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Can Quark-Gluon-Plasma Formation in Relativistic Heavy-Ion Collisions Constrain Inhomogeneous Cosmologies?

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The separation distance between droplets of quark-gluon plasma (QGP) in the early Universe and in relativistic heavy-ion collisions is calculated in isothermal fluctuation theory. It is found that experimental information on the size of the QGP from relativistic heavy-ion collisions has the potential possibility of constraining the degree of inhomogeneity of the baryon-number density distribution and the formation of strange-quark matter at the epoch of the cosmic phase transition in quantum chromodynamics.

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Inflationary cosmology[1,2] is a unique theory which may resolve many fundamental problems of hot big-bang expansion in the early Universe. It requires a marginally closed universe, $\Omega+\Lambda=1$, where Ω is the total universal mass-density parameter and Λ is the cosmological constant. Inflation is now being tested by both theoretical and observational studies of the Ω value, which has recently proved to be $\Omega=0.1-0.4$, while the Λ value is completely unknown.[3] Although the standard big-bang model[4,5] has enjoyed success for primordial light-element abundances, it assumes a homogeneous baryon distribution and infers a rather dilute universe, $\Omega_B \approx 0.04$, at the risk of the missing-mass problem. Inhomogeneous big-bang models[6-10] have suggested a more relaxed Ω_B value which is not very far from unity from the constraint of light-element abundances. Although $\Omega_B=1$ inhomogeneous cosmologies are unfortunately unlikely from nucleosynthesis calculations[11,12] unless the late-time homogenization effect[13] is taken into consideration, the inferred Ω_B value is consistent with the total Ω observed. They still shed light on a possible solution of the missing-mass problem without assuming the existence of unknown nonbaryonic dark matter. In addition, not only the lightest elements ($A \le 7$), but also the rare Li-Be-B species[9] and intermediate to heavy-mass nuclei[6,8,10] could be produced in inhomogeneous proton-neutron distributions, providing a lot of observational tests of cosmological theories.[10] In these models, baryon distributions are assumed to be inhomogeneous as remnants of a first-order[14] cosmic QCD phase transition which took place prior to the nucleosynthesis epoch. All interesting consequences of inhomogeneous cosmologies, however, are subject to unknown fluctuation shapes[7,9,15] of the baryon distribution which was attained at the end of the cosmic QCD phase transition. The purpose of this Letter is to show that the experimental data on quark-gluon-plasma (QGP) formation in relativistic heavy-ion collisions, i.e., pion interferometry,[16] intermittency,[17] and J/ψ suppression,[18] etc., can provide useful information to constrain the degree of baryon inhomogeneities in the early Universe.

Let us start our discussion from the thermodynamics of QCD. The QCD phase transition is specified by two parameters, T_C and σ, where T_C is the coexistence temperature of QGP and hadron phases, and σ is the surface tension of phase boundaries. These two parameters are common to phase transitions in both the early Universe and relativistic heavy-ion collisions. Only the manner of space-time evolution is different in these two cases. The time scale of the cosmic QCD phase transition, ~ 10 μsec, is long enough for the whole Universe to be in thermal equilibrium. We can therefore apply thermodynamical equations to the early Universe and obtain[19]

$$\left(\frac{T}{T_C}\right)^{-2}=\left(\frac{3x}{x-1}\right)^{1/2}\sinh\left[\left(\frac{8\pi GL}{3}\right)^{1/2}t\right] \qquad (1)$$

for QGP in an expanding universe as a solution of the

Einstein equation. We used the MIT bag model as a toy model for describing the QGP phase, and the latent heat is given by $L=4\pi^2/90(g_q-g_h)T_C^4$ so that the pressure equilibrium is realized at $T=T_C$. The quantity $x=g_q/g_h$, G is the gravitational constant, and we used statistical weights $g_q=g_Q+g_{\gamma l}=51.25$ and $g_h=g_H+g_{\gamma l}=17.25$ for QGP and hadron phases, respectively, with $g_{\gamma l}=14.25$ for background photons and leptons.

For linearly expanding QGP formed in relativistic heavy-ion collisions,[20] variation of the temperature in time is described by

$$(T/T_0)^{-3}=t/t_0 \tag{2}$$

in an ideal case without entropy production, where T is the temperature at proper time t. In this equation, T_0 and t_0 are the initial temperature and time of the QGP. The initial entropy density is constrained[21] by the rapidity distribution of pions from the CERN experiment,[22] which gives $t_0T_0^3 \le 1.0\times10^7$ (fm/c) MeV^3. This leads to an initial temperature $T_0\approx 220$ MeV if $t_0\approx 1$ fm/c, being consistent with Bjorken's estimate.[20] We therefore adopted the marginal value $t_0T_0^3=1.0\times10^7$ (fm/c) MeV^3 in the present analysis. The initial energy density >3 GeV/fm^3 is sufficiently high to make it very likely that the system rapidly comes into local thermal equilibrium.[20] Since the time scale, 5–10 fm/c, is comparable to or larger than that of the strong interaction, ~1 fm/c, though shorter than that of electroweak interactions, we assume that QGP initially formed in relativistic heavy-ion collisions is also close to being in quasiequilibrium to the strong interaction. We here intend to adopt the Bjorken conjecture of local thermal equilibrium, though its validity is only partly confirmed experimentally for the initial energy density as discussed above, in order to look for a quantitative connection between the early Universe and relativistic heavy-ion collisions.

Having the relation between temperature and time, we calculate in isothermal fluctuation theory[23] the total number density N of hadron bubbles nucleated in the sea of QGP in the supercooling epoch:

$$N=\int_{t_c}^{t_f} f(t)p(T)dt\,, \tag{3}$$

$$f(t)=\exp\left[-\int_{t_c}^{t} f(t')p(T')dt'\frac{4\pi}{3}V_S^3\left(\frac{T}{T'}\right)^3(t-t')^3\right], \tag{4}$$

$$p(T)\approx T_C^4\exp\left(-\frac{16\pi}{3}\frac{\sigma^3}{T_C L^2\eta^2}\right), \tag{5}$$

where t_c is the time at which the QGP phase was first cooled to $T=T_C$, t_f is the end of the supercooling epoch, $f(t)$ is the fraction of QGP (Ref. 19) unaffected by nucleation at times $t_c\le t\le t_f$, and $p(T)$ is the nucleation rate[23] of hadron bubbles per unit volume per unit time. In these equations $V_S=c/\sqrt{3}$ is the sound velocity. η is the supercooling parameter defined by $\eta=(T_C-T)/T_C$.

We do not care about the prefactor of Eq. (5) because we are interested in the relative difference of N between the two phase transitions of the early Universe and of relativistic heavy-ion collisions. Compared with the rapid increase of $p(T)$ with time, $f(t)$ varies slowly from 1 to 0. Since the nucleation rate $p(T)$ has a maximum value at the lowest temperature $T=T_f$, we expand it around $t=t_f$ as a function of time.[7] We thus obtain

$$N\approx\int_{t_c}^{t_f}\exp\left[-\left(\frac{d\ln p(T)}{dT}\right)_{T_f}\left(\frac{dT(t)}{dt}\right)_{t_f}(t_f-t)\right]p(T_f)dt\approx\frac{2^{12}}{3^6}\frac{\sigma^9}{V_S^3L^6\eta_f^9}\times\begin{cases}\pi^5(41\pi G)^{3/2}T_C^3(1+g)\\ \pi^2T_C^6/(t_0T_0^3)^3\end{cases} \tag{6}$$

for the early Universe (upper) and relativistic heavy-ion collisions (lower), where $g=3x/(x-1)$.

We made here several approximations: First, the function $f(t)$ is taken to be constant of order unity, as shown by the exact solution. Second, we used the condition $1<[d\ln p(T)/dT]_{T_f}[dT(t)/dt]_{t_f}(t_f-t_c)$ in the integration, which is justified by the following discussion: η_f is determined by a condition $f(t_f)=0$. Although $f(t_f)$ is a complicated function of T_f, t_f, T_C, and t_c, the solution is found to be in the form $\eta_f=\gamma\sigma^{3/2}/T_C^{1/2}L$ with dimensionless constants $0.3\le\gamma_U\le0.45$ and $\gamma_U/\gamma_{HI}\approx0.35$ being introduced for a wide range of $50<T_C<250$ MeV and $10^4<\sigma<10^8$ MeV^3. The solution satisfies the required condition,

$$1<[d\ln p(T)/dT]_{T_f}[dT(t)/dt]_{t_f}(t_f-t_c)\,.$$

Hereafter we use the subscript U or HI for expressing the quantities associated with the phase transitions in the early Universe (U) and in relativistic heavy-ion collisions (HI).

The number density of nucleation sites of hadron bubbles is nearly equal to the number density of shrinking QGP droplets at the latter half epoch of the phase transition if their collisions and coalescence do not occur very frequently. It is well known[7,19] that the scale factor increases by about 40% during the cosmic QCD phase transition. Thus we regard N_U as the number density of QGP droplets at the end of the phase transition. Since we can define the end of the phase transition at the time when the speed of the phase boundary reaches sound velocity,[15] the droplets have finite size and eventually form high-baryon-number density zones in the early Universe.

Using Eq. (6), the ratio of mean separation distances between the QGP droplets, $l_U/l_{HI}=(N_U/N_{HI})^{-1/3}$, is calculated to be

$$l_U/l_{HI}\approx6.1\times10^{15}T_C(\gamma_U/\gamma_{HI})^3\,, \tag{7}$$

where T_C is in MeV. This quantity is independent of the surface tension. Let us examine what length scale l_U will result from the calculated value of T_C in a lattice QCD simulation if l_{HI} is observable in relativistic heavy-ion collisions and gives ~ 1 fm. Taking a range of T_C, 100 MeV $\le T_C \le$ 250 MeV, l_U turns out to be $25 \le l_U \le 65$ m. This scale at the epoch of the QCD phase transition is to be compared to ~ 30 m which is an optimal length scale for neutrons to diffuse out of the fluctuations[24] and leads to an interesting inhomogeneous nucleosynthesis. The laboratory experiments will thus provide useful information on the early Universe.

Let us examine the possibility of constraining the baryon inhomogeneities in more detail. The separation distances between the QGP droplets are given by

$$l_U \approx 2.7\times 10^4 \frac{\sigma^{3/2}}{T_C^{13/2}}\ \mathrm{m}, \qquad (8a)$$

$$l_{HI} \approx 1.0\times 10^{-2}(t_0 T_0^3)\frac{\sigma^{3/2}}{T_C^{15/2}}\ \mathrm{fm}, \qquad (8b)$$

where σ is in MeV^3, t_0 in fm/c, T_0 in MeV, and we adopted here the typical values of $\gamma_U = 0.35$ and $\gamma_{HI} = 1$. Figure 1 displays these relations in two-QCD-parameter space. Thick and thin solid curves show l_U and l_{HI}, respectively, and the dotted curve is $\eta_f = 1$, below which a severe supercooling occurs and the present approximations are not justified. We also plot $Rf_V = 20\Omega_B$ (dashed curve) from the theoretical study[15] of time evolution of baryon-number densities during the cosmic QCD phase transition, where R is the ratio of baryon-number density in high-density zones to the baryon-number density in low-density zones, and f_V is the volume fraction of high-density zones at the end of the phase transition. A number of theoretical studies of primordial nucleosynthesis infer that, if the baryon-number density fluctuation is as large as $Rf_V \ge 20\Omega_B$ (below the dashed curve)[7-10,12] and if l_U has a length scale comparable to $10 \le l_U \le 100$ m,[6,24] then the primordial nucleosynthesis is very different from that predicted in the homogeneous standard big-bang model. The $l_{HI} = 1$ and 10 fm lines are located in a very crucial region of cosmological interest (the shaded region) which satisfies both conditions.

There are several data[16,17,22] from relativistic heavy-ion collisions that the size of the QGP-droplet-containing region is 1–7 fm. If the whole central rapidity region forms a single QGP droplet, the rapidity distribution of hadrons will show a rather smooth plateau and l_{HI} must be larger than the inferred size, 4–7 fm, of the droplet. This leads to a marginally important density fluctuation in the early Universe near $l_U \sim 100$ m in Fig. 1. If, on the other hand, there are many small QGP droplets formed in relativistic heavy-ion collisions, evaporating hadrons will show highly fluctuating rapidity distributions like those indicated by the CERN experiments and others. When each droplet is comparable in size to ~ 1 fm, which is extracted from pion intermittency, 1 fm $\le l_{HI}$ leads to possibly large inhomogeneities in the early Universe. Although interpretation of the QGP size is not yet convergent, we can hopefully constrain the T_C-σ plane in Fig. 1 by combining them with forthcoming experiments of dilepton production and classify how large or small the baryon inhomogeneities were in the early Universe.

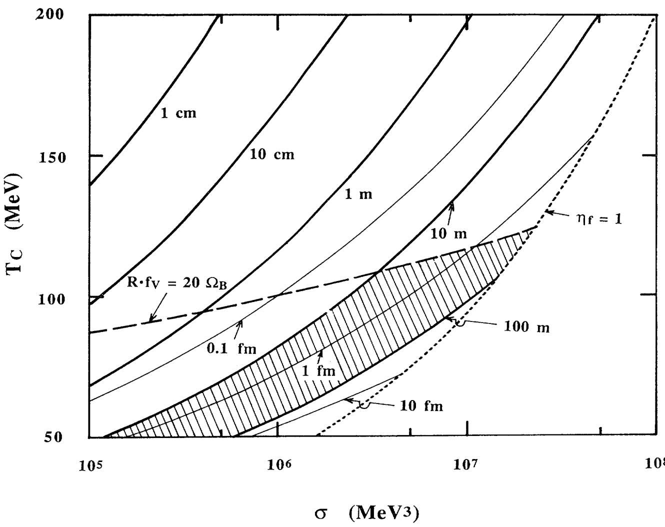

FIG. 1. Calculated mean separation distance between QGP droplets in the early Universe, l_U (thick solid curves), and relativistic heavy-ion collisions, l_{HI} (thin solid curves), as a function of T_C and σ. The dashed curve is for $Rf_V = 20\Omega_B$ with $\Omega_B = 1$ below which baryon-number density fluctuations have appreciable amplitude in the early Universe. $\Omega_B = 0.04$ describes a similar curve slightly above the dashed curve. The dotted curve is for $\eta_{f,U} = 1$ below which severe supercooling occurs. The shaded region leads to interesting inhomogeneous primordial nucleosynthesis.

Recent data of lattice QCD simulations,[25] including the effects of dynamical quarks, suggest $100 \le T_C \le 150$ MeV, although the simulation in pure gauge[14] indicates a little higher temperature, $200 \le T_C \le 250$ MeV. At the former allowable temperature, $100 \le T_C \le 150$ MeV, with $1 \le l_{HI} \le 10$ fm, the inhomogeneous primordial nucleosynthesis could occur if σ were of order $\sim 10^7$ MeV^3. If, however, T_C is relatively high, $200 \le T_C \le 250$ MeV, as indicated from pure-gauge simulation, and $\sigma \sim 10^7$ MeV^3, nothing interesting could happen in primordial nucleosynthesis. The σ value was calculated to be $\sigma = (0.1-1)T_C^3$ in pure-gauge lattice QCD simulations,[26,27] which is marginally consistent with $\sigma = 10^7$ MeV^3. Recent calculations including dynamical quark effects[25,28] have suggested that evidence for first-order character is not easily found although they do not exclude the possibility of very weak first-order phase transition. This means that the σ value should be smaller than 10^7 MeV^3, raising a hope of finding interesting phenomena of inhomogeneous nucleosynthesis. Further large-scale lattice QCD simulations are desirable in order to determine the precise values of these parameters.

Let us finally discuss the strange-quark matter which was suggested by Witten[29] to be formed at the epoch of the cosmic QCD phase transition. It has recently been confirmed[15] quantitatively that strange-quark matter could be really formed at this epoch if $\sigma \leq 5\times10^5$ MeV^3 and $80 \leq T_C \leq 150$ MeV, and the study of the survival[30,31] of such objects until the present time is under way. To confirm this theoretical prediction one needs $l_H \leq 0.1$ fm in relativistic heavy-ion collisions from Fig. 1.

To summarize, we have calculated the mean separation distance between QGP droplets formed in the cosmic QCD phase transition and in relativistic heavy-ion collisions. If the size of the QGP droplets is determined from relativistic heavy-ion collisions, then the two QCD parameters, T_C and σ, will be constrained in the present theoretical calculations of mean separation distance between the droplets. These constrained parameters will also indicate if inhomogeneous primordial nucleosynthesis could occur or strange-quark matter could be formed in the early Universe.

In the present study, we have taken several approximations which are not completely established. One of the biggest assumptions is the local quasiequilibrium condition of QGP initially formed in relativistic heavy-ion collisions. Although this condition has been used in many theoretical studies of QGP formation since the benchmark paper[20] of Bjorken, its applicability must be examined first in future high-energy heavy-ion collision experiments of QGP hunting. Other important physics concerns which have not been included in the present work are complicated hydrodynamical processes such as collisions or merging of QGP droplets near the end of the phase transition. Although it is beyond the purpose of this Letter to refine these approximations, they should be addressed in future experiments and theoretical studies.

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