Emissivity of a Hot Plasma from Photon and Plasmon Decay

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Previous calculations of the emissivity of a plasma due to the decay of transverse photons and plasmons into neutrino pairs have used dispersion relations that are inaccurate at relativistic temperatures or densities. In the high-temperature limit, the use of the correct ultrarelativistic dispersion relations increases the emissivity by a factor of 3.185. This may have a significant effect on the initial cooling rate for the core of a neutron star.

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The emission of neutrinos can be a significant energy-loss mechanism for very hot or very dense stars. In some ranges of temperature and density, the dominant emission process is the decay of a photon or plasmon into a pair of neutrinos: $\gamma \to \nu\bar{\nu}$. This process owes its existence to plasma effects, which generate a mass for transversely polarized photons and also give rise to a propagating longitudinally polarized mode called the plasmon. Previous calculations[1-4] of the emissivity of a stellar plasma due to photon and plasmon decay have relied on dispersion relations that are accurate only at nonrelativistic temperatures and densities. In this Letter, the emissivities are calculated for ultrarelativistic temperatures or densities using the correct dispersion relations. The emissivity in the high-temperature limit is found to be significantly larger than in previous calculations.

An electron plasma is ultrarelativistic if either the temperature T or the chemical potential μ is much greater than the electron mass m_e. This condition corresponds to $T \gg 6\times10^9$ K or $\rho/\mu_e \gg 10^6$ g/cm^3, where ρ/μ_e is the mass density multiplied by the proton to nucleon ratio. Since ρ/μ_e scales like μ^3 at ultrarelativistic densities, we are safely in the ultrarelativistic region if either $T > 6\times10^{10}$ K or $\rho/\mu_e > 10^9$ g/cm^3. At ultrarelativistic temperatures or densities, the plasma frequency m_γ is given by

$$m_\gamma^2 = \frac{2e^2}{3\pi^2}\int_0^\infty dp\, p\left[\frac{1}{e^{(p-\mu)/T}}+\frac{1}{e^{(p+\mu)/T}}\right], \qquad (1)$$

where $e^2/4\pi \simeq 1/137$ is the electromagnetic fine-structure constant. In both the limits $\mu \ll T$ and $T \ll \mu$, (1) reduces to $m_\gamma^2 = e^2T^2/9 + e^2\mu^2/3\pi^2$. The notation m_γ for the plasma frequency is appropriate because it is the rest mass for both transverse photons and plasmons. At ultrarelativistic temperatures or densities, the dispersion relations $\omega_t(q)$ for transverse photons and $\omega_l(q)$ for plasmons depend only on the photon rest mass m_γ. They are the solutions to the transcendental equations[5]

$$\omega_t^2 - q^2 + \frac{3}{2}m_\gamma^2\left[\frac{\omega_t(\omega_t^2-q^2)}{2q^3}\ln\frac{\omega_t+q}{\omega_t-q} - \frac{\omega_t^2}{q^2}\right] = 0, \qquad (2)$$

$$q^2 - \frac{3}{2}m_\gamma^2\left[\frac{\omega_l}{q}\ln\frac{\omega_l+q}{\omega_l-q} - 2\right] = 0. \qquad (3)$$

These dispersion relations are shown as solid lines in Fig. 1.

Previous calculations of the emissivity of a plasma due to photon and plasmon decay have used approximations to their dispersion relations that have a limited range of

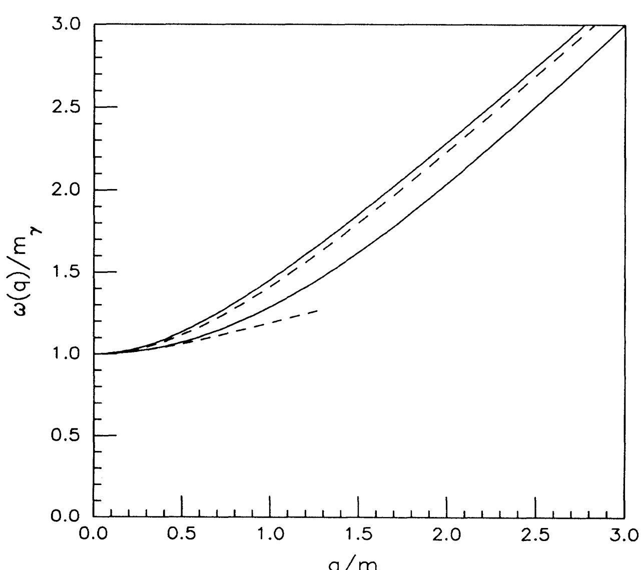

FIG. 1. Ultrarelativistic dispersion relations for transverse photons (upper solid line) and plasmons (lower solid line), compared to the approximate dispersion relations of Ref. 2 (dashed lines).

validity. In their pioneering work, Adams, Ruderman, and Woo[1] used the dispersion relations

$$\omega_t(q)^2 \simeq m_\gamma^2 + q^2, \quad 0 \le q < \infty, \tag{4}$$

$$\omega_l(q)^2 \simeq m_\gamma^2, \quad 0 \le q < m_\gamma. \tag{5}$$

These are accurate only at nonrelativistic temperatures and densities. Beaudet, Petrosian, and Salpeter[2] (hereafter referred to as BPS) included a relativistic correction to the plasmon dispersion relation. In the ultrarelativistic limit, it reduces to

$$\omega_l(q)^2 \simeq m_\gamma^2 + \frac{3m_\gamma^2}{5\omega_l(q)^2} q^2, \quad 0 \le q < (\tfrac{8}{5})^{1/2} m_\gamma. \tag{6}$$

They also wrote down the corresponding correction to the transverse dispersion relation, but did not include it in their calculations. All subsequent calculations have followed BPS in using the approximations (4) for the transverse dispersion relation and (6) for the longitudinal dispersion relation. These dispersion relations are shown as dashed lines in Fig. 1.

We now compare the limiting behaviors of the BPS dispersion relations (4) and (6) and the exact ultrarelativistic dispersion relations (2) and (3). In the small-momentum limit $q \ll m_\gamma$, the exact dispersion relations reduce to $\omega_t^2 = m_\gamma^2 + 6q^2/5$ and $\omega_l^2 = m_\gamma^2 + 3q^2/5$. For plasmons, the BPS dispersion relation (6) gives the correct coefficient for q^2, but not for the q^4/m_γ^2 term. For transverse photons, the BPS dispersion relation (4) gives the wrong coefficient even for q^2. In the large-momentum limit $q \gg m_\gamma$, the exact dispersion relation (2) for transverse photons reduces to $\omega_t^2 = q^2 + 3m_\gamma^2/2$, indicating that high-momentum photons propagate with effective mass $(\frac{3}{2})^{1/2} m_\gamma$. With the BPS dispersion relation (4), the effective mass for high-momentum photons is equal to the rest mass m_γ. The BPS dispersion relation (6) for plasmons involves an even more brutal approximation at large q. It requires the imposition of an artificial upper limit $q < (\frac{8}{5})^{1/2} m_\gamma$ on the momentum of the plasmon in order to maintain the condition $\omega_l > q$. The behavior of the exact dispersion relation (3) at large q is such that $\omega_l^2 - q^2 = 4q^2 \exp(-2q^2/3m_\gamma^2 - 2)$. In the calculation of the emissivity from plasmon decay, a factor of $(\omega_l^2 - q^2)^3$ automatically provides a Gaussian cutoff of order m_γ on the integral over the plasmon momentum q.

The general formulas for the emissivities Q_t and Q_l due to the decays of transverse photons and plasmons, respectively, are

$$Q_t = 2C_V^2 \frac{G_F^2}{24\pi^3 e^2} \int_0^\infty dq\, q^2 Z_t(q) \times [\omega_t(q)^2 - q^2]^3 n_B(\omega_t(q)), \tag{7}$$

$$Q_l = C_V^2 \frac{G_F^2}{24\pi^3 e^2} \int_0^\infty dq\, q^2 Z_l(q) \times [\omega_l(q)^2 - q^2]^3 n_B(\omega_l(q)), \tag{8}$$

where G_F is the Fermi constant and $n_B(\omega) = 1/(e^{\omega/T} - 1)$ is the Bose distribution. The factor C_V^2, summed over the electron, muon, and tau neutrinos, is $\frac{3}{4} - 2\sin^2\theta_W + 12\sin^4\theta_W \simeq 0.911$, where θ_W is the weak mixing angle. We have ignored the axial-vector contribution to the emissivities, since it is always negligible.[6] The residue functions $Z_t(q)$ and $Z_l(q)$ are related to the standard transverse and longitudinal dielectric functions by $Z_t^{-1} = \partial(\omega^2 \epsilon_t)/\partial\omega^2$ and $Z_l^{-1} = (\omega_l^2 - q^2)\partial\epsilon_l/\partial\omega^2$. For an ultrarelativistic plasma, they can be expressed in terms of the dispersion relations[7] defined by (2) and (3) as follows:

$$Z_t(q) = \frac{2\omega_t(q)^2[\omega_t(q)^2 - q^2]}{3m_\gamma^2 \omega_t(q)^2 - [\omega_t(q)^2 - q^2]^2}, \tag{9}$$

$$Z_l(q) = \frac{2\omega_l(q)^2}{3m_\gamma^2 - [\omega_l(q)^2 - q^2]}. \tag{10}$$

For the BPS dispersion relations (4) and (6), the corresponding expressions are $Z_t = 1$ and $Z_l = \omega_l^4/[(\omega_l^2 - q^2) \times (2\omega_l^2 - m_\gamma^2)]$. The emission rate R_t for neutrino pairs from the decay of transverse photons is given by the same integral as (7) except that the integrand is multiplied by $1/\omega_t(q)$. Similarly, the rate R_l from plasmon decay is given by (8) with the integrand multiplied by $1/\omega_l(q)$.

We now compare the emissivities calculated using the exact ultrarelativistic dispersion relations (2) and (3) with the BPS approximations. The integrals in (7) and (8) involve the two energy scales m_γ and T, and analytic expressions can only be obtained in the limiting cases $m_\gamma \ll T$ and $m_\gamma \gg T$. The limit $m_\gamma \ll T$ requires ultrarelativistic temperature and a density that may be high or low, so long as $e\mu/\sqrt{3}\pi \ll T$. In this limit, the integral for the transverse emissivity (7) is dominated by q on the order of T. We can therefore set $\omega_t^2 - q^2 \to 3m_\gamma^2/2$ and $\omega_t \to q$ everywhere else in the integrand. The resulting integral can be evaluated analytically:

$$Q_t = C_V^2 \frac{G_F^2}{24\pi^3 e^2} \frac{27\zeta(3)}{2} m_\gamma^6 T^3, \tag{11}$$

where $\zeta(z)$ is the Riemann zeta function. The emission rate in the limit $m_\gamma \ll T$ is given by $R_t = [\zeta(2)/2\zeta(3)] \times Q_t/T$. Using the BPS dispersion relation (4), the only difference would be that $\omega_t^2 - q^2 \to m_\gamma^2$. The correct result (11) for Q_t is larger than the BPS approximation by a factor of $(\frac{3}{2})^3 = 3.375$. This large difference comes about because Q_t is proportional to the sixth power of the effective mass for high-momentum photons, which is larger than the rest mass by a factor of $(\frac{3}{2})^{1/2}$. Similarly, the correct result for the rate R_t is larger than the BPS approximation by a factor of $(\frac{3}{2})^2 = 2.25$. Actually, according to (1), the closest one can get to the limit $m_\gamma \ll T$ is $m_\gamma/T = e/3 = 0.1009$. Evaluating the emissivities numerically, one finds that the enhancement factor 3.375 is reduced slightly to 3.185.

For the longitudinal emissivity (8) in the limit $m_\gamma \ll T$, the factor $(\omega_l^2 - q^2)^3$ provides a Gaussian cutoff so that the integral is dominated by q on the order of m_γ. The Bose distribution can be approximated by $n_B(\omega_l) \to T/\omega_l$. The integral then involves only the scale m_γ and can be evaluated numerically:

$$Q_l = C_V^2 (G_F^2/24\pi^3 e^2)(0.34909) m_\gamma^8 T. \qquad (12)$$

The corresponding emission rate is $R_l = 0.76926 Q_l/m_\gamma$. With the BPS dispersion relation (6), an artificial upper limit $q < (\frac{8}{5})^{1/2} m_\gamma$ must be imposed on the integral in (8). Setting $n_B(\omega_l) \to T/\omega_l$, the integral reduces to $0.16793 m_\gamma^8 T$. [In the appendix of Ref. 2, the result for this integral is quoted as $\frac{8}{105} m_\gamma^8 T$, but this actually corresponds to the dispersion relation (5).] Thus the correct result for Q_l in (12) is larger than the BPS approximation by a factor of $0.349/0.168 = 2.08$. Note, however, that in the limit $m_\gamma \ll T$, Q_l and R_l are negligible compared to Q_t and R_t, since they are suppressed by powers of m_γ/T.

The limit $T \ll m_\gamma$ corresponds to ultrarelativistic density and temperature low enough that $T \ll e\mu/\sqrt{3}\pi$. In this limit, the integrals in both (7) and (8) are dominated by momenta small compared to m_γ. In (7), we can set $q \to 0$ wherever possible, except that the Bose distribution is approximated by $n_B(\omega_t) \to \exp[-(m_\gamma + 3q^2/5m_\gamma)/T]$. The resulting Gaussian integral can be evaluated analytically:

$$Q_t = C_V^2 \frac{G_F^2}{24\pi^3 e^2} \left(\frac{125\pi}{108} \right)^{1/2} m_\gamma^{15/2} T^{3/2} e^{-m_\gamma/T}. \qquad (13)$$

The emission rate in this limit is $R_t = Q_t/m_\gamma$. With the BPS dispersion relation (4), we would instead set $n_B(\omega_t) \to \exp[-(m_\gamma + q^2/2m_\gamma)/T]$ in (7) and the integral would be proportional to $(2m_\gamma T)^{3/2}$ instead of $(5m_\gamma T/3)^{3/2}$. Thus the correct result (13) for Q_t differs from the BPS approximation by a factor of $(\frac{5}{6})^{3/2} = 0.761$. The longitudinal emissivity is calculated in the same way except that the Bose distribution in (8) is replaced by $n_B(\omega_t) \to \exp[-(m_\gamma + 3q^2/10m_\gamma)/T]$:

$$Q_l = C_V^2 \frac{G_F^2}{24\pi^3 e^2} \left(\frac{125\pi}{54} \right)^{1/2} m_\gamma^{15/2} T^{3/2} e^{-m_\gamma/T}. \qquad (14)$$

The corresponding emission rate is $R_l = Q_l/m_\gamma$. The BPS dispersion relation (6) gives the same result, because it gives the correct coefficient for q^2 in the expansion of $\omega_l(q)$ for small q. Note that in the limit $T \ll m_\gamma$, the emissivity due to plasmon decay is larger than the transverse emissivity: $Q_l = \sqrt{2} Q_t$. The combined emissivity $Q_t + Q_l$ differs from the BPS approximation only by a factor of 0.885.

In recent numerical calculations of these emissivities,[4] the authors have considered temperatures as high as $T = 10^{11}$ K $= 8.6$ MeV, and mass densities ranging up to $\rho/\mu_e = 10^{14}$ g/cm^3, which corresponds to an electron chemical potential of $\mu = 240$ MeV. At this temperature, the photon mass ranges from $m_\gamma = 0.87$ MeV at low densities to $m_\gamma = 13$ MeV at the highest densities considered. The emissivity from plasmon and transverse photon decays have been underestimated at this temperature by a factor that ranges from 3.185 at low densities to 1.64 at $\rho/\mu_e = 10^{14}$ g/cm^3.

The use of the correct dispersion relations may have a significant effect on the cooling rate of the core of a neutron star immediately after it is created in a supernova explosion. The core of the supernova reaches ultrarelativistic temperatures (with T on the order of 60 MeV) and ultrarelativistic electron densities (with Fermi energy μ on the order of 350 MeV). The plasma frequency or photon mass m_γ is on the order of 20 MeV. Under these conditions, the decay of transverse photons is the dominant mechanism for the emission of neutrino pairs,[4] and previous calculations have underestimated the emissivity due to this mechanism by a factor of about 2.70. As the neutron star cools, its core remains at an ultrarelativistic density while the temperature decreases, eventually approaching the limit $T \ll m_\gamma$. When the temperature has decreased by an order of magnitude, the enhancement due to correctly treating the ultrarelativistic plasma effects has decreased to 1.24. Thus the most dramatic effect will be seen at the beginning of the cooling of the neutron star.

In Ref. 2, it was argued that their calculation of the emissivities from photon and plasmon decay would break down at temperatures large enough that $m_\gamma > 2m_e$, since the decay $\gamma \to e^+e^-$ is then kinematically allowed. This statement, which has been repeated in subsequent papers,[3,4] is simply untrue. The plasma effects which generate the photon mass m_γ also generate corrections to the electron mass such that the decay $\gamma \to e^+e^-$ is always kinematically forbidden. For example, at ultrarelativistic temperatures $T \gg \mu$, the effective electron mass[8] is $m_e = 3m_\gamma/\sqrt{8}$, while at ultrarelativistic densities $\mu \gg T$, it is $m_e = \sqrt{3} m_\gamma/2$. In either case, $m_\gamma < 2m_e$ and the decay $\gamma \to e^+e^-$ is forbidden.

We have shown that the approximations to the dispersion relations for photons and plasmons used in Ref. 2 and subsequent papers can lead to significant errors at ultrarelativistic temperatures or densities. We have calculated the emission rates and emissivities due to photon and plasmon decay into neutrino pairs using the correct ultrarelativistic dispersion relations. The combined emissivity $Q_t + Q_l$ changes by a factor that ranges from 3.185 in the limit of high temperature to 0.885 in the limit of low temperature and high electron density. This may have a significant effect on the production rate of neutrinos inside a supernova and on the initial cooling rate of a neutron star. In light of this result, it would be worthwhile examining other plasma effects in supernovas and neutron stars to see if a correct ultrarelativistic treatment gives quantitative differences from previous calculations.

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