Oscillating Soliton Stars

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(Received 26 December 1990)

We study a new type of self-gravitating object, described by a soliton solution to the coupled system of the Einstein equation and a matter field equation. The solution describing the self-gravitating object is not static but, instead, is periodic, with both the spacetime geometry and the matter field oscillating in time. In particular, we show that a system described by a real scalar field $g^{\mu\nu}\phi_{;\mu\nu}-m^2\phi=0$ admits such a solution, which is stable under perturbations. The existence of such objects could have important implications for astrophysics and cosmology.

PACS numbers: 95.30.Sf, 04.20.Jb

It has been long known that classical field theories[1-5] admit nontopological soliton solutions, i.e., solutions which have finite and nonzero masses, confined to finite regions of space for all time, free of singularity, and which are nontopological in nature.[6] The recent surge of interest in these solutions stems from the dark-matter problem in cosmology. It is by now well accepted that visible, baryonic matter can account for only a small fraction of the total mass of the Universe, and there are strong indications that the dark matter is nonbaryonic in nature. Various kinds of nontopological soliton configurations of nonbaryonic matter have been proposed and studied for their possible astrophysical roles. These include[7] (i) Q-balls,[8] (ii) scalar soliton stars,[9] and (iii) boson stars.[10-12] They are configurations made up of complex scalar fields, through nonlinear couplings of the scalar field to itself, to other matter fields, or to gravity. It has been argued that such solitonic solutions to the classical field theories are possible due to the existence of conserved Noether currents[8,9] in the theories. In the cases (i)-(iii) above, the conserved current is a result of the global U(1) symmetry of the complex scalar fields.[13] It has been believed that, e.g., in the case of a real scalar field,[9] because of the absence of such a symmetry there is no nontopological soliton solution.

In this Letter we investigate the existence of soliton solutions for classical field theories without an explicit conserved Noether current. As an example, we show that a massive real scalar field satisfying the Klein-Gordon equation can form a self-gravitating solitonic object when coupled to Einstein gravity. Unlike those in the cases (i)-(iii), this new class of objects is not static, but rather, is periodic in time.[14] We call such objects oscillating soliton stars, emphasizing their possible astrophysical role.

As a simple example of an oscillating soliton star, we consider a massive, real Klein-Gordon scalar field, coupled only to gravity. In the absence of angular momentum, we expect the soliton solution to be spherically symmetric. The metric can then be written in the form

$$ds^2=-N^2(t,r)dt^2+g^2(t,r)dr^2+r^2d\Omega^2 .$$

The coupled Einstein-Klein-Gordon equations lead to

$$(N^2)'=N^2\left[\frac{g^2-1}{r}\right]+4\pi Gr(N^2\phi'^2-N^2g^2\phi^2+g^2\dot\phi^2)\,, \tag{1}$$

$$(g^2)'=-g^2\left[\frac{g^2-1}{r}\right]+4\pi Grg^2\left[\frac{g^2}{N^2}\dot\phi^2+\phi'^2+g^2m^2\phi^2\right], \tag{2}$$

$$\ddot\phi=\frac{(N^2)^{\cdot}}{2N^2}\dot\phi+\frac{(N^2)'}{2g^2}\phi'+\frac{N^2}{g^2}\left[\phi''-\frac{(g^2)'}{2g^2}\phi'-\frac{(g^2)^{\cdot}}{2N^2}\dot\phi\right]+\frac{2}{r}\frac{N^2}{g^2}\phi'-m^2N^2\phi\,, \tag{3}$$

where an overdot denotes $\partial/\partial t$ and a prime denotes $\partial/\partial r$.

It is tempting to search for time-independent solutions. However, the pseudovirial theorem of Rosen[15] implies that no such solution is possible in the Newtonian limit, and in the strong-field case it has been shown numerically in Ref. 4 that no nonsingular solution exists. All known static solutions to the system (1)-(3) either have singularities[16] or are topologically nontrivial.[17]

We find that there exist nontopological solitons described by periodic solutions. The structure of the Eqs. (1)-(3) suggests periodic expansions of the form

$$N^2(t,r)=1+\sum_{j=0}^{\infty}N_{2j}(r)\cos(2j\omega_0t)\,,$$

$$g^2(t,r)=1+\sum_{j=0}^{\infty}g_{2j}(r)\cos(2j\omega_0t)\,,$$

$$\phi(t,r)=\sum_{j=1}^{\infty}\phi_{2j-1}(r)\cos[(2j-1)\omega_0t]\,.$$

We put these expansions into Eqs. (1)-(3), set the coefficients of each Fourier component to zero, and obtain a system of coupled nonlinear *ordinary* first-order differential equations for $N_{2j}(r)$ and $g_{2j}(r)$, and second-order differential equations for $\phi_{2j-1}(r)$. The boundary conditions are given by the following requirements: (i) Asymptotic flatness requires $N_{2j}(\infty)=g_{2j}(\infty)=\phi_{2j+1}(\infty)=0$. (ii) At $r=0$, the absence of a conical singularity implies $g_{2j}(r=0)=0$. (iii) The requirement that the metric coefficients be finite at $r=0$ implies that $(d/dr)\phi_{2j-1}(r=0)=0$. This is an eigenvalue problem: Is there a nontrivial solution to the set of ordinary differential equations satisfying the above boundary conditions for some particular values of $N_{2j}(0)$, $\phi_{2j-1}(0)$, and ω_0? An analytic solution is clearly impossible. To proceed, we truncate the system of equations after a certain maximum $j=j_{\max}$, numerically solve the eigenvalue problem, and study the convergence of the series as a function of $j=j_{\max}$. We find that for each value of $\phi_1(r=0)$ there exists a set of values for the other initial data such that a solution satisfying the appropriate boundary conditions at $r=\infty$ exists. A typical radial metric function $g^2(t=0,r)-1$, for the case of $\phi_1(0)=0.20$, is plotted as a solid line in Fig. 1, while the individual components $g_{2j}(r)$ are plotted as dashed lines for the first few values of j. The convergence of the series expansion is manifest. We also show the scalar field energy densities (as measured by an observer at fixed radius r) at $\omega_0 t=0$, $\pi/2$, and π. In Fig. 2 the mass M of the star is plotted against the radius containing 95% of its mass. This mass curve is similar to those of white dwarfs, neutron stars, and boson stars, with a maximum mass given by $M_c \approx 0.6 M^2_{\text{Planck}}/m$. For a scalar field with a mass $m=10^{-5}$ eV (e.g., an axion), we have $M_c=1.7\times10^{25}$ kg, and the 95% radius is 14 cm.

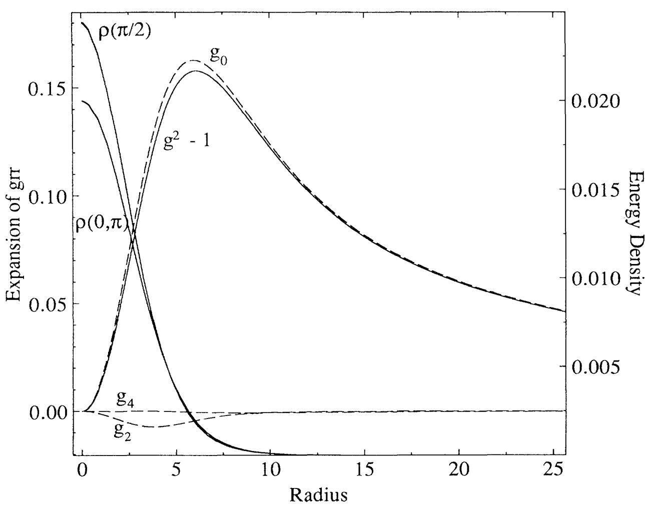

FIG. 1. A typical solution to the truncated eigenvalue equations ($j_{\max}=2$) of the metric quantity g_{rr} is shown (left vertical axis) for a solution with a mass $M=0.562M^2_{\text{Planck}}/m$. The solid line shows $g_{rr}-1$, while the dashed lines show the first three terms of its cosine series expansion. This rapid convergence of the series is typical of all the configurations we have calculated. The energy density ρ is plotted (right vertical axis) at several times $\omega_0 t=0,\pi/2,\pi$ as solid, monotonic decreasing lines, showing explicitly the intrinsic oscillations. The two lines marked $\rho(0,\pi)$, corresponding to times $\omega_0 t=0,\pi$, coincide exactly.

We note that this oscillating soliton solution cannot be obtained as a post-Newtonian expansion, even for those weak-field configurations having small total mass M and large radii. In the Newtonian expansion time derivatives of the metric functions are treated as one order higher in smallness than spatial derivatives. This is not true for oscillating soliton stars, for which temporal derivatives are of zeroth order ($\partial/\partial t \approx \omega_0 \approx m$). The oscillation is an intrinsic character of the solution.

With the construction of the solutions above, two important issues concerning their behavior arise immediately. First, given the explicit construction of the first few terms in the expansion, one would like to investigate the importance of the rest of the terms which were neglected. Second, it is important to know whether the solution is stable with respect to perturbations.

Both of these two issues can be addressed in one stroke, by dynamically evolving the system in time, using the solution to the truncated eigenvalue problem as an initial configuration. Such an initial configuration can be regarded as an exact oscillating soliton solution with a small perturbation resulting from the truncation. We developed a 3+1 numerical relativity code similar to the one described in Ref. 11 for the dynamical evolution. In Fig. 3 we show the results of one such evolution, using

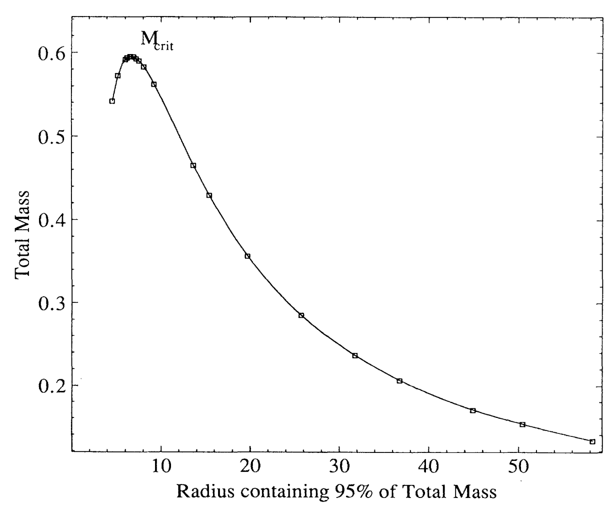

FIG. 2. The total mass M of the oscillating soliton star (in units of M^2_{Planck}/m) is plotted as a function of its radius R (in units of $1/m$). The squares represent actual configurations resulting from solutions to the eigenvalue equations. Configurations to the right of the maximum mass $M_{\text{crit}}\approx0.6$ are stable, while those to the left are unstable.

the truncated solution described in Fig. 1 as the initial configuration. The system settles down after a time $t \sim 300$. The Fourier power spectrum of the resulting configuration is shown, where the Fourier transform of $g^2_{\rm max}$ has been plotted, with $g^2_{\rm max}$ being the spatial maximum value of g^2 on a given time slice (cf. Fig. 1). Compared to the original truncated solution, the first few Fourier coefficients of this final configuration are slightly readjusted, while higher-order coefficients are clearly decreasing rapidly.

To further demonstrate the stability and physical realizability of the oscillating soliton star, we consider a configuration where the scalar field is initially a Gaussian, centered at the origin, with a total mass of $M=0.52$. The subsequent evolution is followed up to $t=14000$. We find that the initial configuration gradually settles into an oscillating soliton star, with a decaying overall vibration superimposed on the intrinsic oscillations. This evolution is illustrated in Fig. 4 where we plot the coefficients a_n and b_n of the expansion

$$g^2_{\rm max}(t) = \sum_{n=0}^{\infty} [a_n \cos(nt) + b_n \sin(nt)] ,$$

with $g^2_{\rm max}(t)$ defined above. The overall vibration of the star has frequency of $\omega_V = 0.0196$ (the leftmost peak), which is much lower than the intrinsic oscillations, $n\omega_0$. From the shift in the positions of the peaks of the sine coefficient b_n with respect to the cosine coefficient a_n, we can read out the decay time scale of this vibration to be $\tau_d \approx 2.0 \times 10^3$. The absence of any shift in the position of the peaks for the intrinsic oscillation frequencies also substantiates that these oscillations are stable, at least as far as $t=14000$, which is many orders of magnitude longer than the intrinsic oscillation time scale 2π.

A few concluding remarks are in order. Although we have not proved analytically that the series represents an exact solution to the Klein-Gordon-Einstein system for a massive scalar field,[18,19] we have given strong evidence that the series indeed converges rapidly, both by constructing numerical solutions to the eigenvalue equations (Fig. 1) and by considering the Fourier spectrum of evolved configurations (Figs. 3 and 4). It is still possible that the solution is not strictly periodic, but just quasiperiodic in time (in the same sense as binary stellar systems), with a secular evolution time scale many orders of magnitude longer than that of the oscillation period. We have demonstrated that the solution is stable with respect to a wide variety of radial perturbations and is the final point of evolution of an initial Gaussian distribution (Figs. 3 and 4). Nonspherical perturbations, which would produce gravitational radiation and which could affect the stability of the star, will be considered in future work. Preliminary study shows that it can be formed under very general initial conditions, e.g., through collapse due to a Jeans instability. Therefore, even if the object is just quasiperiodic, the existence of this new type of self-gravitating object could have significant cosmological and astrophysical implications, as many dark-matter candidates are described by real scalar fields, e.g., the axion[20] and the pseudo Higgs bo-

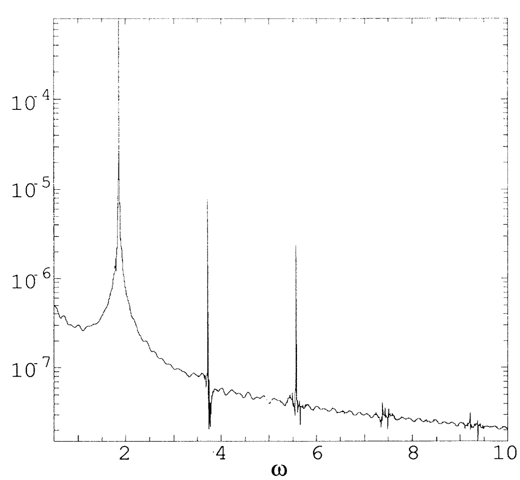

FIG. 3. The Fourier amplitude of the spatial maximum of the radial metric function g_{rr} is plotted for the evolved configuration shown in Fig. 1. There is a strong peak at $\omega=1.87$, followed by successively weaker and weaker peaks at higher frequencies. Rapid convergence is clear.

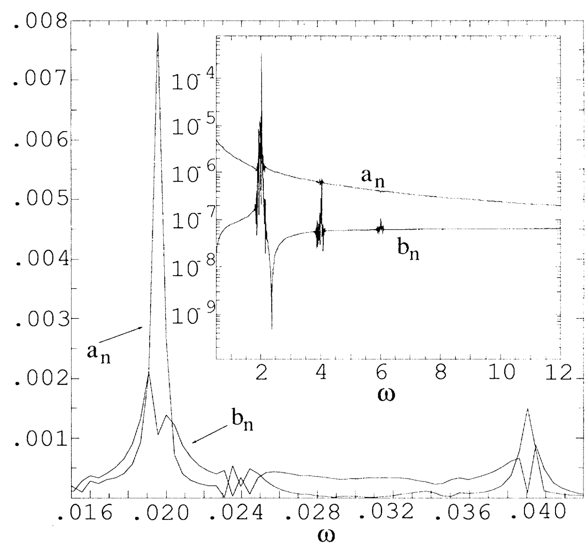

FIG. 4. The absolute value of the Fourier sine and cosine coefficients are shown of $g_{rr\,\rm max}$ for the initial Gaussian configuration discussed in the text. The peaks at $\omega_V=0.0196$ correspond to the lowest vibration frequency of the oscillating soliton star, and the peaks at $\omega=0.39$ correspond to the first overtone. Inset: The higher-frequency peaks are the fundamental oscillations of the solution. The banded structure of these peaks results from the superposition of the vibration, i.e., peaks in the bands have spacing ω_V.

son.[21] On the one hand, the existence of such objects gives rise to the possibility that the dark matter is made up of oscillating soliton stars, and, on the other hand, the condensation of, e.g., axions or pseudo Higgs bosons into very compact, high-density oscillating soliton stars may significantly enhance their annihilation rates, which could in turn rule them out as dark-matter candidates. Whether one of these interesting possibilities turns out to be the case hinges on the formation process of the oscillating soliton stars, and will be discussed elsewhere.

We thank David Hobill, Ian Redmount, Marc Sher, Kip Thorne, and Clifford Will for useful discussions. This work was supported in part by NSF Grant No. PHY89-06286 and National Center for Supercomputer Applications Grant No. 26530. E.S. would like to thank the CNRS at the Observatoire de Paris-Meudon for support while part of this work was done.

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