaluation of I_1 $(=-I_2)$ with Eqs. (4) and (5) shows that for $E_F \gg \Delta E$ one obtains the anticipated result that one electron is transported per flux quantum. When E_F is not much larger than ΔE, the current is less than this.

The currents (per unit energy) $f_1(E)$ and $f_2(E)$ which flow back into reservoirs 1 and 2 are given in Fig. 2(b). From this the power which is dissipated in the reservoirs can be calculated:

$$P_{\mathrm{dis}}=\frac{1}{e}\int_0^\infty 2E[f_1(E)+f_2(E)-2]dE$$
$$=\frac{2(\Delta E)^2}{h}T[\ln(R)]^{-2}\ \text{ for }\ \frac{\Delta E}{E_F}\ll T\ll 1. \qquad (6)$$

The power which is supplied by the magnetic field is given by $P=(d\Phi/dt)\int j_c(E)dE$, with $j_c(E)=j_{c1}(E)-j_{c2}(E)$, the difference between the circulating currents in edge channels 1 and 2 [see Fig. 2(c)]. This is equal to

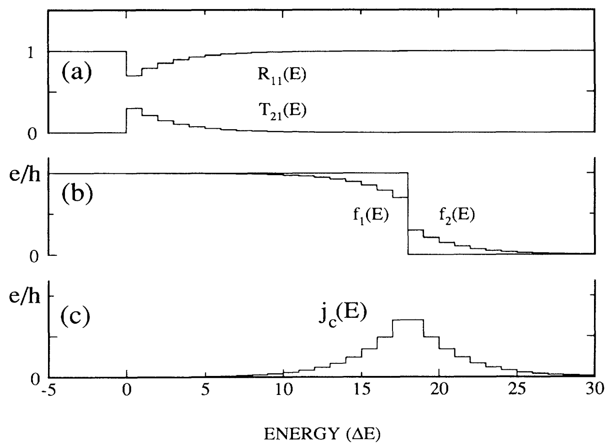

FIG. 2. (a) Reflection probability $R_{11}(E)$ and transmission probability $T_{21}(E)$ for electrons emitted by reservoir 1. (b) Currents (per unit energy) which flow back into reservoir 1 $[f_1(E)]$, and in reservoir 2 $[f_2(E)]$. (c) Circulating current (per unit energy) $j_c(E)$.

the dissipated power, since the energy which is stored in the annulus does not depend on Φ in the classical case.

When $T_3 < 1$, electrons which move in edge channel 1 can be scattered to edge channel 2, and vice versa. Calculations show that in this case the current is reduced below one electron per flux quantum.

I now discuss the transport in the quantum regime. In high magnetic fields[13] the phase ϕ which is acquired in one revolution is related to the total enclosed flux Φ_{tot} ($\Phi_{\text{tot}} = \Phi$ plus the enclosed flux through the 2DEG). For electrons in edge channel 1, $\phi_1 = 2\pi(e/h)\Phi_{\text{tot1}}$ and in edge channel 2, $\phi_2 = -2\pi(e/h)\Phi_{\text{tot2}}$. The energy dependence can then be written as[14] $\phi_1(E) = 2\pi(e/h)\Phi + (2\pi L/hv_D)E$ and $\phi_2(E) = -2\pi(e/h)\Phi + (2\pi L/hv_D)E$. When $T_1, T_2 = 0$, and $T_3 = 1$, electron states are formed at energies for which $\phi_1(E) = \pi/2 + (\text{integer})2\pi$ and $\phi_2(E) = -\pi/2 + (\text{integer})2\pi$.

The effect of the time-dependent flux can be taken into account by a time-dependent phase shift[15] $\Psi_1(t) \to \Psi_1(t)\exp(-i\Delta\omega t)$ for the wave function in edge channel 1 and a phase shift $\Psi_2(t) \to \Psi_2(t)\exp(i\Delta\omega t)$ for the wave functions of edge channel 2. The frequency shift $\Delta\omega$ corresponds with an energy change $\hbar\Delta\omega = \Delta E$. The transmission and reflection probabilities can now be obtained by calculating the propagation of a wave $\Psi_{\text{in}}(x,t) = \exp(ikx - i\omega t)$ which is emitted by a reservoir into the annulus.[16] The injected current per unit energy is then given by $j_{\text{in}} = ev_D|\Psi|^2$. The propagation of this wave in the annulus is calculated by an algorithm which calculates the reflected and transmitted waves by matching the wave functions at the QPCs, as well as taking into account the matching conditions due to the time-dependent flux. The output of the algorithm is the complex amplitudes $t_n(\Phi)$ and $r_n(\Phi)$ of the wave functions

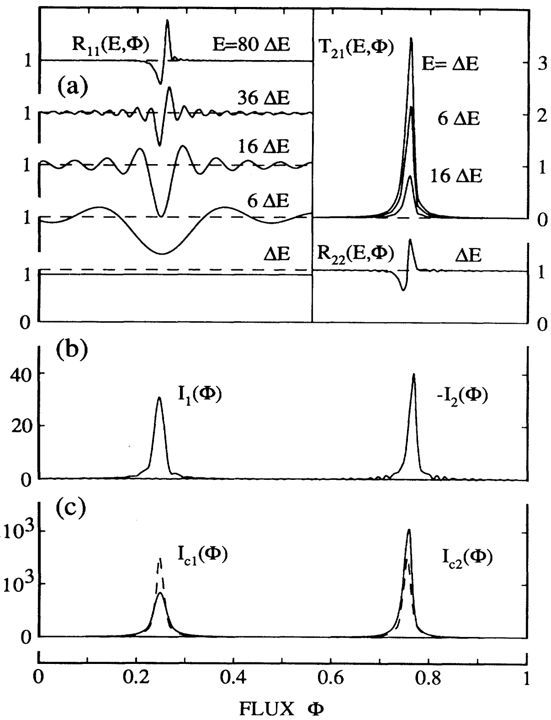

FIG. 3. (a) Top left panel: Flux dependence of $R_{11}(E,\Phi)$. Top right panel: Flux dependence of $T_{21}(E,\Phi)$ and $R_{22}(E,\Phi)$. (b) Flux dependence of the current I_1 through QPC 1 and the current I_2 through QPC 2 (in units $\Delta E\, e/h$). (c) Flux dependence of the circulating currents I_{c1} and I_{c2} (in units $\Delta E\, e/h$) for the static case (dashed lines) and the time-dependent case (solid lines).

with energies $\hbar(\omega + n\Delta\omega)$ which make up the total reflected and transmitted wave functions:

$$\Psi_{\text{ref}}(x,t) = \sum_n r_n(\Phi)\exp[-ik_n x - i(\omega + n\Delta\omega)t]\,,$$
$$\Psi_{\text{tra}}(x,t) = \sum_n t_n(\Phi)\exp[ik_n x - i(\omega + n\Delta\omega)t]\,. \tag{7}$$

The reflected and transmitted currents (evaluated at $t=0$) are now given by $j_{\text{ref}} = ev_D|\Psi_{\text{ref}}|^2 = ev_D|\sum_n r_n(\Phi)|^2$ and $j_{\text{tra}} = ev_D|\Psi_{\text{tra}}|^2 = ev_D|\sum_n t_n(\Phi)|^2$. The reflection and transmission probabilities can then be calculated with Eq. (3).

Figure 3 shows the results. The parameters are $T_1, T_2 = 0.09$, $T_3 = 0$, and $E_b = -5\Delta E$. The energy change is taken to be $\Delta E = (0.001\,25/2\pi)DE$, and is much smaller than the spacing $DE = hv_D/L$ between consecutive electron states in the annulus. This corresponds to the "adiabatic" regime where the relation between the transport current and $d\Phi/dt$ is linear. The results can be understood as follows: A wave with energy ΔE emitted by reservoir 1 has a probability 0.91 of being reflected at QPC 1. The transmitted fraction enters the annulus, loses energy, and cannot be transmitted through QPC 1 anymore. Therefore, $R_{11}(\Delta E,\Phi) = 0.91$, independent of

Φ. Electrons with $E=2\Delta E$ have the possibility of being scattered back after one revolution around the annulus. Therefore, $R_{11}(2\Delta E,\Phi)$ shows structure due to the interference of two waves with energy $2\Delta E$ and ΔE. Note that due to the constructive interference of the waves with different energies, the reflection and transmission "probabilities" can become larger than 1. For electrons with higher incoming energies the backscattered wave consists of many components. The $R_{11}(80\Delta E,\Phi)$ trace shows that when the flux is increased and crosses the value $\Phi=\frac{1}{4}\Phi_0$, the reflection probability first drops below 1 and then rises above 1. This can be understood by the charging and discharging of an electron state in edge channel 1. Figure 3(a) shows that the transmission probabilities $T_{21}(E,\Phi)$ peak at $\Phi=\frac{3}{4}\Phi_0$. The reflection probabilities $R_{22}(E,\Phi)$ do not depend on energy and can be understood in terms of the charging and discharging of electron states in edge channel 2. The transmission probabilities $T_{12}(E,\Phi)$ (not shown) are zero, since no electron wave which is emitted by reservoir 2 can reach reservoir 1.

The transmission and reflection probabilities, averaged over one flux quantum, can be shown to be equal to their classical counterparts shown in Fig. 2(a). The currents through the QPCs, obtained from Eqs. (4) and (5) with $E_F=40\Delta E$, are shown in Fig. 3(b). As anticipated, the total charge which enters the annulus at $\Phi\approx\frac{1}{4}\Phi_0$ and which leaves the annulus at $\Phi\approx\frac{3}{4}\Phi_0$ equals one electron, so that effectively one electron is transported per flux quantum. Similar to the classical case this is only true when $E_F\gg\Delta E$.

Figure 3(c) shows the flux dependence of the circulating currents in edge channels 1 and 2. Also shown is the flux dependence of the circulating (persistent) currents for a time-independent flux.[17] Because of the time-dependent flux I_{c1} is reduced below its static value, while I_{c2} is enhanced above its static value. The imbalance between I_{c1} and I_{c2} implies that the system dissipates power. Its flux-averaged value

$$P_{\mathrm{dis}}=(\Delta E/h)\int_0^{\Phi_0}[I_{c1}(\Phi)-I_{c2}(\Phi)]d\Phi$$

is found to be close to the classical value obtained from Eq. (6) [$20.8(\Delta E)^2/h$ and $20.2(\Delta E)^2/h$, respectively].

When $T_3<1$, the current is found to be less than one electron per flux quantum, similar to the classical case. In this case a further study of the dependence of the current on the parameters of the system is required.

In summary, I have given a description of electron transport generated by a time-dependent flux, in the adiabatic regime. An interesting continuation would be to study the nonadiabatic regime, where Zener tunneling between adjacent electron levels is expected to play an important role.[18]

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[10]Because of the presence of the barrier in QPC 3, the electron states in the bulk of the 2DEG (which have energies $E<E_b$), are not extended around the annulus and do not contribute to the transport. The transport can therefore be described exclusively in terms of edge channels.

[11]In accordance with the static case I will call these "probabilities," although they can be larger than unity.

[12]It can be shown that for the case when the barrier E_b of QPC 3 is higher than that of QPC 1 and QPC 2, the current is carried by electrons with incoming energies near E_b.

[13]The relation between flux Φ and phase ϕ is given by the semiclassical quantization condition for the guiding-center motion: $\phi=2\pi(n+\frac{1}{2})$, with n the number of enclosed flux quanta. For simplicity, I have omitted the $\frac{1}{2}$ in the text.

[14]The change in flux $\Delta\Phi$ can be related to the change in radius Δr of the edge channel: $\Delta\Phi=2\pi Br\Delta r$. A change in energy can be written as $\Delta E=e\mathbf{E}\Delta r$. The combination of these relations leads to the expressions given in the text.

[15]This can be seen by selecting a gauge in which $\mathbf{E}_\theta=-\nabla\phi$ (with ϕ a time-independent scalar potential) and $d\mathbf{A}_\theta/dt=0$ everywhere except in an infinitesimally narrow region between $\theta=0^-$ and $\theta=0^+$. The effect of the time-dependent vector potential $(d/dt)\int_{0^-}^{0^+}\mathbf{A}_\theta dl=d\Phi/dt$ can be accounted for by time-dependent boundary conditions for the wave functions: $\Psi(0^+)=\Psi(0^-)\exp(i\Delta\omega t)$.

[16]It is assumed that ΔE is sufficiently small so that the transverse shift $\Delta y=\Delta E/e\mathbf{E}$ between wave functions with energy difference ΔE is small compared to the magnetic length $l_B=\sqrt{\hbar/eB}$. The transverse part of the wave functions can then be taken identical, and only the longitudinal part of the wave functions has to be considered.

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