Observation of Quantum Interference in Thermoelectric Transport

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We present evidence for the first observation of localization corrections to the thermoelectric coefficient η relating current flow to applied temperature gradient, using the two-dimensional electron gas of a silicon-on-sapphire inversion layer as a test system. We observe an increase in the magnitude of η in a weak, perpendicular magnetic field, which we explain in terms of the suppression of quantum interference effects by the field. Good agreement is found between theory and experiment.

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Within the framework of linear-response theory, the electric current density $\mathbf{j}$ which flows under an applied electric field $\mathbf{E}$ and temperature gradient ∇T may be written as

$$\mathbf{j}=\sigma(\mathbf{E}+|e|^{-1}\nabla\mu)-\eta\nabla T\,, \tag{1}$$

where μ is the chemical potential and $|e|$ the magnitude of the electronic charge. The thermoelectric coefficient η is a fundamental transport parameter like the electrical conductivity σ, but it is seldom discussed directly as usually it is the thermopower $S=\eta/\sigma$ which is measured. It is well known[1] that η (and hence S) provides additional information about a system, beyond that obtained from σ, since η depends upon the derivatives of the scattering lines with respect to energy, rather than just the scattering times themselves. For example, a positive thermopower, recently observed[2] in an n-channel Si metal-oxide-semiconductor field-effect transistor (MOSFET) at low temperatures, has been interpreted in terms of surface roughness scattering, with a scattering time that *decreases* as the energy increases. Measurements of thermoelectric voltages in microstructures have also recently been the focus of interest.[3,4] The determination of absolute thermopower values is, however, rather difficult, and involves quite refined experimental techniques.[5-7] Moreover, comparison with theory is also difficult, due to the sensitivity of η to microscopic details of the system. This is unfortunate since a number of interesting theoretical issues concerning η (and therefore the thermopower) remain to be fully resolved, especially in low-dimensional structures. One such issue is the presence and effectiveness of "phonon drag," by means of which a nonequilibrium phonon distribution (due to the temperature gradient) leads to an excess current.[8] Another is the precise role played by phonon renormalization.[9] A third area, with which we shall concern ourselves here, is the exact form of the localization corrections to η as a function of, say, applied magnetic field $\mathbf{B}$. We shall present what we believe is the first *direct* evidence for the existence of such corrections.

As a starting point for the discussion, we shall explicitly write η as the sum of two parts: $\eta=\eta_d+\eta_g$, where η_d is the diffusive component and η_g is the phonon-drag component. The precise form of the localization correction to η at low temperatures has been the subject of some controversy. In an early paper[10] it was suggested that the correction to η_d was such that the diffusive thermopower remained unaffected by quantum interference, i.e., $\delta S_d/S_d\equiv\delta\eta_d/\eta_d-\delta\sigma/\sigma=0$. This was later corrected by a number of authors,[11-13] who showed that the corrections $\delta\eta_d$ to η_d are in fact quite different in form from the corrections $\delta\sigma$ to the conductivity σ and, as a result, that there are corrections to the diffusive thermopower. One may show by diagrammatic perturbation theory that provided the temperature is sufficiently low,

$$\delta\eta_d(B,T,\varepsilon_F)=-\frac{\pi^2k_B^2T}{3|e|}(1+\lambda)\left[\frac{d}{d\varepsilon}F(B,T,\varepsilon)\right]_{\varepsilon=\varepsilon_F}, \tag{2}$$

where k_B is Boltzmann's constant. At this level of calculation, the electron-phonon interaction leads to a phonon renormalization factor[9] written as $1+\lambda$, and the function $F(B,T,\varepsilon)$ (when evaluated at the Fermi energy $\varepsilon=\varepsilon_F$) is simply the weak-localization correction to the conductivity, $\delta\sigma$. There are, in fact, powerful arguments, based upon an exact theorem originally due to Chester and Thellung, that Eq. (2) may be valid to all orders in the electron-impurity interaction,[12,13] provided that the scattering remains predominantly elastic and that the contributions from electron-electron interactions remain small. The explicit form of this correction to η_d has not been directly verified before. Localization corrections to the *thermopower*, which have recently been observed,[14] have only been interpreted in terms of changes in the conductivity alone, i.e., $\delta S/S\approx-\delta\sigma/\sigma$. This was based upon the fact that $\delta\eta/\eta$ is small, since in most two-dimensional systems of interest, $\eta_g\gg\eta_d$ (i.e., phonon drag is dominant).[5-8] The aim of the present work is to verify the existence of these small (around 1%) corrections to η as indicated by Eq. (2). (The possible role which phonon drag has to play in influencing weak localization is discussed later.)

The silicon-on-sapphire (SOS) MOSFET was the system chosen for these measurements because the large degree of disorder present in this system enhances the effect which we are trying to observe. The measurements were performed on n-channel MOSFET's fabricated on (100) Si on ($\bar{1}012$) sapphire. The mean Si thickness was 0.3 μm and the MOSFET's were of a standard Hall bar geometry, measuring 2000 μm by 300 μm, with an oxide thickness of 2000 Å. This allowed the carrier concentration n_c of the two-dimensional electron gas formed at the Si-SiO_2 interface to be continuously varied from less than 2×10^{11} cm^{-2} up to about 8×10^{12} cm^{-2} before the oxide started to break down.

To mount the sample, the ends of it were wrapped in thin indium foil (to reduce the thermal resistance), and it was clamped between two copper blocks, one of which was in good thermal contact with the ^{4}He in an 8-T cryostat. Resistance and conductance were measured using standard lock-in techniques. To measure the thermopower, a temperature gradient was established across the sample by passing a current through a heater resistor either on one of the copper blocks or on the chip itself. The relative thermopower was measured in both perpendicular and parallel magnetic fields ($B_\perp$ and $B_\parallel$), by applying a low-frequency (2–3 Hz) alternating current to the heater resistor and detecting the thermoelectric voltage using a lock-in technique. This ensures that there is no voltage due to Faraday induction. The accuracy with which the thermopower can be determined depends on the precision with which the temperature difference ΔT along the sample can be measured. Therefore ΔT was found by using the conductance of the device itself as a local probe of the temperature.[6] This involves measuring the conductance across the device at one end point and a *direct* measurement of the difference between the conductances across the device at the two end points.

Measurements of the zero-field thermopower $S(B=0,T)$ were made over the temperature range 1.2–4.7 K. The temperature dependence observed (close to T^3 rather than T)[15] is consistent with the presence of a large phonon-drag component, in agreement with other published work.[2,5–8] The zero-field conductivity $\sigma(B=0,T)$ exhibited the logarithmic temperature dependence characteristic of quantum interference in a two-dimensional system. As a function of electron concentration n_c, both σ and S showed some structure near $n_c\approx4\times10^{12}$ cm^{-2}. A study of the subband structure in the presence of stress arising from the lattice mismatch at the Si-sapphire interface suggests that this value of n_c is consistent with the second subband becoming occupied.[16] The lowest subband has been shown[17] to have a density-of-states effective mass $m^*=0.42m_0$ and a valley degeneracy $g_v=2$.

With only one subband occupied we expect the quantum interference correction to be represented by the conventional expression,[18]

$$\delta\sigma(B_\perp,T)=\frac{g_v\alpha e^2}{2\pi^2\hbar}\left[\Psi\left(\frac{1}{2}+\frac{\hbar}{4eB_\perp L_\phi^2}\right)-\Psi\left(\frac{1}{2}+\frac{\hbar}{4eB_\perp l^2}\right)\right], \quad (3)$$

where L_ϕ is the phase relaxation length, l is an elastic-scattering length, and $\Psi(x)$ is the digamma function. The phenomenological factor $g_v\alpha$ is still not well understood but is expected to vary between g_v and unity as the intervalley scattering rate $1/\tau_v$ increases and becomes greater than the phase relaxation rate.[19] Experimental results for $\Delta\sigma(B_\perp)\equiv\sigma(B_\perp)-\sigma(0)$ for two different electron concentrations are shown in Fig. 1, along with the best-fit solutions resulting from fitting Eq. (3) to the data. The agreement is excellent. Note, however, that the parameter $g_v\alpha$ varies appreciably with n_c (suggesting a variation of τ_v with n_c). It is also less than unity (the theoretically predicted minimum value[19]) but values of $g_v\alpha<1$ have been seen before in the SOS system;[14] this may be due to the breakdown of the condition under which Eq. (3) is strictly valid, namely, $k_Fl\gg1$. There may also be a small correction due to orbital interaction processes,[19] although calculations[14] suggest that this should be small for the electron concentrations used.

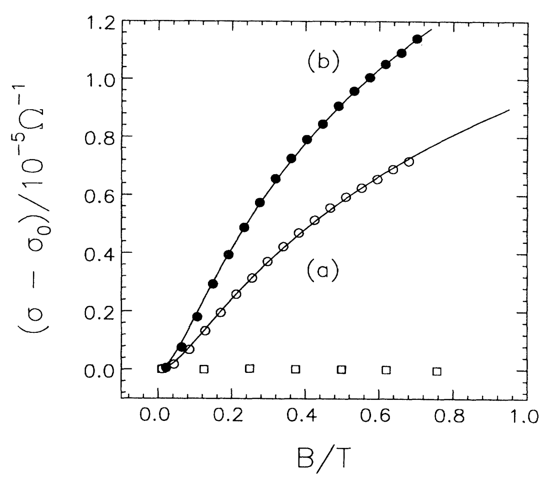

FIG. 1. The conductivity correction plotted against applied magnetic field for an SOS MOSFET. The circles are experimental data for $B_\perp$, a temperature of 1.85 K, and electron concentrations of, curve a, 2.10×10^{12} cm^{-2} ($k_Fl=2.1$) and, curve b, 3.18×10^{12} cm^{-2} ($k_Fl=3.3$). The curves are least-squares fits by Eq. (3) in the text, with the following values for the parameters in each case: curve a, $L_\phi=61.5$ nm, $l=10.1$ nm, and $g_v\alpha=0.55$; curve b, $L_\phi=69.0$ nm, $l=14.3$ nm, and $g_v\alpha=0.74$. The squares are experimental data for $B_\parallel$, a temperature of 1.40 K, and an electron concentration of 2.10×10^{12} cm^{-2}.

Note also from Fig. 1 that $\sigma(B_\parallel)-\sigma(0)$ is of order zero at low fields, reflecting the 2D nature of the system.

Having determined the absolute change with magnetic field of the thermopower and conductivity, we can extract $\Delta\eta(B)\equiv\eta(B)-\eta(0)$ for both $B_\perp$ and $B_\parallel$ and the results are shown in Fig. 2. Theoretically, $\Delta\eta(B_\parallel)=0$ for low fields, which follows from Eq. (2) and the observation that $\sigma(B_\parallel)$ is field independent. The residual variation $\Delta\eta(B_\parallel)$ gives us an estimate of the relative change of ΔT with B, which, even allowing for the lower temperature of the $B_\parallel$ data, turns out to be less than $0.1\%\,\mathrm{T}^{-1}$. This change of ΔT is mainly due to small magnetothermal resistances in various parts of the cryostat which depend only on the magnitude and not the direction of **B**. Our error bars for the $B_\perp$ data allow for this nonzero $\partial(\Delta T)/\partial B$ and there is still clear evidence for a localization correction to η, a view which is reinforced by fitting Eq. (2) to the data. From Eqs. (2) and (3) we see that $\delta\eta$ depends upon the parameters $g_v\alpha$, L_ϕ, and l and their derivatives with respect to energy. The parameters themselves were taken from the fitting to the conductivity data shown. The derivatives were estimated by observing the variation of each parameter with n_c (using extra conductivity data beyond that shown in Fig. 1) and using the relationship $n_c=g_v m^*\varepsilon_F/\pi\hbar^2$ valid for a single parabolic 2D subband.[20] With no evidence to the contrary, we have set λ equal to zero. We know of no detailed theoretical calculations of phonon renormalization effects in two-dimensional electron systems.

For the lower electron concentration, the agreement between theory and experiment is excellent. We expect the agreement to be good for this case because (i) only one subband is occupied, (ii) interaction effects are estimated to be small, and (iii) the magnitude of $\delta\eta$ is large (which reduces the relative experimental uncertainty). For the higher-electron-concentration data, the agreement is less good but still qualitatively reasonable. We show this curve in particular because its shape is interesting, passing as it does through a maximum at about 0.4 T before decreasing in value again. We suspect that this behavior may be linked to the proximity of the second subband (referred to earlier). An accurate fit to this curve using Eq. (2) can, however, only be achieved by using an anomalously large value of $dl/d\varepsilon$. This is another indication that trying to use Eq. (3) for $\delta\sigma$ is not fully valid for the low values of $k_F l$ studied here.

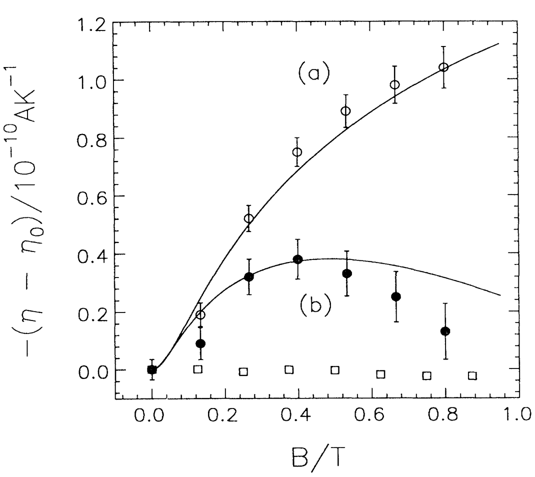

FIG. 2. The correction to the thermoelectric coefficient η plotted against applied magnetic field with the same symbols and parameters as in Fig. 1. The curves are obtained by using the values of L_ϕ, l, and $g_v\alpha$ found by fitting the conductivity correction, and the following values of their derivatives which were estimated from the conductivity data: curve a, $dL_\phi/d\varepsilon=3.8$ nm meV^{-1}, $dl/d\varepsilon=2.0$ nm meV^{-1}, and $d(g_v\alpha)/d\varepsilon=0.11$ meV^{-1}; curve b, $dL_\phi/d\varepsilon=3.4$ nm meV^{-1}, $dl/d\varepsilon=8.6$ nm meV^{-1}, and $d(g_v\alpha)/d\varepsilon=0.04$ meV^{-1}.

In deriving the correction to η we have only allowed for an equilibrium phonon distribution: It is natural to ask how the results will be affected if phonon drag is present. This is an interesting theoretical issue and by no means fully resolved. However, the phenomenon of quantum interference has an underlying universal nature (characterized, for example, in the presence of time-reversal symmetry, by a diffusion pole). Therefore, although phonon drag may alter details of the phase relaxation and phonon renormalization, it should not fundamentally alter the *form* of the localization correction. This is consistent with our experimental results.

We have presented the first experimental evidence for the presence of localization corrections to the thermoelectric coefficient η of a two-dimensional electron system, and shown its magnitude and dependence on magnetic field to be consistent with theoretical predictions.

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