Resonant Tunneling and the Bimodal Symmetric Fission of ^{258}Fm

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The concept of resonant tunneling is invoked to explain the sharp drop in the measured spontaneous-fission half-life when going from ^{256}Fm to ^{258}Fm. Various consequences of such a suggestion on the other observed characteristics of the bimodal symmetric fission of ^{258}Fm are briefly discussed.

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Bimodal symmetric fission of the heaviest known elements has been of considerable interest[1-9] over the past few years. It exhibits two distinctly different components in the total-kinetic-energy (TKE) distributions of the fission fragments occurring concurrently in the symmetric mass division in the spontaneous fission of ^{258}Fm and of a few other elements in this region of the actinide nuclei. Such results are qualitatively understood in terms of two distinguishable modes of spontaneous fission, each of which is derived from the effects of the shell structure. The first mode corresponds to the effects of the shell structure in the parent fissioning nucleus and leads to fission along the so-called "old" valley resulting in liquid-drop-type elongated scission shapes similar to the scission configuration for the lighter actinides, with relatively low TKE values centered around 200 MeV and a broadly symmetric fission-fragment mass distribution. On the other hand, the second mode produces a sharply symmetric fission-fragment mass distribution with rather large values of the TKE of the fragments ($\sim$235 MeV) approaching the Q value for the fission reaction. This mode is believed to occur because of the strong shell effects in the emerging fission fragments approaching the doubly magic ^{132}Sn nuclei with closed proton and neutron shells ($Z=50$, $N=82$). Such fragment-shell-directed symmetric fission is found to occur along the so-called "new" valley leading to very compact scission configuration of two touching spheres. The two such valleys have been found in recent potential-energy-surface calculations[2,4-6] to be separated by a rather high ridge which serves to prevent reequilibration during the descent to scission. Recent results from the neutron multiplicity measurements[10] in the spontaneous fission of ^{260}Md provide rather conclusive evidence in support of explaining the bimodal symmetric fission in terms of two such valleys corresponding to distinctly different scission configurations.

Spontaneous-fission properties of the actinide nuclei are known to vary fairly smoothly as one goes from uranium ($Z=92$) to fermium ($Z=100$). An increase in the proton number of the fissioning nucleus leads to a gradual decrease in fission-fragment mass asymmetry, an increase in fission-fragment kinetic energies, and a decrease in fission half-lives. Except for the fission half-lives these quantities vary fairly slowly with the neutron number of the fissioning nucleus. However, at ^{258}Fm there are sudden dramatic changes in all of these quantities. Fission becomes symmetric with a very narrow mass distribution, the average TKE values are about 35 MeV higher than in the asymmetric fission of ^{256}Fm, and the fission half-life drops by more than 7 orders of magnitude to 0.38 ms for ^{258}Fm compared to 2.86 h for ^{256}Fm. There are also serious difficulties in quantitative understanding of the experimentally observed[1,7,8] coexistence of the above two modes of spontaneous fission with nearly equal probability in the same nucleus exhibiting the bimodal fission. For example, the calculations of Möller, Nix, and Swiatecki[2] suggest that the barrier heights as well as the inertial masses for compact shapes along the new valley are considerably less than those in the trajectory taken by the elongated mode along the old valley. This would greatly increase the fission probability along the new valley and will lead to a much higher rate towards scission for nuclei traveling the less-deformed route. As this prediction is in clear conflict with the measured ratios[2,8] for the relative populations along the two valleys, they have also suggested the possibility of the so-called "switchback" path from the new valley to the old valley in order to account for the approximately equal fission probabilities observed along the two routes. More satisfactory alternative explanations are clearly desirable. In this Letter, we suggest a plausible scenario which could be responsible for the sudden and dramatic decrease of the spontaneous-fission half-life of ^{258}Fm, and may also explain the enhanced rate of fission along the old valley thus competing favorably with that along the new valley. The explanation suggested here is in terms of resonant tunneling from the ground-state level of ^{258}Fm taken to be accidently degenerate with the lowest quasibound state in the second well of the corresponding double-humped fission barrier.

Figure 1 displays the variation of the experimentally measured spontaneous-fission half-lives of the Fm ($Z=100$), No ($Z=102$), and Rf ($Z=104$) nuclei with neutron number. The precipitous drop in fission half-life by more than 7 orders of magnitude is clearly seen for ^{258}Fm, and also for ^{258}No. Such a rapid change in fission half-life when going from ^{256}Fm to ^{258}Fm was

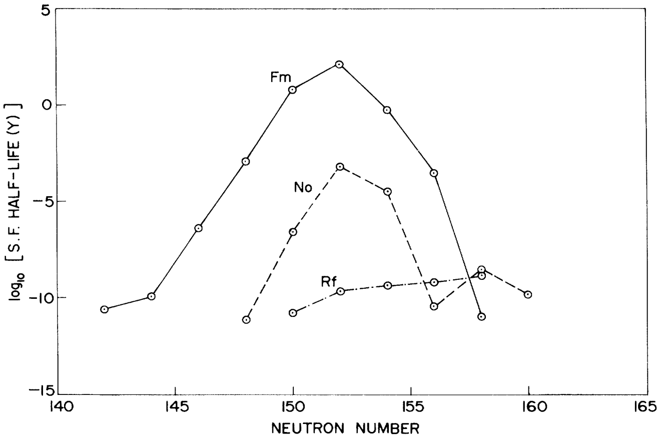

FIG. 1. Variation of the measured spontaneous-fission half-lives of the Fm, No, and Rf isotopes with neutron number. The measured data are taken from those listed in Refs. 2 and 3.

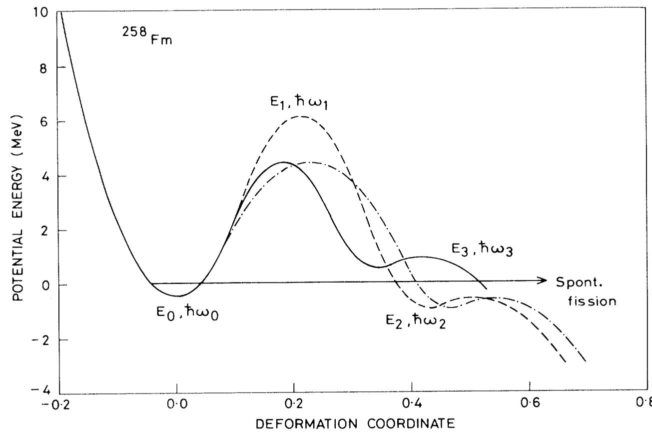

FIG. 2. Various double-humped fission barrier shapes consistent with the experimentally observed spontaneous-fission half-life of ^{258}Fm. The corresponding barrier parameters are as given as follows: Solid line: $E_0=-0.5$ MeV, $\hbar\omega_0=1.0$ MeV, $E_1=4.41$ MeV, $\hbar\omega_1=1.04$ MeV, $E_2=0.50$ MeV, $\hbar\omega_2=1.0$ MeV, $E_3=0.90$ MeV, $\hbar\omega_3=0.60$ MeV; dash-dotted line: $E_0=-0.5$ MeV, $\hbar\omega_0=1.0$ MeV, $E_1=4.41$ MeV, $\hbar\omega_1=0.71$ MeV, $E_2=-1.0$ MeV, $\hbar\omega_2=1.0$ MeV, $E_3=-0.60$ MeV, $\hbar\omega_3=0.60$ MeV; dashed line: $E_0=-0.5$ MeV, $\hbar\omega_0=1.0$ MeV, $E_1=6.08$ MeV, $\hbar\omega_1=1.04$ MeV, $E_2=-1.0$ MeV, $\hbar\omega_2=1.0$ MeV, $E_3=-0.60$ MeV, $\hbar\omega_3=0.60$ MeV.

originally attributed to Randrup *et al.*[11] to the disappearance of the second saddle in the fission barrier below the ground-state energy. While the recent calculations of the potential-energy surfaces of the even-even fermium isotopes by Ĉwiok *et al.*[6] seem to support this earlier hypothesis, those by Möller, Nix, and Swiatecki[2] clearly contradict it. Möller, Nix, and Swiatecki[2] suggest instead that the short half-life of ^{258}Fm is due to the low inertia along the new valley. The near constancy or rather very slow increase of the fission half-lives of the Rf isotopes with neutron number is reasonably understood at least qualitatively in terms of the predicted total disappearance of the outer barrier for the actinide nuclei with $Z\geq 104$, and in terms of the near constancy of the height of the inner barrier.

In Fig. 2 we show a few barrier shapes consistent with the measured spontaneous-fission half-life of ^{258}Fm. The potential-barrier shapes have been parametrized[12] by smoothly joining four parabolic segments, and fission penetrability and fission half-lives have been calculated in the Wentzel-Kramers-Brillouin (WKB) approximation as already reported in some of our earlier publications.[13-17] The mass parameter has been assumed constant with respect to the deformation (fission) coordinate. The inner barrier heights (E_1) as well as the inner- and the outer-barrier curvature parameters ($\hbar\omega_1$ and $\hbar\omega_3$) have been taken to be consistent with the empirical values recommended by Bjørnholm and Lynn[18] in their systematic analysis of the fission characteristics of the even-even actinide nuclei in the region $90\leq Z\leq 98$. The $\hbar\omega_2$ value in the second-well region has been taken equal to 1 MeV. Such a value was found to reproduce reasonably well the observed ground-state spontaneous-fission half-lives as well as the isomeric half-lives of a wide variety of the even-even actinide nuclei in one of our recent works.[17] The height of the outer barrier is varied until the observed spontaneous-fission half-life of ^{258}Fm is reasonably reproduced. The calculated values of the spontaneous-fission half-life through the fission barriers denoted by the solid, the dash-dotted, and the dashed lines in Fig. 2 are 1.191×10^{-11}, 1.253×10^{-11}, and 1.135×10^{-11} yr, respectively. These values are in excellent agreement with the measured value[3] equal to $(1.17\pm0.136)\times10^{-11}$ yr for ^{258}Fm. The fact that the barrier shapes with the outer saddle lying above as well as below the ground-state energy can explain equally well the measured fission half-life of ^{258}Fm seem to suggest that the disappearance of the outer saddle should only lead to a rather gradual decrease in fission half-life with neutron number. The experimentally observed sharp drop in the fission half-life of ^{258}Fm by more than 7 orders of magnitude may then require some other explanation, perhaps in terms of resonant tunneling as described in the following.

Figure 3 shows such a configuration where the ground-state level of ^{258}Fm is seen to be degenerate with the lowest quasibound state in the second well of the corresponding double-humped fission barrier. Such an intermediate situation is expected to occur while the outer saddle as well as the bottom of the second minimum gradually fall below the ground-state energy with an increase in the proton number of the fissioning compound nucleus. We suggest that such a configuration is realized for ^{258}Fm, and perhaps also for ^{258}No. The resonant tunneling through the double-humped fission barrier of

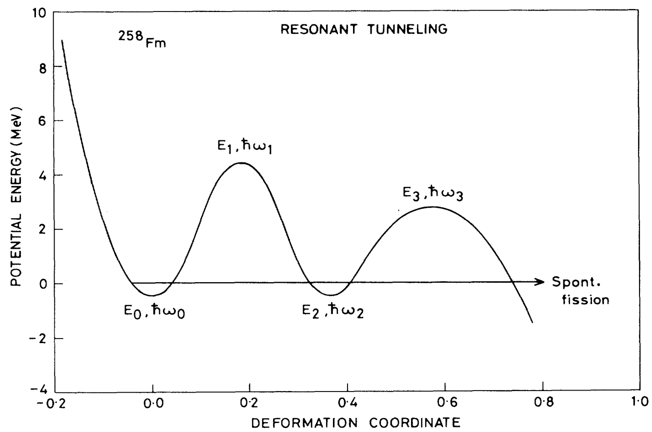

FIG. 3. Resonant-tunneling configuration with the lowest quasibound state in the second well degenerate with the ground-state level of ^{258}Fm. The parameters of the corresponding double-humped fission barrier are $E_0=-0.5$ MeV, $\hbar\omega_0=1.0$ MeV, $E_1=4.41$ MeV, $\hbar\omega_1=1.04$ MeV, $E_2=-0.5$ MeV, $\hbar\omega_2=1.0$ MeV, $E_3=2.76$ MeV, and $\hbar\omega_3=0.60$ MeV.

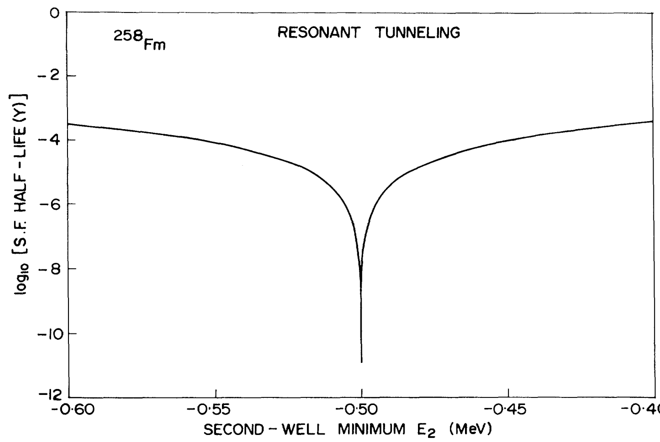

FIG. 4. Variation of the calculated spontaneous-fission half-life of ^{258}Fm with E_2, the bottom of the second well. A steep drop in fission half-life by more than 7 orders of magnitude is clearly realized at the configuration corresponding precisely to the resonant tunneling.

^{258}Fm as shown in Fig. 3 leads to the calculated value of the spontaneous-fission half-life equal to 1.137×10^{-11} yr, which is in excellent agreement with the measured result. Such resonant tunneling is obviously very sensitive to the location of the bottom of the second well on the energy axis. This can be seen in Fig. 4 where a sharp drop in the fission half-life by almost 7 orders of magnitude is clearly evident precisely when the energy of the lowest quasibound state in the second well becomes degenerate with the ground-state level of ^{258}Fm. Such an explanation in terms of resonant tunneling also has the interesting consequence that it predicts an increase in fission half-life as one moves away from ^{258}Fm, for example, to fermium isotopes with neutron numbers equal to or greater than 160 as the tunneling there would be "off resonance." No such information on the measured half-lives of the heavier even-even fermium isotopes is currently available in fission literature. However, such a trend is clearly visible (Fig. 1) in the measured fission half-lives of the nobelium isotopes with $N>156$. It is also interesting to note here that the recent dynamic calculations of the spontaneous-fission half-lives by Patyk *et al.*[19] show that the half-lives for the Fm and No isotopes should increase again for neutron numbers larger than about 158–160, presumably due to the influence of a deformed shell around the neutron number equal to 162–164.

It may seem that the alternative interpretation of the short half-life of ^{258}Fm in terms of resonant tunneling as proposed in the present work is rather speculative particularly because of the low probability of its occurrence in view of the extremely narrow width of the level in the second well, whose energy must be degenerate with that in the first well for the resonant tunneling to occur. Furthermore, several other actinide nuclei in the vicinity of ^{258}Fm are also known to have short spontaneous-fission half-lives, and the probability that all these nuclei also exhibit resonant tunneling may seem incredibly small. A detailed consideration of the various features of the calculated shapes of the double-humped fission barriers in such heaviest actinides helps allay such fears. The results of such potential-energy-surface calculations suggest that while the inner barrier remains almost constant in height, the outer saddle and the second minimum gradually fall below the ground-state energy for the heavier actinide nuclei with increasing proton number of the fissioning compound nucleus. In fact, it is only in this region of the heavier actinide nuclei that the conditions suitable for such resonant tunneling to occur are indeed realized. Keeping the first well and the first or the inner barrier almost fixed, a gradual successive lowering of the second well and of the second or the outer barrier to energies below the ground-state energy in the first well will almost certainly encounter configurations where the energy of the lowest level in the second well shall become degenerate with that in the first well. Once the energies are matched, the narrow width of the level in the second well shall no longer be an impediment to the occurrence of resonant tunneling. The specific nuclei for which such conditions are exactly satisfied shall exhibit short spontaneous-fission half-lives in view of resonant tunneling. While the present work is devoted entirely to the ^{258}Fm nucleus, it will not be surprising if such resonant tunneling may indeed be an appropriate explanation for the short half-lives of some of the other actinide nuclei also in the vicinity of ^{258}Fm.

How does such an explanation of the short fission half-lives of ^{258}Fm and ^{258}No in terms of resonant tunneling fit in with the other observed characteristics of their bimodal fission? Along the new valley the lower inertia may indeed be responsible for the short half-life and consequently for the large rate of fission in terms of compact spherical scission shapes. How do we then explain the experimentally observed approximately equal fission probabilities along the new and the old valleys? We suggest that resonant tunneling as described above occurs along the old valley pertaining to the symmetric deformations, thus resulting in an increase in the fission rate with liquid-drop-type extended scission shapes. The inclusion of the mass-asymmetric deformations might lower the outer saddle of Fig. 3 to energies below that of the ground-state level of ^{258}Fm, and may again result in liquid-drop-type broadly symmetric fission-fragment mass distributions characteristic of the reflection-symmetric inner barrier. Such characteristics of the potential-energy surfaces along the old valley may then increase the rate of fission along this route sufficiently to allow it to compete equally with the large fission rate along the new valley. This may provide an explanation for the experimentally observed nearly equal fission probabilities along these two valleys in the symmetric bimodal fission of ^{258}Fm. In principle, different mass pairs of the fission fragments should lead to different fission barriers for the compound fissioning nucleus. Fission is thus expected to be multimodal. For some such mass pairs the outer saddle in the corresponding double-humped fission barrier may not completely disappear below the ground state and might thus account for the relatively small number of the observed[8] asymmetric fission events in the spontaneous fission of ^{258}Fm at average TKE values less than 200 MeV.

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