Strings from Five-Branes

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A heterotic string emerges as a soliton of the heterotic five-brane. We compare its properties with the fundamental heterotic string and suggest that the two might be identified. The solution relies on the observation that the classical five-brane Lagrangian incorporates string one-loop effects.

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Strominger[1] has recently discovered that the field-theory limit of the heterotic string[2] admits a heterotic five-brane[3] as a soliton solution. In this paper we show the converse, a result which lends further support to the idea of string–five-brane duality. After constructing the solution, we examine its zero modes and suggest that they might correspond to those of the fundamental heterotic string written in a physical gauge.

The bosonic sector of the $D=10$ classical heterotic string action describing the coupling of the supergravity fields g_{MN}, b_{MN}, and ϕ to the SO(32) Yang-Mills field A_M ($M=0,1,\ldots,9$) may be written[4] as

$$S(\text{string})=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2.3!}e^{-\phi}H^2-\alpha' e^{-\phi/2}t^{MNPQ}\operatorname{tr}F_{MN}F_{PQ}+\cdots\right], \tag{1}$$

where F_{MN} are Hermitian matrices in the fundamental representation, $t^{MNPQ}=\frac{1}{2}(g^{MP}g^{NQ}-g^{MQ}g^{NP})$, and the three-form field strength H is given by[5]

$$H=db+2\alpha'\omega_3,\quad d\omega_3=\operatorname{tr}F\wedge F\,. \tag{2}$$

The string tension T_2 is given by $1/\alpha'=2\pi T_2$. We claim that the corresponding heterotic five-brane action, for which the two-form b_{MN} is replaced by a six-form a_{MNOPQR}, is given by

$$S(\text{five-brane})=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2.7!}e^{\phi}K^2-\frac{\beta'}{24}e^{\phi/2}t^{IJKLMNPQ}\operatorname{tr}F_{IJ}F_{KL}F_{MN}F_{PQ}+\cdots\right], \tag{3}$$

where

$$\begin{aligned}t^{IJKLMNPQ}=&-\tfrac{1}{2}(g^{IK}g^{JL}-g^{IL}g^{JK})(g^{MP}g^{NQ}-g^{MQ}g^{NP})\\&-\tfrac{1}{2}(g^{KM}g^{LN}-g^{KN}g^{LM})(g^{PI}g^{QJ}-g^{PJ}g^{QI})-\tfrac{1}{2}(g^{IM}g^{JN}-g^{IN}g^{JM})(g^{KP}g^{LQ}-g^{KQ}g^{LP})\\&+\tfrac{1}{2}(g^{JK}g^{LM}g^{PN}g^{QI}+g^{JM}g^{NK}g^{LP}g^{QI}+g^{JM}g^{NP}g^{KQ}g^{LI}+\text{permutations})\end{aligned} \tag{4}$$

and the seven-form field strength K is given by[5]

$$K=da+\tfrac{1}{3}\beta'\omega_7,\quad d\omega_7=\operatorname{tr}F\wedge F\wedge F\wedge F\,. \tag{5}$$

The five-brane tension T_6 is given by $1/\beta'=(2\pi)^3T_6$. This unconventional quartic Yang-Mills action[3] and the corresponding quartic Chern-Simons terms (5) require some justification. This will necessarily be indirect since, although the super five-brane[6] σ model is well known, the heterotic five-brane σ model has yet to be constructed. Even if we knew it, the quantization of five-branes is still in its infancy, and it is doubtful that (3) could yet be derived as rigorously as (1). The point of view we adopt is that the five-brane action is obtained by dualizing the string action,[3] i.e., by the interchange of field equations and Bianchi identities via $K=e^{-\phi}{}^*H$, where the asterisk denotes the Hodge dual. However, this process does not respect the loop expansion in the Yang-Mills sector and what is a tree-level effect in string perturbation theory may be a one-loop effect in five-brane perturbation theory, and vice versa. To understand this, we recall[7] the relationship between the string loop coupling constant g_2, the five-brane loop coupling constant g_6, and the vacuum expectation value ϕ_0 of the dilaton: $g_6=g_2^{-1/3}=\exp(-\phi_0/3)$. (This implies, in particular, that the strong-coupling limit of the string corresponds to the weakly coupled five-brane, and vice versa.) This, in turn, follows from the relationship between the canonical metric appearing in (1) and (3) and the metrics which appear naturally in the string and five-brane σ models, namely,

$$\begin{aligned}g_{MN}(\text{canonical})&=\exp(-\phi/2)g_{MN}(\text{string }\sigma\text{ model})\\&=\exp(\phi/6)g_{MN}(\text{five-brane}+\sigma\text{ model})\,.\end{aligned}$$

In string variables each term in the string tree-level action S(string) is proportional to $\exp(-2\phi)$ which reveals that the string loop coupling constant is given by $\exp(\phi_0)$. Similarly, in five-brane variables each term in S(five-brane) is proportional to $\exp(2\phi/3)$. Thus the Green-Schwarz[8] anomaly-cancellation term $\mathrm{tr} b\wedge F\wedge F \wedge F\wedge F$ and also the term which in canonical variables looks like

$$\sqrt{-g}\exp(\phi/2)t^{IJKLMPQ}\mathrm{tr}F_{IJ}F_{KL}F_{LM}F_{PQ}$$

have no ϕ dependence in string variables and are therefore seen to be one loop in string perturbation theory. The explicit one-loop calculation of this quartic Yang-Mills action has been carried out by Ellis, Jetzer, and Mizrachi.[9] On the other hand, both these terms are *tree level* in five-brane perturbation theory, because they both behave like $\exp(2\phi/3)$ in five-brane variables and must therefore be included in the five-brane tree-level action, S(five-brane). By the same token the term in S(string) which in canonical variables looks like $\sqrt{-g}\exp(-\phi/2)t^{MNPQ}\mathrm{tr}F_{MN}F_{PQ}$ and the Chern-Simons term in (2) corresponding to $\mathrm{tr}F\wedge F\wedge a$ are one loop in five-brane perturbation theory since they are independent of ϕ when written in five-brane variables. We therefore omit them from S(five-brane). In arriving at (3) and (5) we have also employed the equation[7] $\kappa^2 T_2 T_6=\pi$ which relates the two tensions. Note that T_2 has dimension 2 and T_6 has dimension 6, which is another reason for expecting a quartic classical Yang-Mills action for the five-brane. This causes no problems with unitarity. We emphasize that the *exact* string and five-brane actions are equivalent; it is merely the division into "classical" plus "quantum" which is different in the two cases.

Under the two-parameter rescalings of the background fields[7] $g_{MN}\to\lambda^{1/2}\sigma^{3/2}g_{MN}$, $b_{MN}\to\lambda^2 b_{MN}$, $a_{MNOPQR}\to\sigma^6 a_{MNOPQR}$, $e^\phi\to\lambda^3\sigma^{-3}e^\phi$, the elementary five-brane σ-model action S_6 and the elementary string-σ-model action S_2 scale like $S_6\to\sigma^6 S_6$ and $S_2\to\lambda^2 S_2$. In order that S(string) admit a five-brane as a soliton,[1] it must scale the same way under the σ symmetry, i.e., $S(\text{string})\to\sigma^6 S(\text{string})$. This is indeed the case. Similarly, we are encouraged in our search for a string soliton solution of S(five-brane) by noting that it scales in the right way under the λ symmetry, i.e., $S(\text{five-brane})\to\lambda^2 S(\text{five-brane})$. Without further apology, we now solve the field equations.

We begin by making an ansatz for the $D=10$ supergravity fields corresponding to the most general two-eight split invariant under $P_2\times SO(8)$, where P_2 is the $D=2$ Poincaré group. We split the indices $x^M=(x^\mu, y^m)$, where $\mu=0,1$ and $m=2,3,4,\ldots,9$. We write the line element as

$$ds^2=e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu+e^{2B}\delta_{mn}dy^m dy^n\,, \qquad (6)$$

and $b_{01}=-e^C$ which implies

$$e^{\phi+2A}K^{mnopqrs}=({}^8g)^{-1/2}\epsilon^{mnopqrst}\partial_t e^C\,, \qquad (7)$$

where ${}^8g=\det g_{mn}$. All other components of K and all components of the gravitino ψ^M and dilatino λ are set to zero. P_2 invariance requires that the arbitrary functions A,B,C and the dilaton ϕ depend only on y^m; SO(8) invariance then requires that this dependence be only through $r\equiv(\delta_{mn}y^m y^n)^{1/2}$. One may now show that the four functions A, B, C, and ϕ are reduced to one by the requirement of some unbroken supersymmetry. In other words, there must exist Killing spinors ε satisfying

$$\delta\psi_M=D_M\varepsilon+\frac{1}{2.8!}e^{\phi/2}(3\Gamma_M{}^{NOPQRST}-7\delta_M{}^N\Gamma^{OPQRST})K_{NOPQRST}\varepsilon=0\,, \qquad (8)$$

$$\delta\lambda=-\frac{1}{2\sqrt{2}}\Gamma^M\partial_M\phi\varepsilon-\frac{1}{2.2\sqrt{2.7!}}e^{\phi/2}\Gamma^{MNOPQRS}K_{MNOPQRS}\varepsilon=0\,. \qquad (9)$$

We make the two-eight split of the $D=10$ Dirac matrices $\Gamma_A=(\gamma_\alpha\otimes\Gamma_9, 1\otimes\Sigma_a)$, where A is a tangent space index, and γ_α and Σ_a are the $D=2$ and $D=8$ Dirac matrices, respectively. We also define $\gamma_3\equiv\gamma_0\gamma_1$ and $\Gamma_9\equiv\Sigma_2\Sigma_3\cdots\Sigma_9$. We may then write $\varepsilon=\epsilon\otimes\eta$, where ϵ is a spinor of SO(1,1) and η a spinor of SO(8) which may be decomposed into chiral eigenstates via the projection operators $(1\pm\gamma_3)/2$ and $(1\pm\Gamma_9)/2$, respectively. Since $\Gamma_{11}\varepsilon=\varepsilon$ where $\Gamma_{11}=\gamma_3\otimes\Gamma_9$, the $D=2$ and $D=8$ chiralities are correlated. Substituting our ansatz into (8) and (9), and requiring that the metric be asymptotically Minkowskian, we find $A=3(\phi-\phi_0)/4$, $B=-(\phi-\phi_0)/4$, and $C=2\phi-3\phi_0/2$. Moreover, $\varepsilon=\exp(3\phi/8)\epsilon_0\otimes\eta_0$, where ϵ_0 and η_0 are constant spinors satisfying $(1-\gamma_3)\epsilon_0=0=(1-\Gamma_9)\eta_0$, and hence one-half of the supersymmetries are broken, just as for the solitonic five-brane.[1]

In the Yang-Mills sector we set the gaugino χ to zero and make an ansatz for A_M which preserves both the bosonic and fermionic symmetries. In his solitonic five-brane solution, Strominger found that such a configuration was provided by the instanton[10] in the four transverse dimensions which ensured that

$$\delta\chi=e^{-\phi/4}(t^{MNPQ}\Gamma_{PQ}+\Gamma^{MN})F_{MN}\varepsilon+\cdots=0 \qquad (10)$$

by virtue of the self-duality condition

$$(-{}^4g)^{1/2}t^{mnpq}F_{mn}=\tfrac{1}{2}\epsilon^{mnpq}F_{mn} \qquad (11)$$

and the chirality condition $(1-\Gamma_5)\eta_0=0$. Since they arise from a superstring calculation, the bosonic string one-loop corrections shown in Eq. (3) are undoubtedly part of a supersymmetric action. However, neither the authors of Ref. 9 nor anyone else to our knowledge has written down the explicit transformation rules. Nevertheless, preliminary investigations based on Ref. 11 indicate that the gaugino transformation rule requires that

$$\delta\chi = e^{3\phi/4}(t^{IJKLMNPQ}\Gamma_{PQ} + \Gamma^{IJKLMN})F_{IJ}F_{KL}F_{MN}\varepsilon + \cdots = 0 . \quad (12)$$

Hence, bearing in mind the chirality condition $(1-\Gamma_9)\times\eta_0 = 0$, we require an instanton in the eight transverse dimensions which obeys the condition

$$(-{}^8g)^{1/2}t^{ijklmnpq}F_{ij}F_{kl}F_{mn} = \tfrac{1}{2}\varepsilon^{ijklmnpq}F_{ij}F_{kl}F_{mn} . \quad (13)$$

Such a configuration is provided by the SO(8) instanton in Ref. 12. Explicitly,

$$F_{mn} = (i/2)f(r)\Sigma_{mn}(1-\Gamma_9)/2 ,$$
$$f = 4\rho^2/(r^2+\rho^2)^2 , \quad (14)$$

for which the only nonvanishing components correspond to the SO(8) subgroup of SO(32). The constant ρ is the instanton size. [That these instanton configurations solve the flat-space Yang-Mills equations may be seen by taking the covariant derivative of both sides of Eq. (11) or (13). They also solve the curved-space equations including the dilaton and antisymmetric tensor couplings. In both cases the transverse components of the energy-momentum tensor vanish identically. In arguing that the instantons preserve half the supersymmetry we have consistently ignored the higher-loop corrections in Eqs. (10) and (12) denoted by the centered dots. In particular, this means ignoring the terms cubic in F_{MN} in Eq. (10) relative to the linear terms since they appear with a relative factor $\exp(2\phi)$ in string variables and ignoring the terms linear in F_{MN} in Eq. (12) relative to the cubic terms since they appear with a relative factor $\exp(-2\phi/3)$ in five-brane variables.]

To determine the single unknown function ϕ, we substitute these results into the field equations obtained by varying S(five-brane). They are either satisfied identically or else reduce to the single equation

$$\delta^{mn}\partial_m\partial_n e^{-2\phi} = -420\beta' e^{-3\phi_0/2}f^4(r) , \quad (15)$$

and hence

$$e^{-2\phi} = e^{-2\phi_0}\left[1 + k\frac{r^6+6r^4\rho^2+15r^2\rho^4+20\rho^6}{(r^2+\rho^2)^6}\right] , \quad (16)$$

where the constant k is given by $k = 8\pi T_6^{-1}\exp(\phi_0/2)/3\Omega_7$ and Ω_7 is the volume of the unit seven-sphere. Our solution is nonsingular everywhere and corresponds to an infinite string in the physical gauge $X^0 = \tau$, $X^1 = \sigma$. Remarkably, in the limit $\rho \to 0$, Eq. (16) goes over into the singular elementary string solution of Dabholkar *et al.*,[13] who employed a string-σ-model source as opposed to a Yang-Mills source in the supergravity field equations. This is because, as we shrink the instanton to zero, our quartic Yang-Mills kinetic term and quartic Chern-Simons term in S(five-brane) go over into the kinetic term and Wess-Zumino term of the string σ model. Actually, one obtains a string with 8 times the tension of the elementary string. So, in this sense, our soliton is worth 8 strings. Entirely analogous behavior was found for the solitonic[1] and elementary[13] five-brane solutions and for the quadratic Yang-Mills term in S(string) and the five-brane σ model.[14] Following Ref. 13, we may calculate $\mathcal{M}_2$, the mass per unit length of our infinite string. We find

$$\kappa^2\mathcal{M}_2 = e^{\phi_0/2}Q_2 , \quad (17)$$

where the topological charge Q_2 is given by

$$Q_2 = -\tfrac{1}{2}\int_{S^7}K = -\tfrac{1}{6}\beta'\int_{M^8}\mathrm{tr}F\wedge F\wedge F\wedge F = 8\pi/T_6 . \quad (18)$$

M_8 is the $D=8$ transverse space and S^7 its boundary. This saturates the Bogomol'nyi bound,[13] a result which provides further indirect evidence that, in common with the elementary string solution,[13] the solitonic string preserves half the supersymmetries. These results are in complete agreement with an earlier prediction based on string–five-brane duality, and satisfy the Dirac charge quantization condition $Q_2Q_6 = \kappa^2 n\pi$ with $n=8$, where Q_6 is the Noether charge associated with the elementary five-brane.[7]

Following Strominger[1], we may now count the Bose and Fermi zero modes associated with the instanton moduli. We find eight bosonic translation modes (both left and right movers) and eight fermionic supersymmetry modes (left movers only), one dilation mode and a further 220 modes arising from the embedding of SO(8) in SO(32) associated with the coset SO(32)/SO(24). We have not yet constructed the world-sheet action and it remains to be seen whether such an (8,0) two-dimensional σ model can be identified with the fundamental Green-Schwarz heterotic string in the background of the soliton and written in a physical gauge. In particular, one might ask whether, as for the five-brane, there are any further fermion zero modes coming from the Atiyah-Singer index theorem. A possible explanation for their absence may be provided by the observation that the string one-loop supersymmetric partners of F^4 contain no gaugino kinetic term.[11] We emphasize that the solution of this paper has been established only to tree level in the five-brane field theory and is *a priori* valid only for $\exp(-2\phi/3) \ll 1$. The question of whether it survives loop corrections remains a topic for future research.

Although we omit the string Chern-Simons terms corresponding to $\mathrm{tr}F\wedge F\wedge a$ from our classical five-brane considerations, they play an important role as the five-brane analog of the Green-Schwarz anomaly cancellation term. In the case of strings, the combined gravitational and Yang-Mills anomalies for $N=1$ supergravity coupled to a Yang-Mills supermultiplet (with n left-hand Majorana-Weyl spinors in the adjoint presentation) can be characterized by a certain twelve-form I_{12}. As discussed in Ref. 8, the anomaly can be canceled only if I_{12} factorizes into an expression of the form $I_{12}=dH\wedge X_8$, where X_8 is an eight-form. The necessary and sufficient conditions are

$$n=\dim G=496\,,$$
$$\mathrm{Tr}F^6=\tfrac{1}{48}\,\mathrm{Tr}F^4\mathrm{Tr}F^2-\tfrac{1}{14400}\,(\mathrm{Tr}F^2)^2\,. \tag{19}$$

There are only two solutions: SO(32) and $E_8\times E_8$. The anomaly is then canceled by the addition of a term in the action $b\wedge X_8$. In the case of the five-brane, we would require that I_{12} factorize into an expression of the form $I_{12}=X_4\wedge dK$, where X_4 is a four-form. Assuming that the same I_{12} governs both strings and five-branes, we discover from Ref. 8 that the necessary and sufficient conditions for this to happen are exactly the same as those given in (19). Hence we find SO(32) and $E_8\times E_8$ once more. The anomaly is then canceled by the term $X_4\wedge a$. Thus I_{12} takes on the string–five-brane symmetrical form $I_{12}=dH\wedge dK$. If we include the gravitational terms

$$\frac{1}{2\alpha'}\,dH=\mathrm{tr}F\wedge F-\mathrm{tr}R\wedge R\,,$$
$$\frac{3}{\beta'}\,dK=\mathrm{tr}F\wedge F\wedge F\wedge F-\tfrac{1}{8}\,\mathrm{tr}F\wedge F\,\mathrm{tr}R\wedge R \tag{20}$$
$$+\tfrac{1}{32}\,\mathrm{tr}R\wedge R\wedge\mathrm{tr}R\wedge R+\tfrac{1}{8}\,\mathrm{tr}R\wedge R\wedge R\wedge R\,,$$

one may verify that the gravitational and mixed terms make no contribution to either the five-brane soliton or the string soliton.

Clearly, it is now a matter of some urgency to construct explicitly the heterotic five-brane and to understand its quantization.

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