Collective Excitations in the Ginzburg-Landau Theory of the Fractional Quantum Hall Effect

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The collective excitations of the fractional-quantum-Hall liquid are studied within the Ginzburg-Landau theory. We show that (1) Gaussian fluctuations of the phase of the order parameter correspond to the cyclotron mode with an energy gap of $\hbar\omega_c$ at $\vec{q}=0$ and a contribution to the static structure factor proportional to $|\vec{q}|^2$ as $\vec{q}\to 0$, in accordance with Kohn's theorem, and (2) vortex-antivortex fluctuations give rise to the lowest-Landau-level collective mode with an energy gap that depends only on the Coulomb energy and a static structure factor that vanishes as $|\vec{q}|^4$ as $\vec{q}\to 0$.

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In their seminal works, Girvin and MacDonald[1] and Read[2] discovered a hidden order parameter in the Laughlin state.[3] They have shown that if one views an electron as a hardcore boson with an odd-integer number of Dirac flux quanta attached, the equal-time boson-boson correlation function acquires long-range behavior in the Laughlin state. Subsequently, Zhang, Hansson, and Kivelson[4] and Read[2] constructed a Ginzburg-Landau (GL) theory for the fractional quantum Hall effect (FQHE). It was shown that all the essential features of the FQHE can be derived from the GL theory by looking at the saddle-point solution and the Gaussian fluctuation around it. These successes certainly make the GL approach extremely appealing. However, the GL theory is also faced with some serious problems (in the following we shall focus our discussion on Ref. 4): (1) The GL theory predicts the wrong gap for collective excitations at long wavelength; (2) the static structure factor computed in this theory vanishes like $|\vec{q}|^2$, not like $|\vec{q}|^4$ as obtained from a variational approach;[5] and (3) it fails to explain the existence of the "roton minimum" in the dispersion of the collective excitations.

In general, there are two types of collective excitations at fractional filling factors: One is the cyclotron-resonance mode of the center of mass, which can be viewed as an inter-Landau-level particle-hole excitation, and the other is the intra-Landau-level excitations. On the one hand, by invoking Galilean invariance, Kohn's theorem[6] shows that the frequency of the first mode is exactly $\hbar\omega_c$ at $\vec{q}=0$ and has an oscillator strength in the static structure factor that vanishes as $|\vec{q}|^2$ in the long-wavelength limit. On the other hand, the frequency of the second mode is solely determined by the Coulomb interaction, and at the magic filling factors of the FQHE it gives rise to a correction to the static structure factor proportional to $|\vec{q}|^4$ in the long-wavelength limit. Following Girvin, MacDonald, and Platzman[5] (GMP), we shall call the intra-Landau-level collective mode the single-mode-approximation (SMA) mode. As shown by GMP, the dispersion of the SMA mode displays a magnetoroton minimum at a wavelength roughly equal to the interparticle distance.

In this Letter, we reexamine the problem associated with the collective excitations in the GL theory. Here we summarize our main results as follows. Following Ref. 4, we transform the original fermionic problem into a bosonic problem by introducing a statistical gauge field that has a Chern-Simons action. We then perform a duality transformation,[7] and arrive at an action containing both the Gaussian phase and the vortex degrees of freedom. We first show that the Gaussian-phase fluctuation at $\vec{q}=0$ corresponds to the center-of-mass cyclotron mode with an energy gap of $\hbar\omega_c$. At small $|\vec{q}|$, its contribution to the static structure factor vanishes as $|\vec{q}|^2$. We then integrate out the Gaussian degrees of freedom and obtain an effective action for the vortices. This effective action describes anyonlike vortices that interact via the Coulomb interaction and obey the guiding-center equations of motion. We then quantize this anyon problem and show that both the net vorticity and the total dipole moment of the vortices are conserved. These conservation laws state that only quadrupole (and higher) moment fluctuations can contribute to the long-wavelength dynamical structure factor $S(\vec{q},\omega)$, which leads to a static structure factor proportional to $|\vec{q}|^4$ as $\vec{q}\to 0$. We also show that the creation energy Δ of the vortices is determined purely by the Coulomb interaction, and predict the SMA dispersion relation for large $|\vec{q}|$. We also present a physical picture that explains the existence of the magnetoroton minimum.

We start with the GL Lagrangian (in units in which $c=\hbar=e=1$) written in Euclidean space-time:[4]

$$\mathcal{L}=i\bar{\Psi}\left(\frac{\partial_0}{i}+a_0-A_0\right)\Psi+\frac{1}{2m}\left|\left(\frac{\vec{\partial}}{i}+\vec{a}-\vec{A}\right)\Psi\right|^2+\tfrac{1}{2}(\bar{\Psi}\Psi-\bar{\rho})V(\bar{\Psi}\Psi-\bar{\rho})-\frac{i}{2}\sigma_{xy}\mathbf{a}\cdot\nabla\times\mathbf{a}\,, \tag{1}$$

where $\sigma_{xy}=1/k$ with k being an odd integer, and $\vec{A}=(H_0/2)(x\hat{y}-y\hat{x})$ with H_0 being the external magnetic field. We have also defined $\mathbf{A}$ and $\vec{A}$ as a three-vector and a two-vector, respectively, $\bar{\rho}$ as the average particle density, V as the two-body interaction, and

$$(\bar{\Psi}\Psi-\bar{\rho})V(\bar{\Psi}\Psi-\bar{\rho})\equiv\int d^2r'[\bar{\Psi}(\vec{r})\Psi(\vec{r})-\bar{\rho}]V(\vec{r}-\vec{r}')[\bar{\Psi}(\vec{r}')\Psi(\vec{r}')-\rho].$$

The last term in (1) is the so-called Chern-Simons term; its effect is to attach fluxes in $\vec{a}$ to particles. We then separate the modulus and the phase degrees of freedom by writing $\Psi=\rho^{1/2}\phi$, where $\phi\equiv e^{i\theta}$ is a unimodular field. Substituting this back into (1) and performing a Hubbard-Stratonovich decoupling of the kinetic-energy term we obtain

$$\mathcal{L}=i\mathbf{J}\cdot\left[\bar{\phi}\frac{\mathbf{\nabla}}{i}\phi+\mathbf{a}-\mathbf{A}\right]+\frac{m}{2\rho}|\vec{J}|^2+\frac{1}{2m}|\vec{\partial}\rho^{1/2}|^2+\tfrac{1}{2}(\rho-\bar{\rho})V(\rho-\bar{\rho})-\frac{i}{2}\sigma_{xy}\mathbf{a}\cdot\mathbf{\nabla}\times\mathbf{a}\,, \tag{2}$$

where $\vec{J}$ is the auxiliary field, and $\mathbf{J}\equiv(\rho,\vec{J})$. Here we note that by differentiating (2) with respect to $\mathbf{A}$ we obtain $\mathbf{J}$ as the physical three-current. To calculate the partition function, one has to perform path integrals over $\mathbf{J}$, ϕ, and $\bar{\phi}$.

We split θ into the topologically trivial and nontrivial parts via $\bar{\phi}(\mathbf{\nabla}/i)\phi=\mathbf{\nabla}\theta_g+\bar{\phi}_v(\mathbf{\nabla}/i)\phi_v$, where θ_g is the topologically trivial Gaussian phase and ϕ_v contains the vortex configurations. Of course, in order to have a well-defined Ψ field, ρ should vanish inside the cores of the vortices. We then integrate over θ_g which produces a constraint on $\mathbf{J}$, namely, $\mathbf{\nabla}\cdot\mathbf{J}=0$. We explicitly satisfy this constraint by writing $\mathbf{J}=\mathbf{\nabla}\times\mathbf{b}$ (where $\mathbf{b}$ is an unconstrained field). Substituting this back into (2) and integrating out $\mathbf{a}$ we obtain

$$\mathcal{L}=\frac{m}{2(\mathbf{\nabla}\times\mathbf{b})_0}|(\mathbf{\nabla}\times\mathbf{b})_\perp|^2+\frac{1}{2m}|\vec{\partial}(\mathbf{\nabla}\times\mathbf{b})_0^{1/2}|^2+\tfrac{1}{2}[(\mathbf{\nabla}\times\mathbf{b})_0-\bar{\rho}]V[(\mathbf{\nabla}\times\mathbf{b})_0-\bar{\rho}]+i\mathbf{b}\cdot(\mathbf{J}_v-\mathbf{\nabla}\times\mathbf{A})+\frac{i}{2\sigma_{xy}}\mathbf{b}\cdot\mathbf{\nabla}\times\mathbf{b}\,. \tag{3}$$

Here $\mathbf{J}_v\equiv\bar{\phi}_v(\mathbf{\nabla}/i)\phi_v$ is the vortex three-current and $(\mathbf{\nabla}\times\mathbf{b})_0$ and $(\mathbf{\nabla}\times\mathbf{b})_\perp$ denote the time and space components of the three-vector $\mathbf{\nabla}\times\mathbf{b}$, respectively. Two observations should be made here: (1) $\mathbf{\nabla}\cdot\mathbf{J}_v=0$ and (2) since the winding of ϕ is always an integer, $\mathbf{J}_v$ describes the three-current of integer-quantized point particles. These two facts enable us to write $\rho_v=\sum_i q_i\delta(\vec{r}-\vec{r}_i)$ and $\vec{J}_v=\sum_i q_i\dot{\vec{r}}_i\delta(\vec{r}-\vec{r}_i)$, where q_i is the integer vorticity.

Because of the fact that $\vec{J}=(\mathbf{\nabla}\times\mathbf{b})_\perp=\vec{J}_l(\equiv\hat{z}\times\dot{\vec{b}})+\vec{J}_t(\equiv-\hat{z}\times\vec{\partial}b_0)$ (the first term is the longitudinal current and the second term is the transverse current), it is particularly convenient to work in the Coulomb gauge ($\vec{\partial}\cdot\vec{b}=0$) where $|\vec{J}|^2=|J_l|^2+|J_t|^2$. By integrating out b_0 we obtain

$$\begin{aligned}\mathcal{L}=&\frac{m}{2(\vec{\partial}\times\vec{b})}|\delta\dot{\vec{b}}|^2+\frac{\vec{\partial}\times\vec{b}}{2m}\left|\vec{\partial}G\left[\rho_v+\frac{1}{\sigma_{xy}}\vec{\partial}\times\delta\dot{\vec{b}}\right]\right|^2+\frac{1}{2m}|\vec{\partial}(\vec{\partial}\times\vec{b})^{1/2}|^2\\&+\frac{1}{2}(\vec{\partial}\times\delta\vec{b})V(\vec{\partial}\times\delta\vec{b})+\frac{i}{2\sigma_{xy}}\delta\vec{b}\times\delta\dot{\vec{b}}+i\vec{J}_v\cdot\vec{b}-iA_0(\vec{\partial}\times\vec{b})\,,\end{aligned} \tag{4}$$

where $G\equiv1/|\vec{\partial}|^2$, and we have defined $\vec{b}\equiv\delta\vec{b}+\langle\vec{b}\rangle$ with $\langle\vec{b}\rangle=\frac{1}{2}\bar{\rho}(x\hat{y}-y\hat{x})$.

We first set $\mathbf{J}_v,A_0=0$ and study the Gaussian fluctuations of $\vec{b}$. The physical meaning of $\delta\vec{b}$ becomes clear if we define the displacement field $\vec{u}\equiv\bar{\rho}\hat{z}\times\delta\vec{b}$. Substituting this expression into (4) and linearizing the result at long wavelength we obtain

$$\mathcal{L}=\frac{m}{2}\bar{\rho}|\dot{\vec{u}}|^2=\frac{m}{2}\bar{\rho}\omega_c^2(\vec{\partial}\cdot\vec{u})G(\vec{\partial}\cdot\vec{u})+\frac{\bar{\rho}^2}{2}(\vec{\partial}\cdot\vec{u})V(\vec{\partial}\cdot\vec{u})+\frac{i}{2}\frac{\bar{\rho}^2}{\sigma_{xy}}\vec{u}\times\dot{\vec{u}}. \tag{5}$$

Here $\omega_c\equiv\bar{\rho}/m\sigma_{xy}$ is the cyclotron frequency, and the Coulomb-gauge constraint becomes $\vec{\partial}\times\vec{u}=0$, i.e., $\vec{u}$ describes purely longitudinal displacements. Let us first consider the uniform displacement. In that case, $\vec{\partial}\times\vec{u}=0$ is automatically satisfied; hence there is no further constraint imposed on the uniform displacement. The effective Lagrangian for that case describes the motion of the center of mass and is given by $\mathcal{L}=(M/2)|\dot{\vec{u}}|^2+(i/2)M\omega_c\vec{u}\times\dot{\vec{u}}$, where $M\equiv m\int d^2r\bar{\rho}$ is the total inertial mass of the fluid. This is precisely the Euclidean space-time Lagrangian for a particle of mass M, coordinate $\vec{u}$, moving in an external magnetic field $H=M\omega_c$. Such a Lagrangian implies cyclotron motion of the center of mass with the cyclotron frequency equal to ω_c.

At nonzero wave vector, the term proportional to $\vec{u}\times\dot{\vec{u}}$ in (5) is ineffective due to the constraint $\vec{\partial}\times\vec{u}=0$. The remaining effective Lagrangian gives normal-mode dispersion $\omega_q^2\equiv\omega_c^2+(\bar{\rho}/m)|\vec{q}|^2V(\vec{q})$. Hence for a short-range two-body interaction V, the normal-mode frequency starts out from $\omega=\omega_c$ and disperses quadratically, whereas if $V(r)\propto1/r$ it disperses linearly, in agreement with the random-phase-approximation calculation by Kallin and Halperin.[8] Next we calculate the dynamical structure factor $S(\vec{q},\omega)$ due to this collective mode. To do that we turn A_0 back on and integrate out the displacement field $\vec{u}$. In the resulting action, $\frac{1}{2}S(\vec{q},\omega)$ is the coefficient of the A_0^2 term. Explicit calculation of

$S(\vec{q},\omega)$ gives (after Wick's rotation) $S(\vec{q},\omega)=\bar{\rho}|\vec{q}|^2/m(\omega^2-\omega_q^2)$, which exhibits poles at $\omega=\omega_q$ and has an oscillator strength that vanishes like $|\vec{q}|^2$ as $\vec{q}\to 0$, in accordance with Kohn's theorem. This mode has been mistaken in the literature[4,7] as the lowest-Landau-level collective mode. This misidentification is corrected by our results.

Now we restore the vortex degrees of freedom and study the effects of vortex-antivortex fluctuation on the dynamical structure factor. By assuming that the vortex motion is much slower than the cyclotron frequency, we integrate out $\delta\vec{b}$ in Eq. (4) to obtain

$$\mathcal{L}=-iA_0\bar{\rho}+\tfrac{1}{2}A_0SA_0+i\sigma_{xy}\sum_j q_jA_0(\vec{r}_j)+\sum_j\Delta(q_j)+\tfrac{1}{2}\sigma_{xy}^2\sum_{i\neq j}q_iq_jV(\vec{r}_i-\vec{r}_j)$$
$$+\frac{i}{2}\bar{\rho}\sum_j\vec{r}_j\times\dot{\vec{r}}_j-i\sigma_{xy}\sum_{i\neq j}q_iq_j\dot{\vec{r}}_i\cdot(\hat{z}\times\vec{\partial}_i)G(\vec{r}_i-\vec{r}_j)\,, \tag{6}$$

where $\Delta(q_j)$, the quasiparticle (a quasiparticle is a vortex plus a screening cloud produced by $\vec{\partial}\times\delta\vec{b}$) creation energy, is given by $\Delta(q)\equiv\frac{1}{2}\int d^2r_1d^2r_2V(\vec{r}_{12})g_q(\vec{r}_1,\vec{r}_2)$, in which $g_q(\vec{r}_1,\vec{r}_2)\equiv\langle\vec{\partial}\times\delta\vec{b}(\vec{r}_1)\vec{\partial}\times\delta\vec{b}(\vec{r}_2)\rangle$ is the pair-distribution function in the presence of a vortex with vorticity q situated at the origin. In this work knowing that $\Delta(q)$ is determined solely by the interaction energy is sufficient.

Now we briefly outline the steps leading from (4) to (6). We first consider the case of a single static vortex. Because of the nonlinearity of Eq. (4) and the constraint $\vec{\partial}\times\vec{b}\geq 0$, brute-force integration over $\delta\vec{b}$ is formidable. What we have done instead are the following: (1) recognize that Eq. (4) is the action for a problem of a long-range interacting boson in the presence of an impurity; (2) write down the corresponding Hamiltonian; (3) show[9] that the modulus of the Laughlin quasihole wave function is the ground state of $H_{1/m}$, the part of Hamiltonian that is proportional to $1/m$, and the associated eigenenergy is $N\hbar\omega_c/2$, where N is the total number of particles; and (4) compute $\Delta(+1)$ and show that the total induced charge in the vicinity of the vortex is $+\sigma_{xy}$. The situation is more complicated with an antivortex. This is because in order to diagonalize $H_{1/m}$ and maintain the eigenenergy as $N\hbar\omega_c/2$ a localized vortex-antivortex cloud is induced. However, in regions of space far away from the antivortex, ρ_v is well approximated by a δ function and the asymptotic form of the wave function can be written down easily. Fortunately, knowing the asymptotic wave function is sufficient to show that the total induced charge is $-\sigma_{xy}$. Finally, (5) by considering far separated multivortex configurations and allowing the vortices to move adiabatically, we can show that Eq. (6) yields the action for the path integral for the vortices.

The last term in Eq. (6) describes the Berry phase that vortices experience when they move around each other; hence it describes quasiparticles carrying a fraction (σ_{xy}) of charge with σ_{xy} statistics moving in external magnetic field $\bar{\rho}$, i.e., Laughlin quasiparticles. The goal now is to integrate out the vortex degrees of freedom to obtain the correction to the dynamical structure factor due to the quasiparticles. To do that, it is most convenient to write down the corresponding quantum-mechanical description of the vortices.[10] Equation (6) can also be viewed as the coherent-state path-integral representation of the following quantum Hamiltonian:

$$\mathcal{H}=\tfrac{1}{2}\sigma_{xy}^2\sum_{i\neq j}q_iq_jV'(x_i,p_i)\,, \tag{7}$$

with the commutation relation $[x_i,p_j]=i\delta_{ij}$. Here V' is a normal-ordered operator (normal ordered according to the creation and destruction operators constructed from x and p) such that

$$V'(x_i,p_i)|_{p_i}=V(x_i-x_j,y_i-y_j)\,, \tag{8}$$

where evaluation is at

$$p_i=\frac{\bar{\rho}}{2}q_jy_j-\sigma_{xy}\sum_{k\neq j}q_jq_k\frac{\partial}{\partial y_j}G(\vec{r}_{jk})\,.$$

Although the relation between p_j and y_j is complicated, the Heisenberg equation of motion for $\vec{r}_i$ is simple and is given by $\bar{\rho}\dot{\vec{r}}_j=\sigma_{xy}^2(\hat{z}\times\vec{\partial}_j)\sum_{k\neq j}q_kV(\vec{r}_{jk})$, the same as the Euler-Lagrangian equation derived from (6). This equation of motion enables a direct analysis of the behavior of the structure factor in the $\vec{q}\to 0$ limit. For small $|\vec{q}|$, the vortex density operator $\rho_v(\vec{q})=\sum_jq_je^{i\vec{q}\cdot\vec{r}_j}$ can be expanded in powers of $\vec{q}$. The density-density correlation function after this expansion is given by

$$\langle\rho_v(\vec{q},t)\rho_v(-\vec{q},0)\rangle=\langle Q(t)Q(0)\rangle+\tfrac{1}{3}|\vec{q}|^2\langle\vec{D}(t)\cdot\vec{D}(0)\rangle+\sum_{\alpha,\beta}\frac{(q_\alpha q_\beta)^2}{4}\langle Q^{\alpha,\beta}(t)Q^{\alpha,\beta}(0)\rangle+\cdots\,, \tag{9}$$

where $Q\equiv\sum_iq_i$ is the net vorticity, $\vec{D}\equiv\sum_iq_i\vec{r}_i$ is the total dipole moment, and $Q^{\alpha,\beta}\equiv\sum_iq_ir_i^\alpha r_i^\beta$ is the total quadrupole moment ($\alpha,\beta=x,y$ refer to the space indices). Since the Heisenberg equations of motion explicitly conserve both Q and $\vec{D}$, the first two contributions in (9) vanish identically. This result establishes the fact that the vortex contributions to the dynamical structure factor vanish like $|\vec{q}|^4$ in the $\vec{q}\to 0$ limit.

Finally, we should like to address the issue associated with the dispersion of the SMA mode. First we consider the case of large momentum. It is useful to gain some intuition by looking at the behavior predicted by the classical equation of motion. For simplicity, let us consider a vortex-antivortex pair. Since the vortex and antivortex attract each other, their classical behavior is to form a dipole with size R that drifts with a center-of-mass velocity chosen so that the Lorentz force balances the Coulomb force. For such a pair, small quantum fluctuations do not involve processes in which one particle goes around another. (The same thing cannot be said for two vortices of the same sign. In that case the classical behavior involves one particle going around the other, and fluctuation in the interparticle distance results in changes in the winding number.) Therefore to analyze the pair problem, we drop the Berry-phase term in (6), which leads to a simpler commutation relation $[x_j,(\bar{\rho}/2)q_j y_j]=i$. From this commutation relation and Eq. (7) we see that the components of the relative-position vector $\vec{r}\equiv\vec{r}_1-\vec{r}_2$ commute with each other, and their eigenstates $|r_x,r_y\rangle$ are also eigenstates of the Hamiltonian with eigenvalue $-\sigma_{xy}^2 V(r)$. The density operator $\rho_v(\vec{q})\propto e^{i\vec{q}\cdot\vec{R}}$ $[\vec{R}\equiv(\vec{r}_1+\vec{r}_2)/2]$ is off diagonal in this basis set, since $\vec{R}$ and $\vec{r}$ do not commute. In fact, it can be easily checked from the commutation relation that the operator $e^{iq_xR_x}$ connects an eigenstate $|r_x,r_y\rangle$ to the unique eigenstate $|r_x,r_y+l_0^2/\sigma_{xy}q_x\rangle$, where $l_0=1/(2\pi\bar{\rho})^{1/2}$ is the magnetic length. For large q_x, the dipole configuration created by the density operator has an energy given by

$$E(|q_x|)=2\Delta-\sigma_{xy}^2 V\left(\frac{l_0^2}{\sigma_{xy}}|q_x|\right)=2\Delta-\frac{\sigma_{xy}^3 e^2}{|q_x|l_0^2}, \qquad (10)$$

where we have substituted the long-range part of the static Coulomb interaction for V. Equation (10) agrees exactly with the result of Kallin and Halperin.[8] This is the description of the SMA mode in the region where $q_x>2\pi/l_0$.

For $q_x=0$, we believe that the SMA mode consists of two dipoles, each with a size of roughly l_0, oriented in the $\hat{x}$ and $-\hat{x}$ directions, respectively, and with centers of mass separated by roughly l_0. This configuration has a quadrupole moment, but no net dipole moment, in accordance with our previous analysis for $q=0$. As q_x increases, so does the total dipole moment in the $\hat{y}$ direction, and these two dipoles rotate rigidly in opposite senses as two dumbbells around their individual centers of mass, until they eventually lie in the $\hat{y}$ direction and coalesce into a single dipole when $q_x=2\pi/l_0$. In this process, the electrostatic energy monotonically decreases. As q_x further increases, the size of the remaining dipole further increases in the $\hat{y}$ direction and so does the energy. In this way we can explain the existence of the roton minimum. A disclaimer is in order here. Although we believe that ignoring the Berry-phase term should be a good approximation when $q_x\gg 2\pi/l_0$, we are uncertain about its validity for q_x near or less than $2\pi/l_0$. Finally, this picture also explains why the SMA is not a good approximation for $q_x<2\pi/l_0$. This is because of the relative degrees of freedom involved in the quadrupole configurations.

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