Stabilization of Atoms in Superintense Laser Fields: Is It Real?

J. Grochmalicki, M. Lewenstein, and K. Rzążewski
Institute for Theoretical Physics, Polish Academy of Sciences, 02-668 Warsaw, Poland
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We present an argument based on classical mechanics that stabilization of real atoms in superintense laser fields requires substantially higher frequencies than is suggested by the analysis of simplified one-dimensional models.

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In some situations, forced fast oscillations may be employed to turn an otherwise unstable configuration of a system into a stable one. The possible applications of this phenomenon range from stabilization of the inverted pendulum[1] to the famous Paul trap.[2] Recently, it was suggested[3] that an effect of this kind should also appear in high-frequency strong-field ionization. For a high enough laser-field frequency ω_L, its further increase should lead to a substantial suppression of the ionization, even for ultrastrong intensities.

This fascinating result was derived by Gavrila and collaborators in the framework of quantum mechanics, by means of the so-called Kramers-Henneberger transformation[4] (the space-translation method), which transfers the rapid oscillations of the external field to the atomic potential. Then, by averaging over these oscillations, one finds the effective potential describing the atom in the high-frequency laser field. Furthermore, it turns out that the usual ground state of the atom in such a potential is stable against ionization in the limit of $\omega_L \to \infty$.

The above method relies, in principle, on an expansion in the ratio of the bare ground-state energy to the photon energy, $\hbar\omega_0/\hbar\omega_L$. However, the electron in the ground state of the effective potential has the smaller energy $\hbar\omega_{\text{eff}}$. Accordingly, the approximation used in Ref. 4(a) turns out to be based on an expansion in the rescaled parameter $\omega_{\text{eff}}/\omega_L$, and works better the smaller this parameter is. This parameter, on the other hand, is a decreasing function of the laser intensity I for a fixed ω_L. This implies, according to Gavrila and collaborators, that the atom will remain in the ground state after the turn-off of the field, and that the stabilization will become more effective with growing I.

Immediately, the above prediction was severely criticized, since the described approach ignores the effects connected with introducing the atoms into the oscillating field, and assumes the ideal plane-wave form of the field, rather than a pulse form. Also, as pointed out by Lambropoulos,[5] nonperturbative strong-field effects can hardly be observed in ionization by typical laser pulses. Full ionization takes place in the initial growth phase of the pulse, when the field is still in the perturbative regime. No further absorption or emission processes by the released electron are then possible in the developed intense plane-wave field. Obviously, such criticism does not apply if the turn-on time is sufficiently short.

Recently, an important contribution to resolve the above controversy in favor of the stabilization effect was put forward by Eberly and co-workers.[6] These authors investigated numerically a Schrödinger equation describing the 1D model atom under the influence of a short laser pulse, controlling the turn-on and turn-off times of the pulse. For sufficiently rapid turn-on they observed stabilization of the atom, i.e., a decrease of the ionization probability. They claim that this result shows that a nondestroyed atom can indeed enter into the region of strong field.

The purpose of the present Letter is to demonstrate that the aforementioned effect of atom stabilization has a purely classical counterpart, and that the degree of stabilization depends critically on the dimension of the phase space. We show that the significant stabilization is reproduced in the classical version of the 1D model atom of Eberly and co-workers[7] interacting with short superintense laser pulses. Then, we provide evidence that such an effect will be substantially reduced for true 3D atoms. For real atoms, the phase-space geometry allows for much more complex motion of the electrons. This property of the phase space, accompanied, for yet stronger intensities, by relativistic effects, makes the observation of significant stabilization substantially more difficult.

In order to understand the stabilization of the atom in classical terms, we shall consider an electron in a Coulomb potential $V_{\text{Coul}}(r)$ in the field of a plane wave of frequency ω_L. The classical (nonrelativistic) equations of motion, in the dipole approximation, are

$$\frac{d}{dt}\mathbf{r} = \mathbf{v}, \tag{1a}$$

$$m\frac{d}{dt}\mathbf{v} = -\nabla V_{\text{Coul}}(r) + e\mathcal{E}_L \sin(\omega_L t), \tag{1b}$$

where $\mathcal{E}_L$ is the amplitude of the electric field. The classical Kramers-Henneberger transformation consists in changing the laboratory frame to the one that follows the classical motion of an electron in the plane-wave field,

$\mathbf{x}=\mathbf{y}-(e\mathcal{E}_L/m\omega_L^2)\sin(\omega_L t)$, $m\,d\mathbf{y}/dt=\boldsymbol{\pi}$, so that

$$\frac{d}{dt}\boldsymbol{\pi}=-\nabla V_{\mathrm{Coul}}\left[\mathbf{y}+\frac{e\mathcal{E}_L}{m\omega_L^2}\sin(\omega_L t)\right]. \tag{2}$$

For large ω_L the potential in Eq. (2) may be replaced by its average V_{eff} [Ref. 4(b)],

$$V_{\mathrm{eff}}(\mathbf{y})=\frac{\omega_L}{2\pi}\int_0^{2\pi/\omega_L}dt\,V_{\mathrm{Coul}}\left[\mathbf{y}+\frac{e\mathcal{E}_L}{m\omega_L^2}\sin(\omega_L t)\right]. \tag{3}$$

Note that $V_{\mathrm{eff}}(\mathbf{y})$ is time independent, and has typically two attraction centers, at $\pm e\mathcal{E}_L/m\omega_L^2$. The corrections to $V_{\mathrm{eff}}(\mathbf{y})$ (the higher-order Fourier components) are rapidly oscillating functions of time. Such an effective potential was discussed for the 1D model atom in Ref. 7, with the regularized Coulomb potential $V_{\mathrm{reg}}(x)=-1/(1+x^2)^{1/2}$. The amplitude of the lowest-order correction to $V_{\mathrm{eff}}(x)$ decreases as a function of ω_L, and saturates as a function of I. Therefore, the overall impact of the correction remains negligible. The same conclusion holds also in three dimensions, $D=3$.

The classical stabilization of an atom can be explained as follows. A classical phase-space distribution of electrons that mimics the quantum ground state in the potential V_{eff} will be concentrated in configuration space around the two minima of V_{eff}. After transforming it back to the laboratory frame, such a distribution will oscillate with the frequency ω_L. Its zeroth Fourier component describes the effective distribution of particles (i.e., the distribution that measures how often a given region of configuration space is visited by particles from the sample). Obviously, such an effective distribution will have three peaks, with symmetric maxima at $\mathbf{x}=\pm e\mathcal{E}_L/m\omega_L^2$, and a maximum around $\mathbf{x}=0$ roughly twice that large. Thus, for such a distribution nearly half of the population will be frequently very close to the Coulomb force center, and may remain trapped after the field is rapidly turned off.

The above statements can be formulated also in the quantum theory, provided that we apply them to the wave functions rather than to the probability distributions in the configuration space. In this case, however, additional quantum interference effects are possible. They can limit the validity of our argument.

We have verified the above heuristic arguments using the classical simulation of the ionization process. This approach was introduced by Leopold and Percival,[8] and is widely known as a phase-space averaging method. Here, the quantum-mechanical initial state of an atom is approximately represented by a statistical sample of points in the phase space. It is constructed as a microcanonical ensemble, by means of reversing the Einstein-Brillouin-Keller quantization scheme. Then, this sample is subject to the classical evolution laws [Eqs. (1) or their relativistic counterpart], and values of the observables are established through ensemble averaging. For instance, the degree of ionization (after termination of the laser pulse) is determined as the fraction of the sample with negative energies (electrons bounded to the center). Such a method proved fruitful for the description of the microwave ionization of highly excited hydrogen atoms,[8,9] and has been successfully extended[10] to describe strong-field above-threshold ionization.

We have applied the phase-space averaging method to study the stabilization of atoms. First, we have investigated the case of the one 1D potential of Ref. 7. For this potential, the quantum-mechanical ground-state energy was $E=-0.6698$, in the standard atomic units. As an initial distribution we chose the microcanonical ensemble corresponding to this energy. We then performed a dynamical Monte Carlo simulation of the motion in the presence of a short, intense laser pulse. The duration of the pulse T_D was twenty optical periods, while its frequency was varying. The pulse shape was trapezoidal,

$$\mathcal{E}_L(t)=\mathcal{E}_L f(t)\sin(\omega_L t+\phi), \tag{4a}$$

where

$$f(t)=\begin{cases} t/T_{\mathrm{on}}, & \text{for } 0\le t\le T_{\mathrm{on}},\\ 1, & \text{for } T_{\mathrm{on}}\le t\le T_D-T_{\mathrm{off}},\\ 1-(t-T_D+T_{\mathrm{off}})/T_{\mathrm{off}}, & \text{for } T_D-T_{\mathrm{off}}\le t\le T_D. \end{cases} \tag{4b}$$

T_{on} and T_{off} describe the turn-on and turn-off times of the pulse, respectively, and ϕ is the random, uniformly distributed phase.

In Fig. 1 we present the ionization probability as a

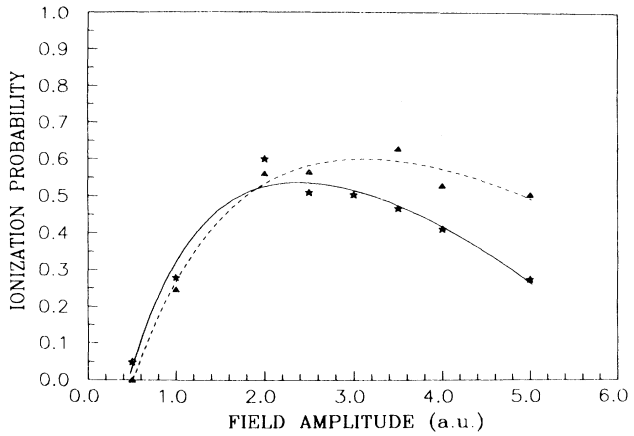

FIG. 1. The dependence of the ionization probability on the laser field amplitude $\mathcal{E}_L$, in the 1D model atom, for the "half-photon" ionization ($\hbar\omega_L=2E$). Stars (solid-line fit) correspond to the fast turn-on (and turn-off) of the pulse, 5% of the pulse duration (one period) each. Triangles (dashed-line fit) represent the "adiabatic" pulse, with 40% of the pulse duration turning on and off. The points were obtained in the numerical experiment, and are endowed with a statistical error of the order of a few percent.

function of the laser pulse amplitude (in a.u.), representing the "half-photon" ionization, $\hbar\omega_L = 2E$. Right above the breakdown of the perturbative regime, stabilization of the atom is evident. The results are a direct counterpart of those of Ref. 6. Moreover, a comparison of the curves shows that the stabilization is stronger for shorter turn-on times (solid curve), in accordance with Ref. 5. Furthermore, we have also observed complete cessation of the ionization, above the (more or less) "quarter-photon" ionization ($\hbar\omega_L \gtrsim 4E$), for fixed $\mathcal{E}_L = 2$. This thresholdlike behavior is quite similar to the one observed already in Ref. 8.

For the 3D case we have performed similar simulations. The atomic Hamiltonian was that of a hydrogen atom, while the initial state was a classical stochastic representation of its quantum-mechanical ground state [for details see Ref. 10(b)]. We have compared two kinds of dynamics: (a) nonrelativistic dynamics with the dipole approximation [Eq. (1)] and (b) true relativistic dynamics, containing the magnetic-field effects, the relativistic mass increase, and the propagation of the plane-wave pulse, without the dipole approximation. The relativistic equations were integrated in proper time, tied to the pulse [cf. Ref. 10(c) for more details]. The temporal (spatiotemporal) pulse shape was in both cases trapezoidal, as described by Eqs. (4).

The results corresponding to the half-photon case, are presented in Fig. 2. Apparently no stabilization occurs already for case (a). For relativistic dynamics, case (b), the ionization is even more efficient. The analogous results were also obtained for the case of quarter-photon ionization. Here, however, the ionization is a little smaller than in Fig. 2, which may suggest that stabilization is perhaps possible for higher ω_L.

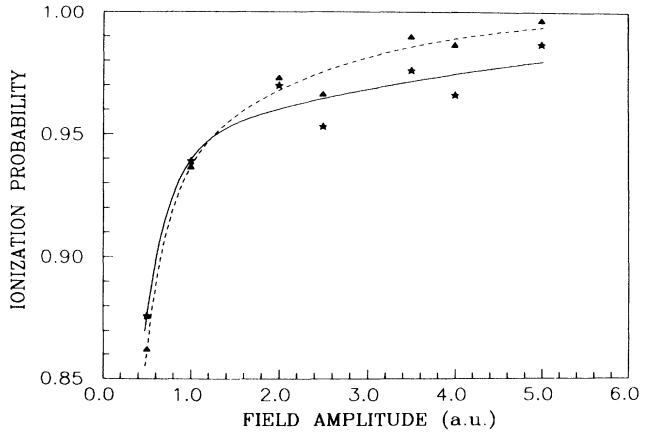

FIG. 2. The ionization probability as a function of $\mathcal{E}_L$, in the 3D hydrogen atom, for the "half-photon" ionization ($\hbar\omega_L = 2E$). Triangles (dashed-line fit) represent the relativistic dynamics, while stars (solid-line fit) the nonrelativistic one. In both cases the pulses were turned on and off rapidly, as in Fig. 1.

In Fig. 3 we present a comparison of the high-frequency limit for the 1D and 3D cases. Here, we plot probability of ionization as a function of the effective "photon number," $n = E/\omega_L$, for $\mathcal{E}_L = 2$. There is a tremendous difference between these two cases. For the 1D case the ionization drops to zero for $\hbar\omega_L \gtrsim 4E$, whereas for the 3D case complete stabilization[11] demands $\hbar\omega_L \simeq 80E$. These frequencies become unrealistically large if one considers processes starting from the atomic ground state. However, if only the dipole approximation is valid (in the nonrelativistic case), one could make use of the well-known scaling properties of the classical hydrogen atom (cf. Ref. 8). In effect, by means of the substitutions $\omega_L \to \lambda\omega_L$, $E \to \lambda^{2/3}E$, and $\mathcal{E}_L \to \lambda^{4/3}\mathcal{E}_L$, one can perhaps obtain the stabilization effect for somewhat lower frequencies, provided that we identify the initial state of the system with an appropriate Rydberg level.

The interpretation of these results is straightforward: Stabilization occurs for $D = 1$ and is much more difficult to achieve for $D = 3$. The reason is that the region of phase space accessible for the electron trajectories in 3D is much larger. Note that our classical argument explaining stabilization is based exclusively on the analysis of the configurational space, and works for particles with $\pi \approx 0$. In 1D, the particles move necessarily along the direction of the electric-field polarization. Here, our classical argument holds, provided the velocities of the electrons in the sample are small in comparison to those they gain under the direct influence of the electric field. In the 1D model, the initial velocities randomize the phase of the oscillatory motion driven by $\mathcal{E}_L(t)$. Each

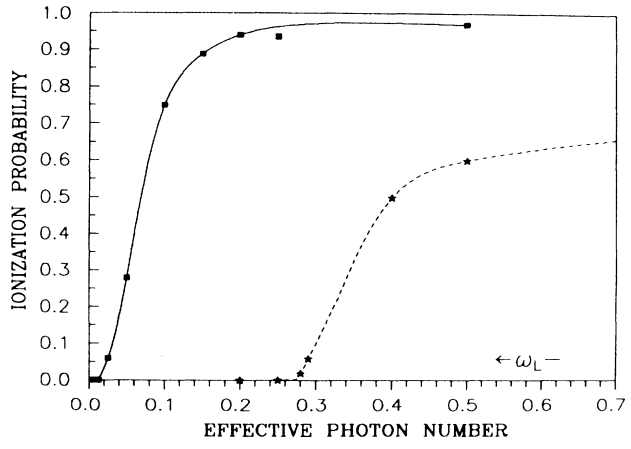

FIG. 3. The dependence of the ionization probability on the effective photon number, $n = E/\omega_L$. The 1D case is represented by stars (with dashed line), whereas squares (with solid line) correspond to $D = 3$. In both cases $\mathcal{E}_L = 2$. Here, for the 3D case, the relativistic and nonrelativistic dynamics yield the same results [the importance of the relativistic corrections scales as I/ω^2, cf. Ref. 10(c)]. The stabilization of the true atom becomes significant only for ultrahigh frequencies.

time the potential center is crossed, additional randomization of the phase, caused by Coulomb forces, takes place [cf. Refs. 10(b) and 10(c)]. In effect, after a few passes a stationary distribution in the Kramers-Henneberger frame is attained.

For $D=3$ the initial velocities are not necessarily directed along the polarization axis. During the very initial phase of the motion (close to the nucleus) quite large accelerations of the electron are possible [Ref. 10(c)]. The angular momentum typically increases during the multiphoton absorption. Such effects would cause the following: In the moving frame the stationary distribution would not be so well localized around $\mathbf{y}=\pm e\mathcal{E}_L/m\omega_L^2$, but rather diffused over a much larger region of phase space. The diffusion effects should, in fact, be larger for longer pulses. Similarly, such diffused distributions when transformed to the laboratory frame will not exhibit any particular maximum for slow (i.e., trapped) electrons close to the nucleus. The destabilization effects of diffusion in the nonrelativistic case are enhanced in the relativistic case, due to the action of the Lorentz force that additionally drags the electron in the direction of the incident pulse, away from the nucleus.

Summarizing, we have presented a simple argument based on classical mechanics that stabilization of real atoms in superintense laser fields is extremely difficult. Our theory supports, however, the prediction that it is present in 1D model atoms, as in Ref. 7. The stabilization of 3D atoms in short laser pulses would require either an ultrashort wavelength of the laser light or special mechanisms of a quantum interference origin.[12] Alternatively, one could also look for ways of initial preparation of atoms that could compromise between a low level of excitation and constraining the electron motion to only one dimension,[13] or, on the other hand, that could result in preparing high-lying Rydberg levels.

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[1]L. D. Landau and E. M. Lifshitz, *Mechanics* (Addison-Wesley, Reading, MA, 1960).

[2]W. Paul, O. Osberghaus, and E. Fisher, Forshungsber. Wirtsch. Verkehrsminist. Nordheim-Westfalen **415**, 42 (1959).

[3]M. Gavrila and J. Z. Kamiński, Phys. Rev. Lett. **52**, 613 (1984); for the earliest reference see I. Gersten and M. H. Mittleman, J. Phys. B **9**, 2561 (1976); M. Pont, N. R. Walet, M. Gavrila, and C. W. McCurdy, Phys. Rev. Lett. **61**, 939 (1988); M. Gavrila, in Proceedings of the Conference on Super-Intense Laser-Atom Physics, Rochester, New York, June 1989 (to be published).

[4](a) H. A. Kramers, in *Les Particules Elementaires,* Proceedings of the Eighth Solvay Conference, edited by R. Stoops (Wiley, New York, 1950); W. C. Henneberger, Phys. Rev. Lett. **21**, 838 (1968); C. K. Choi, W. C. Henneberger, and F. C. Sanders, Phys. Rev. A **9**, 1895 (1974). (b) See also P. L. Kapitza, Zh. Eksp. Teor. Fiz. **21**, 588 (1951); Y. Pomeau, Ann. Inst. Henri Poincaré **45**, 29 (1986).

[5]P. Lambropoulos, Phys. Rev. Lett. **55**, 2141 (1985).

[6]Q. Su, J. H. Eberly, and J. Javanainen, Phys. Rev. Lett. **64**, 862 (1990); Q. Su and J. H. Eberly, J. Opt. Soc. Am. B **7**, 564 (1990).

[7]J. Javanainen, J. H. Eberly, and Q. Su, Phys. Rev. A **38**, 3430 (1988); J. H. Eberly, Q. Su, and J. Javanainen, J. Opt. Soc. Am. B **6**, 1289 (1989).

[8]J. G. Leopold and I. C. Percival, Phys. Rev. Lett. **41**, 944 (1978); I. C. Percival, in *Advances in Chemical Physics,* edited by I. Prigogine and S. A. Rice (Wiley, New York, 1977), Vol. 36.

[9]See, for instance, G. Casati, B. V. Chirikov, D. L. Shepelyansky, and I. Guarneri, Phys. Rep. **154**, 77 (1987).

[10](a) J. Mostowski and K. Życzkowski, Z. Phys. D **5**, 293 (1987); G. A. Kyrala, J. Opt. Soc. Am. B **4**, 731 (1987); G. Bandarage, A. Maquet, and J. Cooper, Phys. Rev. A **41**, 1744 (1990); (b) J. Grochmalicki, J. Mostowski, and M. Trippenbach, J. Phys. B **21**, 1673 (1988); (c) J. Grochmalicki, M. Lewenstein, M. Wilkens, and K. Rzążewski, J. Opt. Soc. Am. B **7**, 607 (1990).

[11]Note that a weaker notion of stabilization requires somewhat smaller frequencies. For instance, assuming that it occurs only if the ionization probability $\leq \frac{1}{2}$, we have the condition $\hbar\omega_L \gtrsim 14E$ $(D=3)$ vs $\hbar\omega_L \gtrsim 2E$ $(D=1)$.

[12]M. V. Fedorov and A. M. Movsesian, J. Opt. Soc. Am. B **6**, 928 (1989); J. Parker and C. R. Stroud, Jr., Phys. Rev. A **41**, 1601 (1990); B. Piraux, E. Huens, and P. L. Knight (to be published).

[13]J. E. Bayfield, in *Quantum Measurement of Chaos,* edited by E. R. Pike and S. Sarkar (Plenum, New York, 1987), and references therein.