Critical Conductivity Exponent for Si:B

Peihua Dai, Youzhu Zhang, and M. P. Sarachik
City College of the City University of New York, New York, New York 10031

(Received 19 December 1990)

We have determined the critical exponent which characterizes the approach of the zero-temperature conductivity to the insulating phase from measurements down to 60 mK of the resistivity of a series of just-metallic uncompensated *p*-type Si:B samples with dopant concentrations near the critical concentration for the metal-insulator transition. Our results indicate a critical exponent for Si:B of $0.65^{+0.05}_{-0.14}$, which is close to the "anomalous" values near $\frac{1}{2}$ found for the uncompensated *n*-type silicon-based semiconductors Si:P, Si:As, and Si:Sb. This implies that, despite strong spin-orbit scattering, Si:B belongs to the same universality class as other silicon-based systems.

PACS numbers: 71.30.+h

The critical exponent ν which characterizes the approach of the zero-temperature conductivity, $\sigma(T=0)=\sigma_0(n/n_c-1)^\nu$, to the metal-insulator transition has been the subject of considerable attention, both theoretically[1] and experimentally.[2] Here n is the net carrier concentration and n_c is the critical concentration for the metal-insulator transition. An exponent $\nu \approx 1$ has been found in most amorphous metal-insulator alloys [Kr:Bi,[3] Si alloyed with metals such as Nb (Ref. 4) or Au,[5] Ge:Au,[6] Ge:Mo,[7] and many others[2]] and for many compensated [e.g., Ge:Sb,[8] Si:(P,B),[9] *n*-Ga:As (Ref. 2)] and uncompensated [Ge:Sb (Ref. 10)] crystalline doped semiconductors. On the other hand, an exponent close to $\frac{1}{2}$ has been found in all the uncompensated *n*-type silicon-based crystalline semiconductors, namely, Si:P (Ref. 11) and Si:As,[12,13] as well as Si:As+P,[14] and probably Si:Sb.[15] Most recently, Zint, Rohde, and Micklitz[16] have reported a conductivity exponent of $\frac{1}{2}$ in amorphous Ar:Ga.

The difference between these two apparently distinct types of behavior remains a puzzle, and various conjectures and possible explanations have been advanced to resolve it. Spin-flip scattering in the presence of localized magnetic moments has been suggested[2,17-19] as a possible source for the "anomalous" value $\nu=\frac{1}{2}$ in the case of Si:P and Si:As. Spin-orbit scattering has also been proposed as a factor which may be responsible for the difference. Based on Fermi-liquid theory applied to these materials, Castellani, Kotliar, and Lee[19] have claimed that in the case of strong spin-orbit scattering, the metal-insulator transition is driven by electron-electron interactions with a critical exponent $\nu=1$ rather than being a true localization transition where the diffusion constant $D\rightarrow 0$ and the density of states at the Fermi level remains finite. The difference between exponents may thus be associated with the relative importance of spin-orbit scattering.

In contrast with *n*-type Si:P and Si:As, spin-orbit scattering is expected to play an important role[20,21] in *p*-type Si:B. Whereas the six degenerate conduction-band minima are at different equivalent points in the Brillouin zone, silicon has degenerate light- and heavy-hole $J=\frac{3}{2}$ valence-band maxima at $k=0$ and a spin-orbit-split $J=\frac{1}{2}$ band at $k=0$ at an energy 0.044 eV below these. The scattering by impurities causes transitions among states with different J_z values between the degenerate heavy- and light-hole bands at a rate comparable with ordinary potential scattering.[20,21] The importance of spin-orbit scattering in Si:B is evidenced by a g value[22,23] of 1.2, which is substantially different from the value near 2 found for Si:P. Additional experimental support is provided by the fact that, unlike *n*-type silicon where the magnetoresistance is negative[11] at low fields due to the delocalizing effect of a magnetic field,[24] the magnetoresistance of Si:B is positive[25] as is expected[21,26] when spin-orbit scattering is the dominant phase-breaking mechanism.

In this paper we present resistivity studies of a series of just-metallic Si:B samples which indicate that despite the presence of spin-orbit scattering the critical exponent in Si:B is $0.65^{+0.05}_{-0.14}$, a value which is quite close to the critical exponents 0.55 ± 0.1, 0.60 ± 0.05, and 0.49 ± 0.09 found for Si:P,[11] Si:As,[13] and Si:Sb,[15] respectively.

Wafers of boron-doped silicon roughly 0.3 mm thick were obtained commercially from Pensilco Corporation. The samples were etched in CP4 and thin leads of gold containing 2% tin were attached by spark welding. Room-temperature resistivities and resistivity ratios $\rho(4.2\text{ K})/\rho(300\text{ K})$ were determined in the van der Pauw[27] geometry. A semilogarithmic plot of the resistivity ratio versus room-temperature resistivity is shown in Fig. 1(a), while Fig. 1(b) shows the ratio plotted as a function of carrier concentration deduced from the calibration of Thurber *et al.*,[28] again on a semilogarithmic scale. Samples used for measurements below 1 K were cut in a bar-shaped configuration roughly 1.5 mm×8 mm; their boron concentrations were determined from measurement of the resistivity ratio and use of Fig. 1(b). Boron-ion implants at the contact points were required

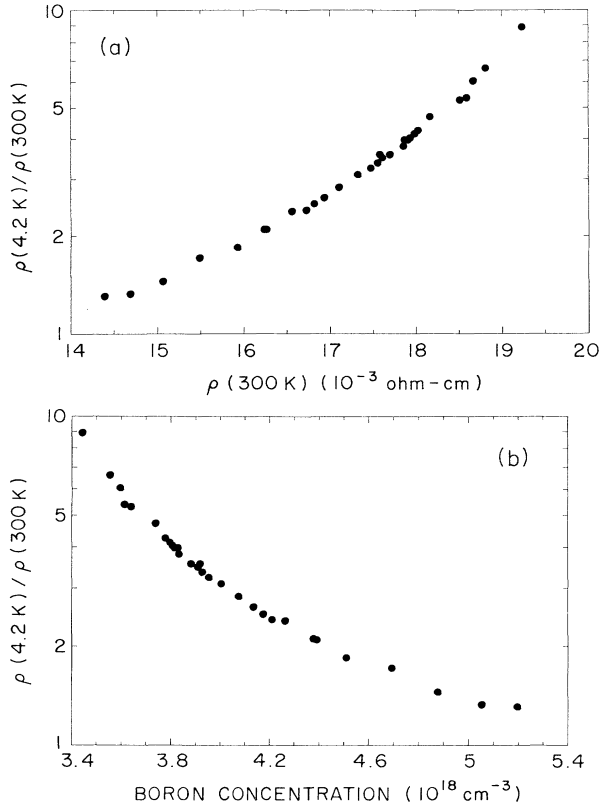

FIG. 1. (a) Resistivity ratio ρ(4.2 K)/ρ(300 K) plotted as a function of the room-temperature resistivity ρ(300 K) on a semilogarithmic scale. (b) Resistivity ratio vs boron concentration, obtained using Ref. 28.

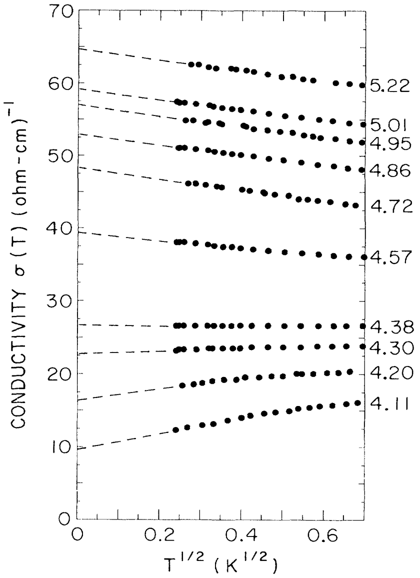

FIG. 2. Conductivity plotted as a function of $T^{1/2}$ for Si:B. Boron concentrations are indicated next to each curve in units of 10^{18} cm^{-3}. The dashed lines represent linear-regression fits to the data.

for the measurements below 1 K to insure Ohmic behavior and to minimize self-heating. Standard low-frequency four-terminal ac measurements were made in an Oxford model 75 dilution refrigerator with an RV-Elektroniikka Oy model AVS-46 ac bridge and with an EG&G PAR model 124A lock-in amplifier. Since small stresses associated with attaching the Si:B samples to a sample holder with grease were found to produce very large changes in the measured conductivity, thermal contact was maintained by immersing the samples directly in the helium-3–helium-4 mixture.

The conductivities of ten samples used in these studies, containing different boron concentrations as labeled, are plotted as a function of $T^{1/2}$ in Fig. 2. Zero-temperature extrapolations were obtained by fitting the data by $\sigma(T) = \sigma(0) + m(n)T^{1/2}$, where the temperature-dependent term is associated with electron-electron interactions.[29] We disregard deviations from this simple form due to localization, which become increasingly important at higher temperatures, by restricting the range of the fits to data below 500 mK ($T^{1/2} = 0.71$) for all samples except the three closest to the transition, for which only data below 200 mK ($T^{1/2} = 0.45$) were used. As has also been found in other systems such as Si:P (Ref. 11) and Ge:Sb,[30] the slope $m(n)$ varies with concentration and changes numerical sign.

The intercepts $\sigma(0)$ deduced from linear-regression fits of the data by the above expression are plotted as a function of dopant concentration in Fig. 3. A nonlinear

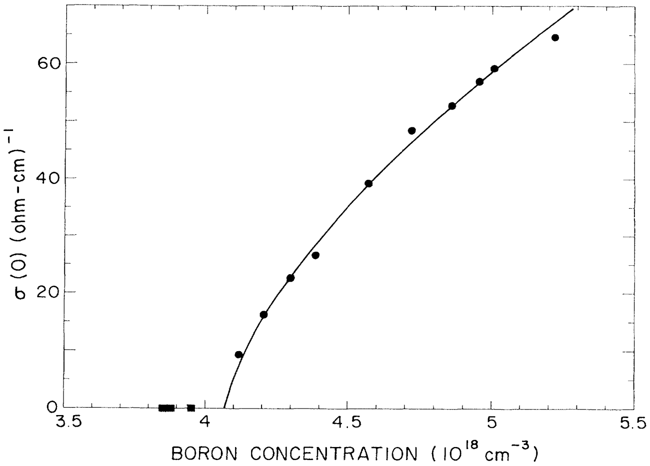

FIG. 3. Zero-temperature conductivity $\sigma(T=0)$ vs boron concentration. The circles represent metallic samples, while the three squares indicate samples (not shown in Fig. 2) which were found to exhibit hopping conductivity characteristic of insulators. The solid line is a best fit for the metallic samples by $\sigma(0) = \sigma_0(n/n_c - 1)^{\nu}$, with $\nu = 0.65$. Note that a lower bound on n_c is set by the insulating sample with boron concentration $n = 3.95 \times 10^{18}$ cm^{-3}.

least-squares fit of the resultant $\sigma(0)$ values by the expression $\sigma(0)=\sigma_0(n/n_c-1)^{\nu}$ yields a prefactor $\sigma_0=152 \pm 10$ $(\Omega\,\mathrm{cm})^{-1}$, a critical concentration $n_c=(4.06 \pm 0.02)\times 10^{18}$ cm^{-3}, and an exponent $\nu=0.65\pm 0.05$, where the quoted errors for each parameter correspond to 1 standard deviation with the other two parameters allowed to vary freely. One should note, however, that $\sigma(0)$ becomes increasingly uncertain near the metal-insulator transition, owing to the fact that progressively lower temperatures are required for a reliable determination of $\sigma(0)$ as the transition is approached, and to a lack of a complete theoretical understanding of the temperature dependence expected for the conductivity very near the transition. Maliepaard *et al.*[31] have found, for example, that samples of *n*-type compensated GaAs very near the transition exhibit a $T^{1/3}$ rather than $T^{1/2}$ dependence; analysis of our data using this form for the three samples with the lowest dopant concentrations yields essentially the same value $\nu=0.64$. Omitting the two points closest to the transition entirely due to this uncertainty yields $\nu=0.51$. If one enlarges the error bars to include this possibility, one obtains $\nu=0.65^{+0.05}_{-0.14}$, $\sigma_0=152^{+10}_{-18}$ $(\Omega\,\mathrm{cm})^{-1}$, and $n_c=(4.06^{+0.12}_{-0.02})\times 10^{18}$ cm^{-3}. It is clear in any event that an exponent of 1 is inconsistent with the experimental data.

Spin-orbit scattering is expected to be important in *p*-type materials such as Si:B. Experimental corroboration is provided by the fact that, in contrast with Si:P, the magnetoresistance of Si:B is found to be positive[25] for all dopant concentrations even in very small magnetic fields over the range of temperatures between 80 mK and 4.2 K measured to date. Despite this, the critical exponent for the conductivity of Si:B is close to $\frac{1}{2}$, and very similar to the value for all other uncompensated silicon-based doped semiconductors. Thus, although the spin-orbit scattering determines the sign of the magnetoresistance, it does not appear to determine the nature of the transition. Our results imply that the physical processes which govern the transition must be the same or similar for uncompensated *n*-type and *p*-type silicon-based semiconductors. Further, they must be different in the case of Ge:Sb,[10] which has an exponent $\nu=1$, but where the magnetoresistance is negative[32] indicating that spin-orbit scattering is relatively weak.

We are grateful to Alice White of AT&T Bell Laboratories for the ion implantations which were essential for making contact to our samples for the low-temperature measurements. We are indebted to R. Richardson and E. Smith of Cornell, whose gift to us of a calibrated thermometer for use in our dilution refrigerator made these experiments possible. We thank David Schmelzer, Subir Sachdev, and Miguel Levy for enlightening discussions. We also thank H. Z. Cummins for his invaluable advice on the error analysis. This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-84ER45153. Partial support was also provided by the Professional Staff Congress, City University of New York.

---

[1] See, for instance, P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985), and references therein.

[2] See, for example, G. A. Thomas, in *Localisation and Interaction in Disordered Metals and Doped Semiconductors,* edited by D. M. Finlayson (Scottish Universities Summer School in Physics, Edinburgh, 1986), p. 172, and references therein; Philos. Mag. B **52**, 479 (1985). Also references listed below.

[3] M. Rohde and H. Micklitz, Phys. Rev. B **36**, 7572 (1987).

[4] G. Hertel, D. J. Bishop, E. G. Spencer, J. M. Rowell, and R. C. Dynes, Phys. Rev. Lett. **50**, 743 (1983).

[5] M. Yamaguchi, N. Nishida, T. Furubayashi, K. Morigaki, H. Ishimoto, and K. Ono, Physica (Amsterdam) **118B**, 694 (1983).

[6] W. L. McMillan and J. Mochel, Phys. Rev. Lett. **46**, 556 (1981).

[7] S. Yoshizumi, D. Mael, T. H. Geballe, and R. L. Greene, in *Localization and Metal-Insulator Transition,* edited by H. Fritzsche and D. Adler (Plenum, New York, 1985), p. 77.

[8] G. A. Thomas, Y. Ootuka, S. Katsumoto, S. Kobayashi, and W. Sasaki, Phys. Rev. B **25**, 4288 (1982).

[9] M. J. Hirsch, U. Thomanschefsky, and D. F. Holcomb, Phys. Rev. B **37**, 8257 (1988).

[10] Y. Ootuka, H. Matsuoka, and S. Kobayashi, in *Anderson Localization,* edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1988), p. 40.

[11] T. F. Rosenbaum, R. F. Milligan, M. A. Paalanen, G. A. Thomas, R. N. Bhatt, and W. Lin, Phys. Rev. B **27**, 7509 (1983).

[12] P. F. Newman and D. F. Holcomb, Phys. Rev. B **28**, 638 (1983).

[13] D. W. Koon and T. G. Castner, Phys. Rev. B **40**, 1216 (1989).

[14] P. F. Newman and D. F. Holcomb, Phys. Rev. Lett. **51**, 2144 (1983).

[15] A. P. Long and M. Pepper, J. Phys. C **17**, L425 (1984); Solid State Electron. **28**, 61 (1985).

[16] Th. Zint, M. Rohde, and M. Micklitz, Phys. Rev. B **41**, 4831 (1990).

[17] S. Hikami, Prog. Theor. Phys. **64**, 1466 (1980).

[18] M. A. Paalanen and R. N. Bhatt (to be published).

[19] C. Castellani, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **59**, 323 (1987).

[20] R. J. Elliott, Phys. Rev. **96**, 266 (1954).

[21] A. Kawabata, J. Phys. Soc. Jpn. **55**, 3299 (1986), and references therein; in *Proceedings of the Fifteenth International Conference on the Physics of Semiconductors, Kyoto, 1980* [J. Phys. Soc. Jpn. **49**, Suppl. A, 375 (1980)].

[22] G. Feher, J. C. Hansel, and E. A. Gere, Phys. Rev. Lett. **5**, 309 (1960).

[23] J. M. Cherlow, R. L. Aggarwal, and B. Lax, Phys. Rev. B **7**, 4547 (1973).

[24] A. Kawabata, Solid State Commun. **34**, 431 (1980); J. Phys. Soc. Jpn. **49**, 628 (1980).

[25] P. Dai, Y. Zhang, and M. P. Sarachik (to be published); H.

Roth, W. D. Straub, W. Bernard, and J. E. Mulhern, Jr., Phys. Rev. Lett. **11**, 328 (1963).

[26] S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980); B. L. Al'tshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel'nitskii, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)].

[27] L. J. van der Pauw, Philips Res. Rep. **13**, 1 (1958).

[28] W. R. Thurber, R. L. Mattis, Y. M. Liu, and J. J. Filiben, J. Electrochem. Soc. **127**, 2291 (1980).

[29] B. L. Al'tshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)]; Pis'ma Zh. Eksp. Teor. Fiz. **30**, 514 (1979) [JETP Lett. **30**, 482 (1979)].

[30] G. A. Thomas, A. Kawabata, Y. Ootuka, S. Katsumoto, S. Kobayashi, and W. Sasaki, Phys. Rev. B **26**, 2113 (1982).

[31] M. C. Maliepaard, M. Pepper, R. Newbury, and G. Hill, Phys. Rev. Lett. **61**, 369 (1988).

[32] T. A. Polyanskaya and I. I. Saidashev, Zh. Eksp. Teor. Fiz. **84**, 997 (1983) [Sov. Phys. JETP **57**, 578 (1983)].