

## Charge-Transfer Gap and Superexchange Interaction in Insulating Cuprates

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A cluster-model analysis is made on the material dependence of the optical charge-transfer gap and antiferromagnetic superexchange interaction of a variety of insulating cuprates. It is shown that the electronic structure of cuprates typically of the charge-transfer type is characterized by the unique energy-level separation that reflects the three dimensionality of the crystal via the long-range Madelung potential; such characteristics are absent in the Mott-Hubbard regime.

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The electronic structure of insulating cuprates has been understood to be in the charge-transfer (CT) regime rather than in the Mott-Hubbard (MH) regime.<sup>1</sup> Their energy gap should thus be determined basically by the energy-level separation of orbitals between copper and ligand oxygens, and the separation may well be characterized by the ligand electronegativity and the Madelung potential, not by the on-site Coulomb interaction of copper. Quantitative studies of the electronic structure of CT insulators have not, however, been explored much until the recent intensive investigation of the cuprate high- $T_c$  superconducting materials.

Experiments on the optical conductivity and magnon Raman scattering spectra recently made by two groups<sup>2,3</sup> for a large variety of insulating cuprates have shed light on their electronic structure and provided an important opportunity for a deeper understanding of the CT insulators. Explicit information from such studies also contributes to establishing a quantitative basis for the description of the electronic states of the high- $T_c$  superconducting cuprates.

In this Letter, we analyze the material dependence of the reported CT gap  $E_g$  and superexchange interaction  $J$  by means of the combined ionic- and cluster-model approach, and deduce the characteristics of the electronic structure of the cuprates representative of the CT insulators. We will show that the material dependence of  $E_g$  is predominantly determined by the unique energy-level separation between copper and in-plane oxygen which reflects the three-dimensionality of the crystal via the long-range Madelung potential. The small material dependence of  $J$  is also due to the effect. We will see that such characteristics become apparent in the CT insulators but are absent in the MH regime.

The electronic structure of cuprates has two features: (i) the relevant electrons are highly correlated and (ii) the ligand oxygens play an explicit role. To describe the dynamics of the electrons of such a system, we take an accepted view<sup>4</sup> that the  $3d_{x^2-y^2}$  orbital of copper and the  $2p_\sigma$  orbitals of in-plane oxygens are essential. The electronic parameters characterizing the system are then as follows:<sup>5</sup> (i) The on-site Coulomb interactions  $U_d$  and  $U_p$ . They are of the atomic character and assumed in-

dependent of the neighbors. (ii) The hopping interactions  $T_{pd}$  and  $T_{pp}$ . They measure the overlap of the wave functions and are assumed to depend only on the bond length between two atoms. (iii) The level separation  $\Delta$  between the  $3d_{x^2-y^2}$  and  $2p_\sigma$  orbitals. This parameter has a long-range nature as discussed below, in contrast to the parameters in (i) and (ii). We also examine the contributions of the Cu  $3d_{3z^2-r^2}$  and apex-oxygen  $2p_z$  orbitals.

First, we specify the level separation  $\Delta$  and demonstrate its unique variation. Considering that the system in the insulating state exhibits no metallic screening, we take the ionic model and write<sup>6</sup>

$$\Delta = \Delta V_M / \epsilon(\infty) + \Delta_0, \quad (1)$$

where  $\Delta V_M$  is the difference in the Madelung site potentials ( $V_i$  at the  $i$  site) for a hole between the copper and in-plane oxygen: i.e.,  $\Delta V_M = V_O - V_{Cu}$ .<sup>7</sup> The dielectric constant  $\epsilon(\infty)$  represents the nonlocal charge relaxation at infinite frequencies due to core polarization of the background ions.  $\Delta_0$  involves the second electron affinity of the  $O^{2-}$  ion and second ionization energy of the  $Cu^{2+}$  ion, representing the atomic limit of the level separation.  $\Delta V_M$  turns the large negative  $\Delta_0$  into the small level separation  $\Delta$  of positive sign. This treatment of  $\Delta$  is our basic assumption.  $\Delta$  coincides with  $E_g$  if there are no hopping interactions. We calculate  $\Delta V_M$  for a variety of compounds by the Ewald method using the reported crystal structures.<sup>8</sup> The formal ionic charge is assigned to each ion. Note that  $\Delta V_M$  does not scale with the inverse of the lattice constant among the compounds since it directly reflects the out-of-plane structure (as well as the volume) of the crystal. This is the case even within the  $T'$ -phase compounds because by different rare-earth substitutions the lattice is about twice as easy to expand or contract along the  $c$  axis as along the  $a$  and  $b$  axes. A characteristic material dependence thus comes out in  $\Delta V_M$ . In Fig. 1, we examine the correlation between the calculated  $\Delta V_M$  and observed  $E_g$  and find that  $E_g$  scales predominantly with  $\Delta V_M$ . This result suggests that the Madelung potential plays an important role in the CT gap energy of the cuprates. Actually, the variation of  $\Delta$  appearing via Eq. (1) offers,<sup>9</sup> as we will show, the origin

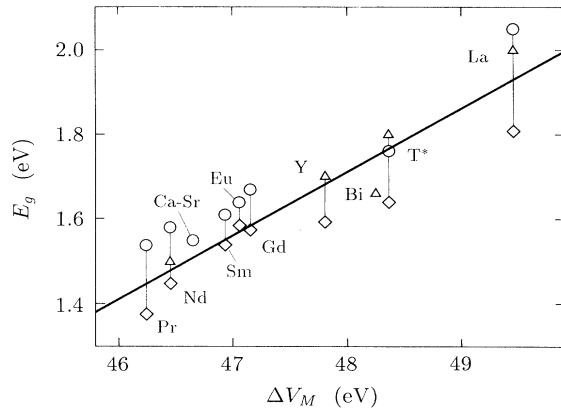


FIG. 1. The correlation between the observed  $E_g$  and the calculated  $\Delta V_M$ . Collected are the data for  $T'$  phase  $L_2\text{CuO}_4$  ( $L=\text{Pr, Nd, Sm, Eu, and Gd}$ ),  $T$  phase  $\text{La}_2\text{CuO}_4$ ,  $T^*$  phase  $(\text{La-Ga-Sr})_2\text{CuO}_4$  and  $(\text{La-Tb-Sr})_2\text{CuO}_4$  (where the same crystal structure in Ref. 8 is assumed for the calculation of  $\Delta V_M$ ),  $(\text{Ca-Sr})\text{CuO}_2$ ,  $\text{Bi}_2\text{Sr}_2\text{YCu}_2\text{O}_8$ , and  $\text{YBa}_2\text{Cu}_3\text{O}_6$ .  $\diamond$  after Ref. 2;  $\circ$  after Ref. 3; and  $\triangle$  after Tokura *et al.*, Tajima *et al.*, and Terasaki *et al.* of Ref. 15. The solid line is a guide to the eye.

of the material dependence of  $E_g$  of this class of compounds.

We now explore the calculation including the hopping interactions. A quantitative treatment is essential because the detailed changes of the parameters contribute to the material dependence of  $E_g$  and  $J$ . We follow the prescription proposed by Tohyama and Maekawa,<sup>10</sup> who have calculated the doping effects on the optical conductivity of high- $T_c$  materials. The real part of the optical conductivity,  $\sigma(\omega)$ ,<sup>11</sup> is given by the Kubo formula as

$$\sigma(\omega) = -\frac{1}{\omega} \text{Im} \langle 0 | j_x^\dagger \frac{1}{\omega + i0 + E_0 - H} j_x | 0 \rangle, \quad (2)$$

where  $|0\rangle$  is the ground state with energy  $E_0$  of the Hamiltonian  $H$  and  $j_x$  is the  $x$  component of the current operator  $\mathbf{j}$ . We take the cluster-model approach. The current operator is then defined as

$$\mathbf{j} = ied_{\text{Cu-O}} \left( T_{pd} \sum_{mls} \mathbf{u}_{ml} (p_{ls}^\dagger d_{ms} - d_{ms}^\dagger p_{ls}) + \sqrt{2} T_{pp} \sum_{l's} \mathbf{v}_{l'l} p_{ls}^\dagger p_{l's} \right), \quad (3)$$

where  $e$  is the electron charge,  $d_{\text{Cu-O}}$  is the Cu-O bond length,  $\mathbf{u}_{ml}$  and  $\mathbf{v}_{l'l}$  are, respectively, the unit vectors directed from the  $m$  and  $l'$  sites to the  $l$  site, and  $p_{ls}^\dagger$  ( $p_{ls}$ ) and  $d_{ms}^\dagger$  ( $d_{ms}$ ) are the creation (annihilation) operators for the O  $2p_\sigma$  and Cu  $3d_{x^2-y^2}$  orbitals with spin  $s$ , respectively. The  $m$ ,  $l$ , and  $l'$  summations are over the orbitals on the neighboring sites. We calculate  $\sigma(\omega)$  by the continued-fraction expansion of Eq. (2) via the Lanczos algorithm.<sup>12</sup> The ground state is evaluated by the modified Lanczos method.<sup>12</sup> The CT gap energy  $E_g$

is obtained as the energy of the lowest peak of  $\sigma(\omega)$ . The  $\text{Cu}_4\text{O}_{13}$  cluster with the free boundary condition is employed, where four  $\text{CuO}_4$  clusters are linked linearly in the  $x$  direction by sharing the corner oxygens. This cluster is sufficient in size to examine the material dependence of  $E_g$ . The superexchange interaction  $J$  is calculated as the difference between the lowest energies of the singlet and triplet spin states of the coupled  $\text{Cu}_2\text{O}_7$  cluster.<sup>13</sup> The clusters contain one hole per Cu ion to simulate the insulating state. We choose the values of the parameters as follows:  $U_d=8.5$  eV and  $U_p=4.1$  eV.  $T_{pd}=0.873$  eV and  $T_{pp}=0.366$  eV for  $\text{Sm}_2\text{CuO}_4$  and are assumed to have the standard bond-length ( $d$ ) dependence of  $d^{-4}$  and  $d^{-3}$ , respectively, for the other compounds.  $\Delta$  is given by Eq. (1). Values of  $\epsilon(\infty)$  and  $\Delta_0$  are chosen to give an overall agreement with the observed values of  $E_g$ ;  $\epsilon(\infty)=3.5$  and  $\Delta_0=-10.88$  eV, independent of the compound,<sup>14</sup> which are consistent with the work in Ref. 6. Note that the material dependence of  $\Delta$  then comes only through  $\Delta V_M$ .

The contribution from the hopping interactions through the apex-oxygen  $2p_z$  and Cu  $3d_{3z^2-r^2}$  orbitals is examined first of all. We expect that their contributions to  $E_g$  and  $J$  are small because the symmetry of the undoped ground state is orthogonal to the symmetry of these orbitals in the  $\text{CuO}_5$  cluster. Calculations in the extended clusters actually show that the presence or absence of these orbitals changes  $E_g$  and  $J$  by less than 3% and has a negligible effect. This result leads to a statement that the contribution to  $E_g$  and  $J$  from the three dimensionality (or out-of-plane structure) of the crystal enters solely through  $\Delta$ .

Calculated results for  $E_g$  and  $J$  are compared with the experimental results<sup>2,3,15</sup> in Figs. 2 and 3 for a variety of insulating cuprates. A good agreement is obtained. Noted in particular is a comparatively small (large) value of  $E_g$  ( $J$ ) in  $(\text{Ca-Sr})\text{CuO}_2$  which has the smallest  $d_{\text{Cu-O}}$  among the compounds with no apex oxygens. The material dependence of  $E_g$  and  $J$  may be further examined by a decomposition into three contributions,  $\Delta$ ,  $T_{pd}$ , and  $T_{pp}$ , as shown in Fig. 4, where each of the parameters is varied independently from the  $\text{Sm}_2\text{CuO}_4$  reference. We note that the effects of  $T_{pd}$  and  $T_{pp}$  on the material dependence of  $E_g$  are one order of magnitude smaller than the effect of  $\Delta$ .<sup>16</sup> The effect of the level repulsion due to band hybridization is mostly canceled out by the  $d$ -band-width effect due to the indirect  $d$ - $d$  hopping. The effect of oxygen bandwidth is also very small. We thus conclude that the material dependence of  $E_g$  is caused only by  $\Delta$ . Remember that  $\Delta$ , as given by Eq. (1), reflects the three dimensionality of the crystal, namely, its volume and out-of-plane structure, via the long-range Madelung potential. The short-range contributions specified by the bond lengths are of little significance. The pressure coefficient of  $E_g$ ,  $dE_g/dP$ , may then be positive in cuprates (due to the negative

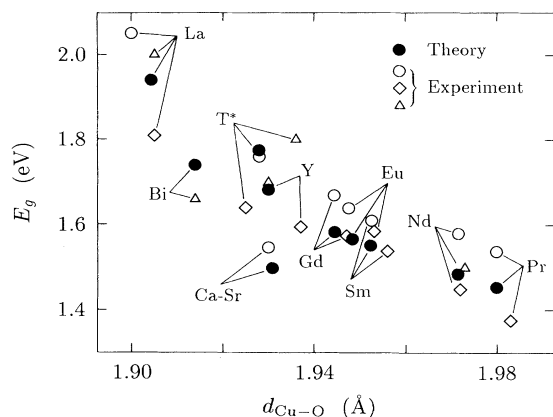


FIG. 2. The calculated CT gap energy  $E_g$  compared with the data of the optical conductivity measurements.  $\diamond$  after Ref. 2;  $\circ$  after Ref. 3; and  $\Delta$  after Tokura *et al.*, Tajima *et al.*, and Terasaki *et al.* of Ref. 15.

volume derivative of  $\Delta$ ), which is in contrast to the MH insulators where the pressure-induced metal-insulator transition has been observed.<sup>17</sup> This contrast has not yet been well recognized, but it is not surprising that the positive pressure coefficient of the energy gap may be a common feature of the typical CT insulators. Experimental studies on this point are desirable. For the superexchange interaction  $J$ , the most important effect is that the strong positive contributions of  $T_{pd}$  and  $T_{pp}$  to the bond-length dependence of  $J$  are canceled largely by the negative contribution of the above-mentioned  $\Delta$  (see Fig. 4), to give a small material dependence of  $J$ . A recent magnon Raman scattering measurement under pressure for  $\text{La}_2\text{CuO}_4$  (Ref. 18) supports this result. The MH insulators, in contrast, exhibit a strong bond-length dependence of  $J$  due to the  $d$ - $d$  hopping contribution.<sup>19</sup> Thus, the small material dependence of  $J$  may also be characteristic of the CT insulators. We have assumed the standard bond-length dependence of the hopping interactions throughout the work; the possibility of an anomalously large or small bond-length dependence is ruled out because such a dependence does not explain the behaviors of  $E_g$  and  $J$  on the same footing.

We see that among transition-metal oxides cuprates are unique in the sense that the Madelung potential plays an explicit role in their electronic structure via the level separation  $\Delta$ , and its importance was recognized first after the intensive examination of the high- $T_c$  cuprates. The highest phonon frequency of the measured optical-phonon spectra assigned as the Cu-O-bond-stretching mode has been reported<sup>20</sup> to show an anomalously strong bond-length dependence, whereby its strong interaction with the electronic system has been suggested. We believe that this anomaly is also due to the above-mentioned uniqueness of the cuprates where the virtual CT excitation with energy characterized by  $\Delta$  can

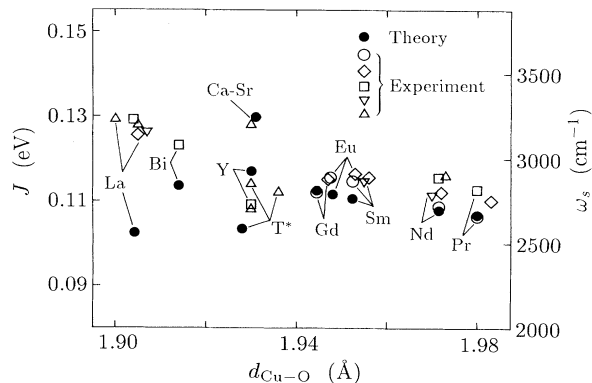


FIG. 3. The calculated superexchange interaction  $J$  compared with the two-magnon Raman peak position  $\omega_s$ . Two scales on the ordinate are given by assuming an approximate relation  $\hbar\omega_s = 3.1J$  (see Ref. 21).  $\diamond$  after Ref. 2;  $\circ$  after Ref. 3;  $\square$  after Sugai *et al.* of Ref. 15;  $\nabla$  after Sulewski *et al.* of Ref. 15; and  $\Delta$  after Tokura *et al.* and Tajima *et al.* of Ref. 15.

couple to the phonons of the bond-stretching mode. Since our combined ionic and cluster model is applicable to the intermediate and other classes<sup>1</sup> of compounds, a future accumulation of experimental data will enable us to clarify what are the crucial parameters for their electronic structure.

In conclusion, we have analyzed the material dependence of  $E_g$  and  $J$  for a variety of the insulating cuprates quantitatively via the combined ionic- and cluster-model approach, and have shown that their electronic structure is predominantly characterized by the unique level separation  $\Delta$  that reflects the three dimensionality of the crystal through the long-range Madelung potential. Such characteristics become apparent in the CT insula-

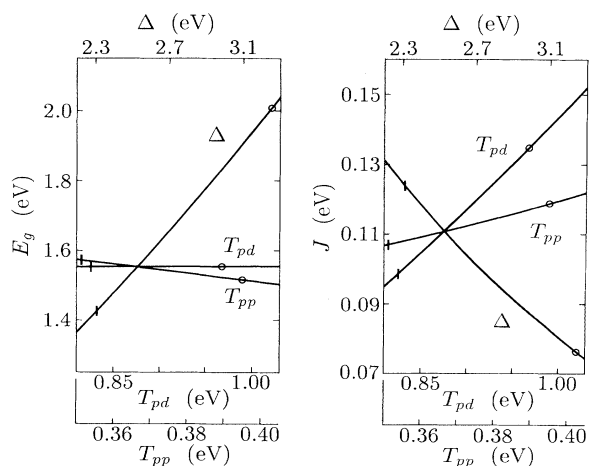


FIG. 4. The contributions to  $E_g$  (left-hand panel) and  $J$  (right-hand panel). The crossing point of the curves corresponds to  $\text{Sm}_2\text{CuO}_4$ , and circles and bars on the curves indicate the parameter values for  $\text{La}_2\text{CuO}_4$  and  $\text{Pr}_2\text{CuO}_4$ , respectively.

tors but are absent in the MH insulators.

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<sup>4</sup>See, for example, *Strong Correlations and Superconductivity*, edited by H. Fukuyama, S. Maekawa, and A. P. Malozemoff, Springer Series in Physics Vol. 89 (Springer-Verlag, Heidelberg, 1989).

<sup>5</sup>The subscripts  $d$  and  $p$  of  $U$  and  $T$  stand for the  $3d_{x^2-y^2}$  and  $2p_\sigma$  orbitals, respectively.

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<sup>7</sup>J. B. Torrance and R. M. Metzger, Phys. Rev. Lett. **63**, 1515 (1989).

<sup>8</sup>The crystal structures used for calculation are summarized in Ref. 6, except the  $T'$ -phase structure where the data of F. Izumi, Y. Matsui, H. Takagi, S. Uchida, Y. Tokura, and H. Asano [Physica (Amsterdam) **158C**, 433 (1989)] are used with a modification for the different rare-earth compounds. We use the lattice constants measured by H. Okada, M. Takano, and Y. Takeda [Physica (Amsterdam) **166C**, 111 (1990)] and Y. Tokura *et al.* (to be published).

<sup>9</sup>Equation (1) produces a strong material dependence of  $\Delta$  from  $\Delta V_M$ . These dependences, if they could be regarded as functions only of the bond length  $d$ , would roughly be  $\Delta V_M \propto d^{-1-d^{-2}}$  and  $\Delta \propto d^{-7-d^{-8}}$ .

<sup>10</sup>T. Tohyama and S. Maekawa, J. Phys. Soc. Jpn. **60**, 53 (1991).

<sup>11</sup> $\sigma(\omega)$  stands for the  $xx$  component of the optical conductivity tensor. The  $x$  direction will be taken along the -Cu-O-Cu-bond.

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<sup>14</sup> $\epsilon(\infty)$  is assumed material independent, which may be justified by the dielectric similarity of the cuprates; oxochloride and oxobromide are then not included in this work.

<sup>15</sup>P. E. Sulewski, P. A. Fleury, K. B. Lyons, S.-W. Cheong, and Z. Fisk, Phys. Rev. B **41**, 225 (1990); Y. Tokura, S. Koshihara, T. Arima, H. Takagi, S. Ishibashi, T. Ido, and S. Uchida, Phys. Rev. B **41**, 11657 (1990); S. Sugai, M. Sato, T. Kobayashi, J. Akimitsu, T. Ito, H. Takagi, S. Uchida, S. Hosoya, T. Kajitani, and T. Fukuda, Phys. Rev. B **42**, 1045 (1990); S. Tajima, S. Uchida, S. Ishibashi, T. Ido, H. Takagi, T. Arima, and Y. Tokura, Physica (Amsterdam) **168C**, 117 (1990); I. Terasaki, T. Nakahashi, S. Takebayashi, A. Maeda, and K. Uchinokura, Physica (Amsterdam) **165C**, 152 (1990).

<sup>16</sup> $E_g$  itself involves the contribution from the hopping interactions, which may be seen by comparing, e.g.,  $E_g = 1.55$  eV with  $\Delta = 2.53$  eV of  $\text{Sm}_2\text{CuO}_4$ .

<sup>17</sup>See, for example, D. B. McWhan, A. Menth, J. P. Remeika, W. F. Brinkman, and T. M. Rice, Phys. Rev. B **7**, 1920 (1973).

<sup>18</sup>M. C. Aronson, S. B. Dierker, B. S. Dennis, S.-W. Cheong, and Z. Fisk (unpublished).

<sup>19</sup>See, for example, L. J. de Jongh and R. Block, Physica (Amsterdam) **79B**, 568 (1975).

<sup>20</sup>S. Tajima, T. Ido, S. Ishibashi, T. Itoh, E. Eisaki, Y. Mizuo, T. Arima, H. Takagi, and S. Uchida, Phys. Rev. B (to be published).

<sup>21</sup>For a quantitative discussion on obtaining an accurate value of  $J$ , see R. R. P. Singh, P. A. Fleury, K. B. Lyons, and P. E. Sulewski [Phys. Rev. Lett. **62**, 2736 (1989)].