Propagating Fronts near a Lifshitz Point

Dee and Saarloos[1] found in an extended Fisher-Kolmogorov (EFK) equation the transition from a uniformly translating front to a pattern-generating envelope front. A new $\epsilon^{3/4}$ scaling of the front velocity is described and electrohydrodynamic convection (EHC) in planar nematic liquid crystals is proposed as an appropriate system for investigating some aspect of this front type.

In EHC one expects for some nematic materials as a function of the frequency ω of the applied voltage a continuous transition form oblique to normal-oriented convection rolls.[2-6] Near the transition frequency ω_z (Lifshitz point) the weakly nonlinear behavior is described by the amplitude equation[3-5]

$$\tau_0\partial_t A=[\epsilon+\xi_1^2\partial_x^2-iZ\xi_1\xi_3^2\partial_x\partial_y^2+W\xi_3^2\partial_y^2-\xi_3^4\partial_y^4-|A|^2]A\,; \qquad (1)$$

$\epsilon=(V^2-V_c^2)/V_c^2$ measures the distance from the normal-roll threshold V_c. Numerical values for the relaxation time τ_0 and the coherence lengths ξ_i and Z are given elsewhere.[3,5,6] W is proportional to $\omega-\omega_z$. For periodic solutions $A=E\exp[i(Qx+Py)]$, with

$$E^2=\epsilon-\xi_1^2Q^2-(Z\xi_1Q+W)\xi_3^2P^2-\xi_3^4P^4\,,$$

the absolute threshold for oblique rolls is $\epsilon_c=-W^2/(4-Z^2)$ at $Q_c=ZW/(4-Z^2)\xi_1$ and $P_c^2=-2W/(4-Z^2)\xi_3^2$ and $\epsilon_c=0$ at $Q_c=P_c=0$ for normal rolls. The stability of these solutions as well as zigzag and undulated solutions have been investigated.[4,5] Equation (1) is more general then the EFK equation in Ref. 1. It is an equation for a complex field A instead of a real field and includes two space dimensions.

For real y-dependent functions Eq. (1) reduces to the EFK equation in Ref. 1, however, the scaling used there is restricted to $W>0$ and $\epsilon=1$. Using the ansatz $A=\exp[i(Qx+\phi)]B(y)$, with B a real function, then the marginal-stability analysis, described, for example, in Refs. 1 and 7, gives the velocity for fronts propagating in the y direction (parallel to the convection rolls). For $U>\sqrt{12\epsilon}$ and $U=W+QZ$ the uniformly translating fronts move with the velocity

$$v_1=\frac{2}{\sqrt{54}}[36\epsilon U+U^3-(U^2-12\epsilon)^{3/2}]^{1/2}\frac{\xi_3}{\tau_0} \qquad (2)$$

and for $U<\sqrt{12\epsilon}$ the envelope fronts move with velocity

$$v_2=\frac{2}{\sqrt{54}}\frac{24\epsilon+2U^2-FU}{\sqrt{4U+F}}\frac{\xi_3}{\tau_0}\,, \qquad (3)$$

with $F=(7U^2+24\epsilon)^{1/2}-3U$. The wavelength of the periodic patterns behind the front for $U<\sqrt{12\epsilon}$ is, according to the arguments used in Refs. 1 and 7,

$$\lambda=\frac{8\pi}{3}\left(\frac{2}{3}\right)^{1/2}\frac{24\epsilon+2U^2-FU}{12\epsilon-U^2-FU}\xi_3\,.$$

This wavelength λ diverges for $U\to\sqrt{12\epsilon}$. For $W<0$ and $Q=Q_c$ at $\epsilon\gtrsim\epsilon_c$, however, $P=2\pi/\lambda$ reaches the critical wave number P_c of the oblique periodic solution. One could call $U=\sqrt{12\epsilon}$ the Lifshitz point for the transient periodic pattern the behind fronts, similar to $W=0$ for the stationary periodic pattern.

For $U=0$ the first nonvanishing derivative with respect to y is of fourth order in Eq. (1) and then $v_2=\frac{8}{3}\times(\frac{3}{2})^{3/4}\epsilon^{3/4}\xi_3/\tau_0$. Very remarkable is the $\epsilon^{3/4}$ scaling, which is different to the well-known front velocity $v_x=2\xi_1\sqrt{\epsilon}/\tau_0$ perpendicular to the convection rolls and to $v_2\sim(\epsilon-\epsilon_c)^{1/2}$ for $U\gg0$ and $U\ll0$ parallel to the rolls. The measurement of this new $\epsilon^{3/4}$ scaling seems more easily feasible than the transient periodicity behind the front. A possibility to detect this ϵ law is to measure the growth velocities $(v_x,v_y=v_2)$ of nuclei of convection rolls, as done in Ref. 8 apart from the Lifshitz point. Another way is to investigate fronts in samples, where the electrodes are narrow in the x direction but extended in the y direction.

In addition, the measurement of the ϵ dependence of the front velocity seems sometimes an appropriate tool for the determination of the transition (bifurcation) type. This was, e.g., tried in Ref. 9 where we found experimentally for the front velocity roughly $v\sim\epsilon^{0.7}$, however, this has perhaps a different reason than discussed here.

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