Collisionless Reconnection and the Sawtooth Crash

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A model of the sawtooth crash is presented in which electron inertia combined with anomalous diffusion of the current replace classical resistivity in allowing magnetic reconnection to occur.

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Experimental measurements of the sawtooth crash time τ_c in the TFTR and JET tokamaks range from 20 to 2000 μsec.[1,2] The crash time, based on a Sweet-Parker model of reconnection by the $m=1$ tearing mode, is of order 2000 μsec, which is short enough to explain the slow crashes, but not the fast crashes.[3]

In this paper we show that the electric field E induced during magnetic reconnection during the sawtooth crash greatly exceeds the Dreicer field E_D, and, therefore, classical resistivity cannot play a role in this process. Including electron inertia in Ohm's law allows reconnection to proceed. We present numerical simulations which demonstrate that the current layer during collisionless reconnection using this new Ohm's law collapses to a thickness which is much smaller than the electron collisionless skin depth $\delta = c/\omega_{pe}$, contrary to conventional scientific wisdom.[4] This narrow current layer throttles the nonlinear reconnection rate and, as a consequence, inertial reconnection cannot, by itself, cause the fast sawtooth crash. The extremely narrow current layers induced during collisionless reconnection are strongly unstable to the collisionless current convective instability.[5] We estimate the anomalous current diffusion rate due to this instability to be

$$D_\perp = \rho_e \delta \nabla_\perp v_{\parallel e} . \tag{1}$$

Electron inertia combined with anomalous current diffusion given by the expression (1) produce sawtooth crash times which are in reasonable agreement with observations.

Under the assumption that the sawtooth crash is caused by Kadomtsev-like magnetic reconnection,[6] the parallel inductive electric field can be readily estimated. The helical magnetic flux which reconnects during the crash is of order $\psi^* = \Delta q r_1^2 B_\phi / R$, where B_ϕ and B_θ are the toroidal and poloidal magnetic fields, R is the major radius, r_1 is the radius of the $q=1$ surface, $\Delta q = 1 - q(0)$, and $q(r) = rB_\phi / RB_\theta$. The induction field $E \sim \psi^*/c\tau_c$ so that $E \sim \Delta q r_1^2 B_\phi / c\tau_c R$. The Dreicer electric field is given by $E_D = m_e v_{te} \nu_{ei}/e$, with v_{te} the electron thermal velocity and ν_{ei} the electron-ion collision rate. Thus,

$$E/E_D = \Delta q r_1^2 / \tau_c R \nu_{ei} \rho_e , \tag{2}$$

with ρ_e the electron Larmor radius. For parameters characteristic of TFTR ($B=4$ T, $T=7$ keV, $n=5\times10^{19}/\text{m}^3$, $r_1=0.2$ m, $R=2.6$ m, $\tau_c=35$ μsec),[1] $E/E_D = 3.5\times10^3 \Delta q$. Thus, $E > E_D$ even for small values of Δq ($\sim 10^{-2}$). For $E > E_D$, classical collisions are not effective in limiting the electron response to the parallel electric field. Classical parallel resistivity can therefore not be the dissipation mechanism for magnetic energy during the sawtooth crash.[7]

Ohm's law in resistive MHD, $E_\parallel = \eta J$, is simply the parallel component of the electron momentum equation. When classical collisions are absent or weak, a generalized Ohm's law can be obtained by simply including electron inertia,[4,8]

$$E_\parallel = \eta J + (4\pi/\omega_{pe}^2) dJ/dt . \tag{3}$$

As $\eta \to 0$ electron inertia prevents J from becoming singular. In 2D MHD the generalized Ohm's law becomes an evolution equation for the flux function ψ,

$$\frac{d}{dt}(\psi - \delta^2 \nabla_\perp^2 \psi) = \eta \nabla_\perp^2 \psi . \tag{4}$$

Equation (4) has been written in dimensionless units: $a\nabla_\perp \to \nabla_\perp$, $t/\tau_A \to t$, $\psi/Ba \to \psi$, $\delta/a \to \delta$, and $\eta c^2 \tau_A / 4\pi a^2 \equiv \tau_A/\tau_r \to \eta$.

The electron inertia introduces a new scale length, the skin depth δ, into the equation. It has been assumed in the past that the scale size of the "dissipation" region during collisionless reconnection is δ.[4] In the linear regime δ is the scale size,[8] although the finite ion Larmor radius may broaden the layer somewhat.[9] To explore the nonlinear behavior, we have written a 2D code which advances the isothermal compressible MHD equations with the generalized Ohm's law in (4). The equations are solved on a Cartesian grid using a fourth-order-accurate finite-difference scheme with grid scale hyperviscosity.[10] Time stepping is with a second-order-accurate leapfrog trapezoidal scheme.[11] The sawtooth crash time depends on the nonlinear structure of the dissipation region. To simplify the geometry, we have studied the structure of the dissipation region during the merging of two isolated circular flux bundles of radii $r_0 = 0.3$, peak magnetic field $B_\theta = 0.7$, and Alfvén time $\tau_A = r_0/B_\theta = 0.39$ as shown in Fig. 1. The system is taken to be symmetric about $x=0$, so the second flux bundle can be obtained by reflection. The sequence of flux contours are from a run with $\delta=0.1$ on a 1000×132 grid. Shown in Fig. 2 are cuts of $J(x)$ through the x line over the interval $0\le x\le 0.05$ for the times shown in Fig. 1. The entire region shown is 0.5δ in the x direction. A current layer is formed around the x line with an initial scale length along x of order 0.5δ. As reconnection proceeds the x scale length L of the layer shrinks. The collapse of the current layer continues until reconnection ends. At smaller values of δ, the current layer collapses until our grid can no longer resolve the structure. Physically, the electron fluid is carried into and then out of the acceleration region where $E_{\parallel}\neq 0$. Finite electron inertia combined with the finite transit time through the acceleration region limits the electron current. However, the x line is a stagnation point of the flow so that electrons close to the x line remain in the acceleration region for a long time; i.e., electrons very close to the x line are accelerated as long as reconnection continues. The result is a highly localized current layer.

The collapse of the current layer greatly reduces the rate of reconnection in the nonlinear regime since a simple Sweet-Parker scaling argument yields peak reconnection velocities $v_x \propto L$. Thus, contrary to previous speculations,[12] electron inertia by itself cannot produce fast sawtooth crashes consistent with observations.

That a localized current layer with a scale size L much smaller than δ could survive in a real 3D system seems implausible. We expect that this narrow layer will be strongly unstable well before it collapses to the scale length shown in Fig. 2 and that the resultant transport of electron parallel momentum will prevent the collapse of the current layer. We have investigated the stability of the electron current layer in the regime $L\sim\delta$. The dominant instability is the current convective instability with unmagnetized ions.[5] This instability is electrostatic for wave numbers $k>\delta^{-1}$ and electromagnetic in the opposite limit. The characteristic growth rate is $\gamma=(k_{\parallel}v_{\parallel e}\omega_{*v}-k_{\parallel}^2v_{es}^2)^{1/2}$, where $\omega_{*v}=k_ycT_i/eBL$ is the diamagnetic frequency based on the current gradient scale length $L=|v_{\parallel e}/v'_{\parallel e}|$, $v_{es}^2=(T_e+T_i)/m_e$, and $k_{\parallel}$ is the parallel wave vector. The growth rate peaks at $k_{\parallel 0}=v_{\parallel e}\omega_{*v}/v_{es}^2$ with $\gamma=(v_{\parallel e}/v_{es})\omega_{*v}/\sqrt{2}$. The electron current layer produces a local shear in the magnetic field which could impact the instability. This local shear is unimportant if $k_{\parallel 0}>\Delta k_{\parallel}\sim k\Delta B_y/B$, where $\Delta B_y\sim 4\pi nev_{\parallel e}L/c$ is the jump in the magnetic field across the layer. Inserting $k_{\parallel 0}$ into this inequality, we find that local magnetic shear can be neglected for $L<\delta$. For $L>\delta$ the shear localization distance Δ is given by $\Delta\sim\delta^2/L$.

The current convective instability has substantial similarity to the slab η_e instability.[13] In both instabilities the ions are unmagnetized and respond adiabatically to the perturbed electrostatic fields and the parallel compression of electrons plays a central role in driving the instability. The set of nonlinear equations describing both instabilities are identical. Previous fluid simulations of the η_e instability[14] provide a guide in estimating transport by the current convective instability. In the limit of weak shear, transport is dominated by the longest wavelengths in the system. For $L<\delta$, $k\sim L^{-1}$, and the radial step size $\Delta\sim L$. Thus, $D_{\perp}\sim\Delta^2\gamma\sim v_{\parallel e}\rho_e$. When the shear is strong, $\Delta\sim\delta^2/L$ and $D_{\perp}\sim\Delta^2\gamma\sim\rho_e\delta^2v_{\parallel e}/L^2$. As a simple model for local transport, we use the expression given in (1), which approximates the expression for $D_{\perp}$ in both limits if L is not too different from δ. Our evolu-

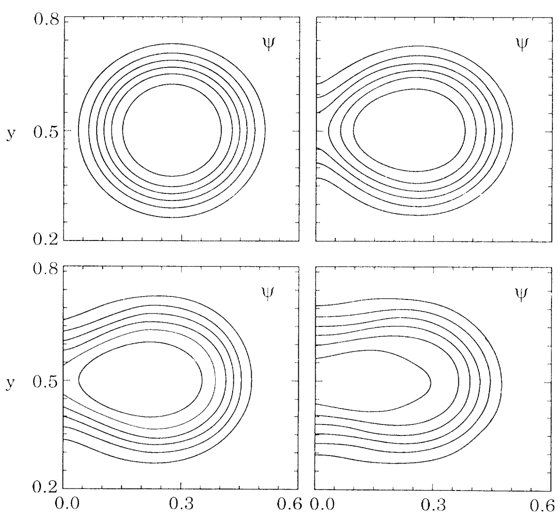

FIG. 1. Flux contours ψ in the (x,y) plane at times $t/\tau_A=0$, 3.7, 4.1, and 4.7 from a simulation of collisionless reconnection with $\delta/a=0.1$.

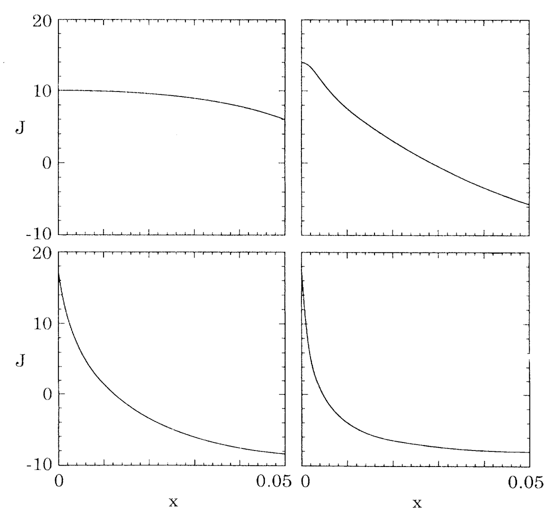

FIG. 2. Cuts of J along the midplane over the interval $0\le x\le 0.05$ at the times shown in Fig. 1.

tion equation for the magnetic flux now takes the form

$$\frac{d\psi}{dt} - \delta^2 \left(\frac{d}{dt} - \nabla_\perp \cdot \mathbf{D}_\perp \cdot \nabla_\perp \right) J = \eta J \,, \tag{5}$$

with

$$\mathbf{D}_\perp = \delta^3 \hat{\beta}^{1/2} (\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} |\partial J/\partial x| + \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} |\partial J/\partial y|)$$

and $\hat{\beta} = \beta m_i/m_e$.

A series of simulations have been completed with the flux evolution equation given in (5) for a range of δ $(2.5\times10^{-6} < \delta^2 < 10^{-3})$. In Fig. 3 we show contour plots of ψ, J, v_x, and v_y, $t=6.1$, from a run with $\delta = 10^{-2}$ and $\hat{\beta} = 18.36$ on a 1000×132 grid. At this point in time about half of the magnetic flux in Fig. 3(a) has reconnected and a narrow current layer has formed around the magnetic x line. A cut of J along the midplane is shown in Fig. 4. During the entire run the half-width L of the current layer remains nearly constant with $L \sim 2.5\delta$. The anomalous current diffusion prevents the spatial collapse of the current. At the time corresponding to Figs. 3 and 4, the integrated current across the layer has reached a maximum, i.e., the jump in the magnetic field across the layer is a maximum, $\Delta B_y \sim 0.7$, which is also the peak Alfvén velocity in our dimensionless variables. The contours of v_x in Fig. 3(c) illustrate that the magnetized flux bundle is moving toward the x line with a nearly uniform velocity, $v_x \simeq -0.20$. This plasma is ejected at high velocity along the y direction [Fig. 3(d)] with a maximum speed of $v_y \sim 0.71$, which is basically the Alfvén speed.

The structure of the reconnection region in Figs. 3 and 4 is consistent with a Sweet-Parker-like model.[15] By balancing the various terms in the Ohm's law in Eq. (5), we can derive the scaling of the width L of the current

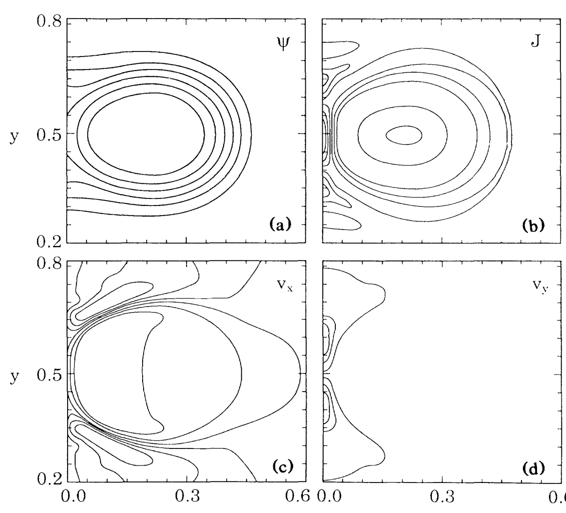

FIG. 3. Contour plots of ψ, J, v_x, and v_y at $t = 6.1\tau_A$ from a simulation with $\delta/a = 10^{-2}$ and $\hat{\beta} = 18.36$.

layer and the inflow velocity v_x with δ and $\hat{\beta}$. The convection of the magnetic flux into the dissipation region must balance the current diffusion,

$$v_x \frac{\partial \psi}{\partial x} \sim \frac{v_x \psi}{L} \sim \delta^2 \nabla_\perp \cdot \mathbf{D}_\perp \cdot \nabla_\perp J$$

$$\sim \delta^2 \frac{1}{L} \left(\delta^3 \hat{\beta}^{1/2} \frac{1}{L} J \right) \frac{1}{L} J$$

and since $J \sim \psi/L^2$ and $JL \sim \Delta B_y \sim 1$,

$$v_x \sim \delta^5 \hat{\beta}^{1/2}/L^5 \,.$$

The flow in and out of the dissipation region is nearly incompressible. Thus, $v_x \sim v_y L$, where the inflow is over the length of the dissipation region along y which is of order 1. Since $v_y \sim c_A \sim 1$, $v_x \sim L$ and

$$L \simeq 1.1 a^{1/6} \delta^{5/6} \hat{\beta}^{1/12} \tag{6}$$

with

$$v_x \simeq 4.8 c_A (\delta/a)^{5/6} \hat{\beta}^{1/12} \,, \tag{7}$$

where the coefficients 1.1 and 4.8 are determined from the simulations. It should be emphasized that L and v_x are very insensitive to the precise form of the anomalous diffusion rate (note the factor $\hat{\beta}^{1/12}$). Thus, any difference between the expression for $D_\perp$ in (1) and the actual expressions for $D_\perp$ in the limits $\delta \lessgtr L$ does not significantly affect v_x or L.

With the expression for v_x in (7) with a given by r_1, the sawtooth inversion radius, and c_A the helical Alfvén velocity, we can estimate the time required for a collisionless Kadomtsev-like reconnection during the sawtooth collapse. For TFTR (with numbers given previously) $\delta/r_1 \simeq 4\times10^{-3}$ and $\tau_c \sim r_1/v_x \simeq (4\ \mu\text{sec})/\Delta q$. For $\Delta q \sim 0.1$, $\tau_c \sim 40\ \mu$sec, consistent with the observations of fast sawtooth crash.

Although the motivation of the present calculation was the apparently collisionless sawtooth crashes on TFTR (Ref. 1) and JET,[2] the expressions for v_x and L also apply to more collisional machines with $E < E_D$ if L in (6)

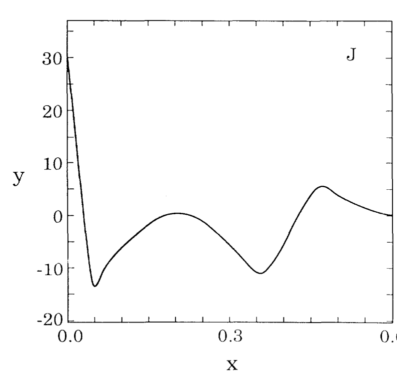

FIG. 4. Cut of J along the midplane from the simulation in Fig. 3.

exceeds the resistive layer width $L_r \sim (\tau_A/\tau_r)^{1/2}a$ of resistive MHD. Namely, the current convective instability and associated diffusion of current are insensitive to collisions even if collisions play a role in the parallel electron dynamics on the longer reconnection time scale. Aydemir has shown that a generic enhancement of electron viscosity can increase the rate of reconnection in the resistive limit.[16]

An important conclusion of the experiments is that the sawtooth crash times can vary significantly even from one sawtooth event to the next, while the basic discharge parameters have hardly changed.[1] One possible explanation is that the threshold for the onset of the current convective instability is sufficiently high that during some sawtooth crashes the mode is not excited and the crash is therefore slow.[3] The threshold for the instability appears to be easily exceeded for the localized currents which are produced during reconnection in high-temperature tokamak discharges. This explanation for the variability of the crash time therefore does not appear likely. On the other hand, the crash time remains quite sensitive to $\Delta q = 1 - q(0)$. Small changes in q from one sawtooth crash to the next could lead to significant variation of the crash time, e.g., for $\Delta q = 0.01$, $\tau_c \sim 400$ μsec. A definite conclusion on this issue must await a more complete understanding of the onset of the sawtooth crash, which may be linked to Δq.

Perhaps a more direct test of our model would be to investigate whether enhanced density fluctuations with scale lengths of the order of δ develop during the sawtooth collapse. A more detailed investigation of the nonlinear broadening of the electron current layer is being undertaken.

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