Controlling Chaos in Spin-Wave Instabilities

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A microwave-pumped spin-wave-instability experiment is used to demonstrate that chaos can be controlled by a small periodic perturbation of an available system parameter, as recently proposed by Ott, Grebogi, and Yorke. The experiment is performed in an yttrium-iron-garnet sphere in the subsidiary-resonance configuration with a small modulation in the applied magnetic field. Observation of the Fourier spectrum of the low-frequency auto-oscillations and measurement of the attractor dimension and metric entropy demonstrate clearly that the chaotic attractor becomes periodic when the modulation frequency and amplitude are carefully chosen.

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Spin-wave instabilities driven by microwave fields constitute one of the best physical systems for studying nonlinear dynamic phenomena. A rich variety of behavior has been observed in these systems, including self-oscillations, spiking, period multiplication, intermittency, and chaos.[1-18] One of the main features of spin-wave nonlinearities is that they can be modeled by nonlinear equations derived from microscopic Hamiltonians with well-known parameters, thus providing a theoretical framework to interpret or predict experimental results. On the experimental side, several parameters can be easily controlled by the experimenter in a manner not attained in other systems. As a result of combined experimental and theoretical work, many features of the nonlinear dynamics of spin-wave instabilities are presently well understood.[7-20] However, much like in other physical systems, this is true for the prechaotic region and for the borderline chaos, but not for the fully chaotic regime.

In this paper we report experiments with spin-wave instabilities that demonstrate the feasibility of controlling chaos by a small time-dependent perturbation of the biasing magnetic field applied to the sample. This is the first observation of the control of chaos by small perturbations in an accessible system parameter, following the recent proposal of Ott, Grebogi, and Yorke.[1] The method is based on the fact that a chaotic attractor usually has embedded within it an infinite number of unstable periodic orbits. By applying a small time-dependent modulation to a conveniently chosen parameter one can stabilize some of these unstable orbits to achieve control of the chaotic state, without having to make large variations in the system parameters. In order to understand the basic principles of the method, consider a dissipative dynamical system described by N nonlinear differential equations of the form $d\mathbf{x}/dt = F(\mathbf{x},\mu)$, where $\mathbf{x}$ is the dynamical variable vector and μ represents the various system parameters. When the system is excited by some external source (the microwave radiation in the case of the spin-wave instability), $\mathbf{x}$ describes a continuous-time orbit which appears as a discrete-time series of points in a conveniently chosen Poincaré surface in the N-dimensional space. If the attractor is chaotic and chaos has been approached by a cascade of period-doubling bifurcations, the time interval between two consecutive piercings of the surface is nearly constant and given by $T_0 = f_0^{-1}$, where f_0 is the fundamental frequency. Assume now that some system parameter $p = \mu_i$ can be modulated periodically about its mean value, with period $T_1 \simeq nT_0/m$, where n and m are integers. The periodic change in p can be tailored in such a manner that the unstable orbit is synchronously pushed into describing the same path as in the previous cycles during the time interval mT_1, thus resulting in a controlled trajectory. Ott, Grebogi, and Yorke[1] have given a prescription for finding the amplitude p^* of the parameter variation necessary to achieve control of chaos. Their approach relies on the knowledge of the system equations or the time series of an experimental variable. Here we determine p^* experimentally by observing the behavior of the chaotic system response with increasing parameter modulation.

The experiments were carried out with a polished sphere (diameter 1.0 mm) of the "prototype ferromagnet" yttrium iron garnet (YIG) at room temperature, in the perpendicular-pumping, "subsidiary-resonance" configuration. The sample is located at the center of a critically coupled rectangular TE_{102} microwave cavity ($Q = 2000$, $f_p = \omega_p/2\pi = 8.87$ GHz) placed between the poles of an electromagnet, so that the microwave magnetic field $\mathbf{h}$ is perpendicular to the biasing field $\mathbf{H}$, as in the usual setup.[12,19] However, we have added inside the cavity a loop of diameter 1.5 cm made with a 0.5-mm copper wire to allow the modulation of the sample biasing field $H = H_0 + \delta H \cos(2\pi f_1 t)$ over a broad frequency range 0–10 MHz, typically with $\delta H/H_0 \sim 10^{-4}$. The microwave radiation is provided by an X-band backward-wave oscillator with frequency stabilized by an external crystal oscillator and manually adjusted to the center of the cavity resonance. The radiation is amplified by a 1.8-W traveling-wave tube, attenuated with a variable precision attenuator, and directed by a circulator to the resonant cavity where it drives spin

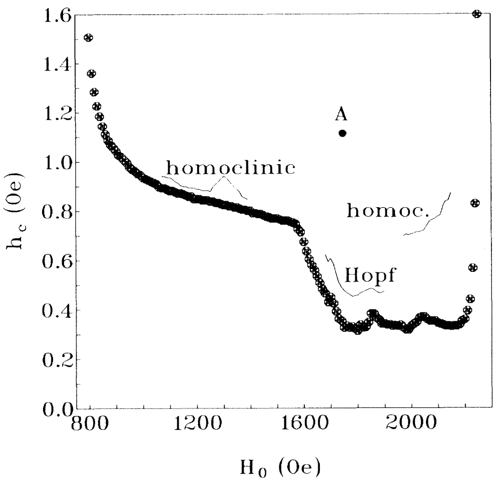

FIG. 1. Data for the threshold field h_c vs H_0 for subsidiary-resonance spin-wave instability in a 1-mm YIG sphere driven by a microwave field h with frequency 8.87 GHz in a static field H_0 applied along the [110] axis. The solid lines represent the threshold boundaries for self-oscillations characterized by Hopf and homoclinic bifurcations. The experiments on the control of chaos described here were done at the indicated point A: $H_0=1750$ Oe, $h=1.1$ Oe.

waves in the sample. The nonlinear interaction between spin waves results in a low-frequency auto-oscillation of the microwave absorption which is detected with a sensitive Schottky-barrier diode at the output port of the circulator and recorded in a digital storage oscilloscope. In order to avoid sample heating we use pulsed microwave radiation (duration 100 μs at 100 pulses/s) but we can record the steady-state regime of the signal since the pulse length is much longer than the transient time and the fundamental period T_0 (few μs) of the auto-oscillation.

The usual experiment to study spin-wave phenomena is done with fixed values of H_0 and varying microwave power. At low power levels the steady-state reflection from the critically coupled cavity is negligible. As the driving field h is increased, an abrupt change in reflection occurs at the Suhl threshold h_c, due to the parametric excitation of a magnon pair with frequency $\omega_k \simeq \omega_p/2$ and wave vectors $\mathbf{k}$ and $-\mathbf{k}$. The value of k is determined by the frequency ω_p, the field H_0, and the condition for minimum threshold, which depends on the pumping configuration. Figure 1 shows data for h_c vs H_0 obtained for the sample oriented with the [110] crystal axis along the applied field. As h increases further, at

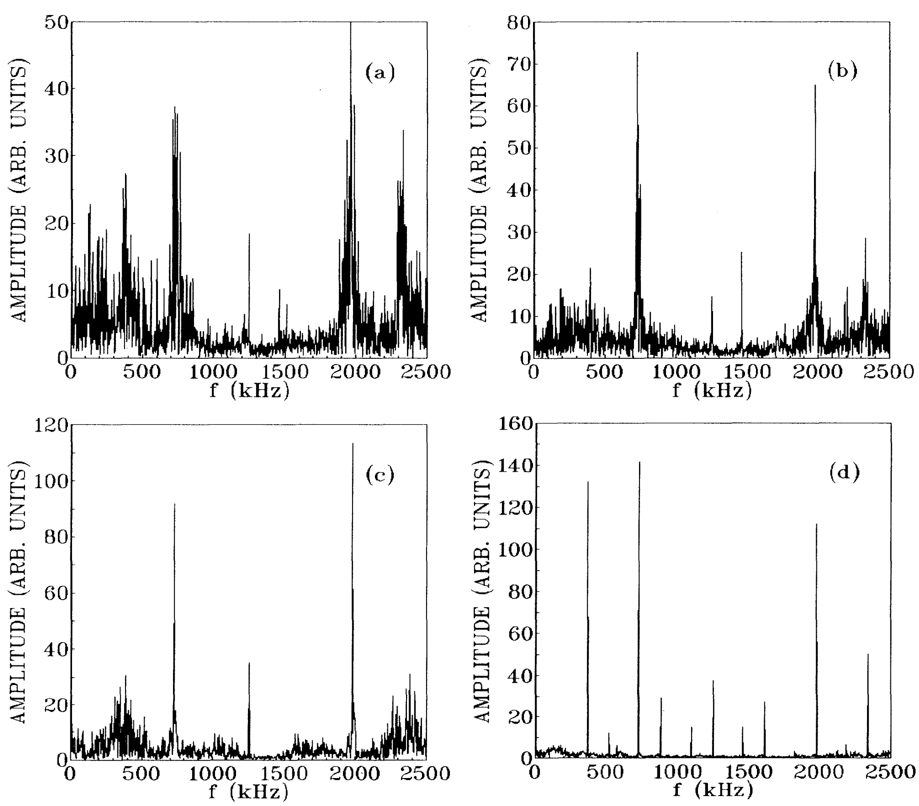

FIG. 2. Evolution of the power spectra of the auto-oscillation observed at point A (Fig. 1) with increasing amplitude δH of the field modulation with frequency $f_1=1480$ kHz. The spectra in (a)-(d) were obtained with $\delta H=0$, 140, 287, and 435 mOe, respectively. Spectrum (d) corresponds to a periodic signal with fundamental frequency $f_0=740$ kHz and a subharmonic component at $f_0/2$, superimposed on another frequency of 1975 kHz.

some threshold $h_c' > h_c$ the amplitude of the reflected microwave radiation suddenly develops a low-frequency ($f_0 = 100$ kHz–1 MHz) auto-oscillation, arising from the nonlinear interaction between collective spin-wave modes. This is described by trajectories in a low-dimensional phase space of slowly varying spin-wave variables with time evolution governed by autonomous differential equations[8,12-16] in which the microwave frequency enters only in a detuning parameter $\Delta\omega_k = \omega_k - \omega_p/2$, where $\Delta\omega_k \sim (10^{-5}\text{-}10^{-4})\omega_p$ is of the order of the auto-oscillation frequency $2\pi f_0$. This low-frequency spontaneous oscillation may exhibit a variety of bifurcations as the system parameters are changed, including intermittency and period-multiplication routes to chaos.[2-20] The solid lines in Fig. 1 represent the threshold boundaries for Hopf and homoclinic bifurcations[18] which lead to self-oscillations. The chaotic regime sets in not far above these boundary lines and is characterized by an attractor with fractal dimension d in the range $1.6 < d < 2.0$ near the onset of chaos.[19,20]

By applying a small modulation to the magnetic field H with appropriate wave shape, frequency, and amplitude we have been able to control the chaotic states in most regions of Fig. 1. The field modulation results in a corresponding modulation in the spin-wave frequency ω_k, and so in the detuning parameter $\Delta\omega_k$, providing a handle to control the orbits. Figures 2–4 show the results obtained with a sinusoidal field modulation applied after the system is driven to a fully chaotic regime with $h = 1.1$ Oe and $H_0 = 1750$ Oe, represented by point A in Fig. 1. Figure 2 shows the evolution of the power spectra, obtained from the digitally stored signals, with increasing field modulation δH at a frequency $f_1 = 1480$ kHz. For $\delta H \le 200$ mOe the spectra display broadband features characteristic of chaos as shown in Fig. 2(a) for $\delta H = 0$. As δH increases above this value the spectrum becomes progressively cleaner, with sharp lines characteristic of a periodic signal. This is shown in Figs. 2(b)–2(d) for $\delta H = 140$, 287, and 435 mOe, respectively. The spectrum in (d) corresponds to a periodic signal with fundamental frequency $f_0 = 740$ kHz and a subharmonic component at $f_0/2$, characteristic of a period-doubled oscillation, superimposed on another signal with frequency 1975 kHz.

The control of chaos is further demonstrated by the variation of the information dimension D_1 and the metric entropy K of the attractor with increasing field modulation. These quantities have been obtained with the embedding technique[21] from the time-delayed digitized signals. We employ the method of Badii and Politi[19,22,23] to calculate $D_1 \equiv D(0)$ and $K \equiv K(0)$ from 2048 data points and obtain good convergence for embedding dimension $E > 8$. Figure 3 shows that for $\delta H < 200$ mOe $D_1 \simeq 3.5$ and $K \simeq 0.20$, characterizing a fully chaotic regime. Strikingly, for a modulation amplitude above the critical value $\delta H^* \simeq 200$ mOe, D_1 and K decrease

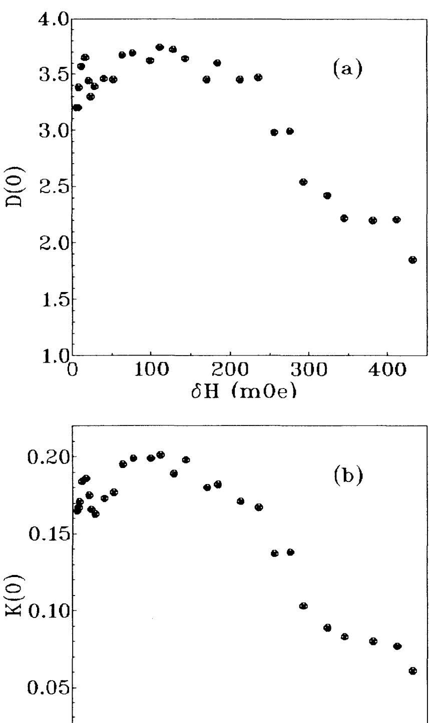

FIG. 3. (a) Information dimension $D(0)$ and (b) metric entropy $K(0)$ vs field modulation amplitude δH. These quantities have been obtained with the embedding technique from the time-delayed digitized signals. Note that for $\delta H > 200$ mOe the values of $D(0)$ and $K(0)$ decrease towards the values $D(0) = 1$ and $K(0) = 0$ appropriate for periodic signals.

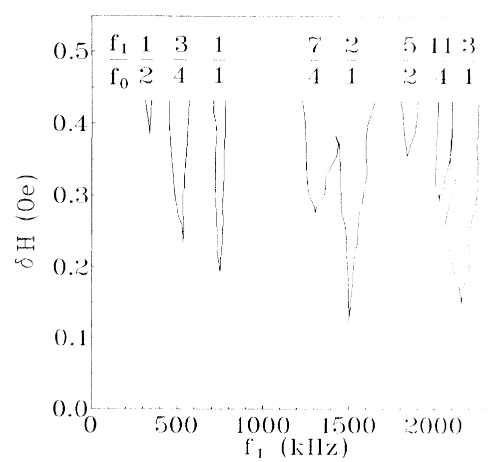

FIG. 4. Critical modulation amplitude δH^* vs modulation frequency f_1. The boundaries between the chaotic and controlled regions have minima at values of f_1 commensurate with the fundamental frequency f_0 at ratios indicated at the top.

with increasing δH towards the values $D_1 = 1$ and $K = 0$, characteristic of a periodic trajectory. This result confirms that the control of chaos is achieved by a small periodic modulation of the applied field. In regard to the data of Figs. 2 and 3 we note that with the application of the field modulation the auto-oscillation enters a steady-state regime only after a time τ has elapsed. The spectra of Fig. 2 and the time series used to obtain D_1 and K were calculated from the steady-state portion of the signal. The delay time τ decreases rapidly with increasing amplitude modulation δH, in qualitative agreement with the results of Ott, Grebogi, and Yorke.[1] However, we could not obtain a critical exponent relating τ and the increment amplitude $\varepsilon = \delta H - \delta H^*$ because of the difficulty in measuring τ and δH^* with adequate precision.

Finally, in Fig. 4 we show the variation of the critical amplitude δH^* necessary to control chaos with the modulation frequency f_1, measured at point A in Fig. 1. The measurements were done by scanning f_1 at constant δH and observing the change from chaotic to periodic behavior directly at the oscilloscope. Notice that δH^* has minima at values commensurate with the fundamental frequency f_0, i.e., $f_1 = mf_0/n$, increasing rapidly as f_1 departs from these values. Hence, the boundaries between the chaotic and controlled regions have the appearance of "tongues," similar to those observed in phase-locking phenomena.[24] This is not surprising since we expect the chaotic motion to be suppressed when the unstable orbit becomes phase locked to the external parameter modulation. Although the phase-locking phenomenon has been widely studied, it seems that this is the first demonstration that it can be used to control chaos.

In conclusion, we remark that we have been able to suppress the chaotic behavior of spin-wave instabilities over a wide region in the phase diagram of Fig. 1 using a small modulation in the biasing magnetic field, not only with a sinusoidal wave shape, but also with a square-wave or pulse modulation. In some points of the h_c vs H_0 diagram only a specific wave shape results in a controlled signal, whereas in others they are all equally effective. In each case, however, the resulting periodic signal has a different wave form, corresponding to a different trajectory. This confirms that by using a carefully chosen small parameter perturbation, it is possible to control a chaotic trajectory and create a variety of attracting periodic orbits, as proposed by Ott, Grebogi, and Yorke.[1]

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