# Yrast-Yrare Interaction Strength

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An accurate particle-number-conserving calculation shows that in a single-$j$ ($j=\frac{13}{2}$) model the yrast-yrare interaction $V$ is always strong and no periodic oscillation of $V$ with Fermi energy $\lambda$ is obtained. However, in a two-$j$ model or realistic nuclei sharp backcrossings do occur for appropriate single-particle level distribution and oscillation of $V$ with increasing $\lambda$ is found. The occupation-number distribution for the high-$j$ shell plays a key role in understanding the backcrossing properties.

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The cranked shell model[1] (CSM) and its generalization[2-4] have proven to be very fruitful in describing the yrast spectra of deformed nuclei. The eigenvalue problem of the CSM Hamiltonian $H_{\rm CSM}$ is usually treated by using the Hartree-Fock-Bogoliubov (HFB) approximation. General considerations show that the HFB approximation is very suitable for a system of a large number of particles. The question is, however, how reliable is the HFB approximation for treating the eigenvalue problem of $H_{\rm CSM}$. The crucial problem is that the number of nucleons in a nucleus ($\sim 10^2$), particularly, the number of valence nucleons ($\sim 10$) which dominate the behavior of low-lying excited states, is very limited. Therefore, serious defects (particle-number nonconservation, excessive spurious states, blocking effect, etc.) should be taken into account, and the conclusions drawn from the HFB approximation need reexamination. A well-known example is that in all self-consistent solutions to the cranked HFB equation a pairing collapse is found,[5] but calculations with particle-number projection before variation show that the gap parameter decreases very slowly and no sharp phase transition is found.[6]

The sharpness of the first backbending in the nuclear yrast spectrum depends on the magnitude of the interaction strength, $V$, between the yrast and yrare bands. The HFB calculation for a single-$j$ ($i\frac{13}{2}$) CSM (Ref. 7) shows that $V$ is a periodic function of the degree of shell filling, and thus a sharp backbending may be expected not only at the bottom of the neutron $i\frac{13}{2}$ shell but also may be obtained for appropriate configurations even near the top of the shell. This striking feature has attracted much attention.[8-11]

In the present Letter the yrast-yrare interaction is calculated using the particle-number-conserving (PNC) approach,[12] where a many-particle-configuration (MPC) truncation[13] is adopted instead of the usual single-particle-level truncation. As usual, the CSM Hamiltonian is expressed as[4,7]

$$H_{\rm CSM}=H_{\rm intr}+H_{\rm C},\quad H_{\rm intr}=H_{\rm SP}+H_{\rm P}, \tag{1}$$

where $H_{\rm SP}$ is the single-particle Hamiltonian, $H_{\rm P}$ the pairing interaction with strength $G$, and $H_{\rm C}=-\omega J_x$ the Coriolis interaction with rotating frequency $\omega$. A calculation in a single-$j$ ($j=\frac{13}{2}$) CSM was carried out with the single-particle energy[4,7]

$$\varepsilon_{|\Omega|}=\kappa\frac{3\Omega^2-j(j+1)}{j(j+1)}+e_0,\quad |\Omega|=\tfrac{1}{2},\tfrac{3}{2},\ldots,j. \tag{2}$$

We diagonalize $H_{\rm CSM}$ in a truncated MPC space with $E_c=3.5\kappa\sim 10$ MeV. The pairing interaction strength is reasonably, but somewhat arbitrarily, chosen as $G/\kappa=0.15$. A calculation in the full (untruncated) MPC space has also been carried out to judge the degree of accuracy of the MPC truncation. The results show that the solutions obtained in the truncated MPC space are accurate enough and the conclusions thus drawn are reliable.

Now let us focus on the variations of the yrast-yrare interaction strength $V$ and the band-crossing frequency $\omega_c$ with the Fermi energy $\lambda$. In the PNC formalism the yrast-yrare interaction strength $V$ is defined as one-half of the minimal separation between the yrast and yrare bands where the cranking frequency is referred to as the band-crossing frequency $\omega_c$. At the Fermi energy $\lambda$ the pair occupation probability is 50% for the yrast state (at $\omega=0$) for a given $N$ (particle number) system. Calculation shows that in the single-$j$ model the value of $\lambda$ is very insensitive to the magnitude of $G$. In other words, once the particle number $N$ is given, $\lambda$ is in fact fixed. This situation is quite different from that in the HFB calculation where $\lambda$ is allowed to vary continuously, and hence to give rise to a periodic oscillation of $V$.

The calculated $V$ and $\omega_c$ for $N=2$, 4, 6, 8, and 10 particle systems are shown in Fig. 1. It is seen that in the single-$j$ model (a) $V$ is always rather strong for various $N$-particle systems, and (b) $V$ and $\omega_c$ increase monotonically with increasing $\lambda$ (or $N$), and no periodic oscillation of $V$ is found.

These results can be understood from the configuration structure of the yrast and yrare states.[14] When

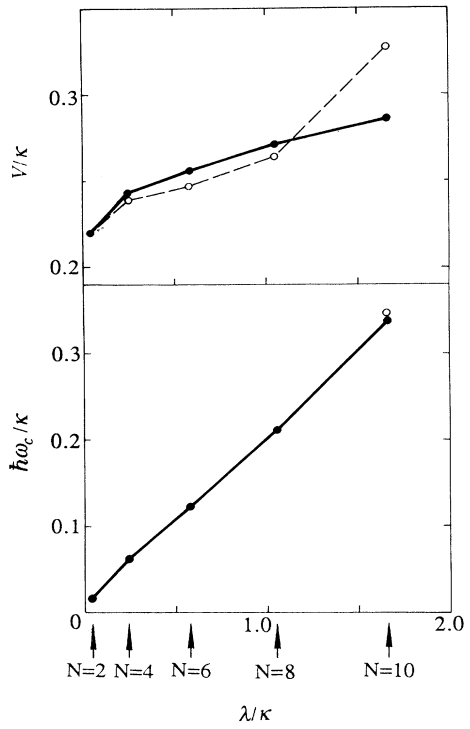


FIG. 1. The variations of the yrast-yrare interaction $V$ and the band-crossing frequency $\omega_c$ with the Fermi energy $\lambda$ in a single-$j$ model ($G/\kappa=0.15$). The values from the MPC truncation ($E_c/\kappa=3.5$) and the untruncated ($E_c=\infty$) calculations are denoted by open and solid circles, respectively. The band-crossing frequencies calculated in these two cases are almost the same except for $N=10$.

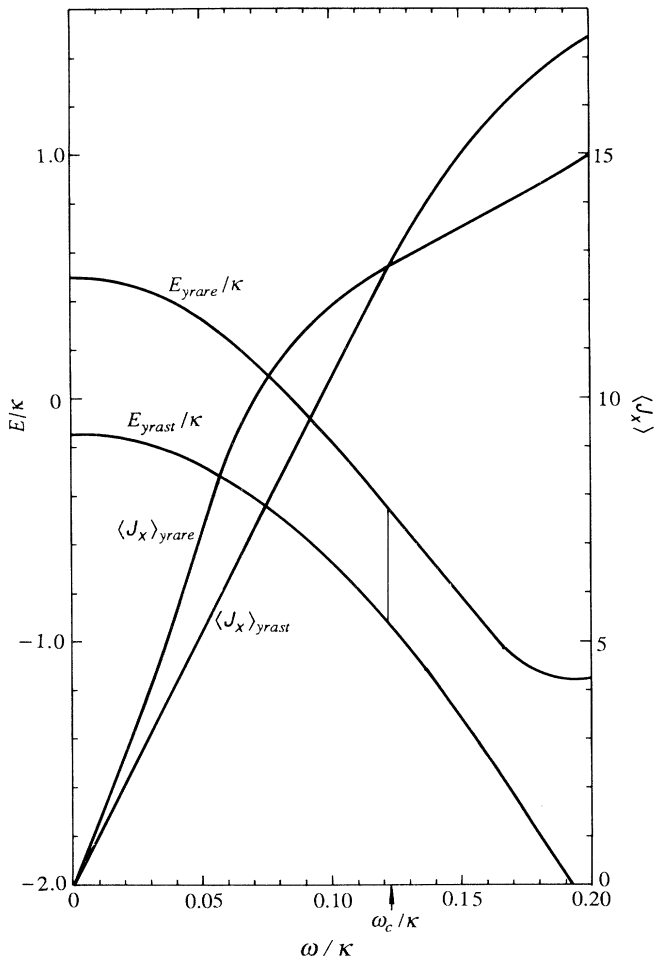


FIG. 2. The eigenenergies and spin alignments of the yrast and yrare bands for a six-particle system in the single-$j$ model ($E_c/\kappa=3.5$, $G/\kappa=0.15$). The constant $E_0-NG/2$ is removed in the figure, where $E_0$ is the energy of the lowest configuration.

$\omega=0$, the lowest eigenstate of $H_{\text{intr}}$, which is usually referred to as quasiparticle vacuum, is a coherent superposition of a large number of fully paired (seniority $v=0$) $K^\pi=0^+$ configurations. Above an energy gap there exist many pair-broken (seniority $v=2$) eigenstates of $H_{\text{intr}}$. In the single-$j$ model the lowest excited (yrare) state of an even-$N$ system is $K^\pi=1^+$, which is also a coherent superposition of many $K^\pi=1^+$ configurations. When the Coriolis interaction is switched on ($\omega\neq0$), the pair-broken ($v=2$) eigenstates of $H_{\text{intr}}$, particularly, the low-lying $K^\pi=1^+$ eigenstates, will be gradually mixed into the yrast band of $H_{\text{CSM}}$ due to large $J_x$ matrix elements.[14] The strong mixing results in a large amplitude of $K^\pi=1^+$ configuration in the yrast band, hence a large spin alignment and a rapid drop of the eigenenergy, as shown in Fig. 2. Meanwhile, the yrare state of $H_{\text{intr}}$ is strongly mixed with the densely distributed neighboring eigenstates, and then gets a larger spin alignment than the yrast band. However, with increasing $\omega$, the increasing pace of $\langle J_x\rangle_{\text{yrare}}$ gradually slows down due to destructive interference. When $\omega$ reaches a certain value, $\omega_c$, the first band crossing occurs. The magnitudes of $V$ and $\omega_c$ depend on the distribution of the low-lying eigenspectrum of $H_{\text{intr}}$, especially on the position of the lowest $K^\pi=1^+$ state, which increases monotonically with $N$ in the single-$j$ model. This is just the reason why $V$ and $\omega_c$ increase monotonically with increasing $N$. The PNC calculation in the single-$j$ ($j=\frac{13}{2}$) CSM shows that there exists no such yrast band which has no (or only a very weak) Coriolis response because all the particles in a high-$j$ shell have strong Coriolis response, and no abrupt exchange of features between the yrast and yrare bands occurs around $\omega\sim\omega_c$. Therefore, *no weak yrast-yrare interaction is found.*

However, weak yrast-yrare interactions do occur in some realistic nuclei. We believe that the single-$j$ model is too simplified to account for the observed sharp backbending observed. In fact, apart from the high-$j$ intruder orbits, there also exist many normal orbits of opposite parity, which have relatively weak response to the

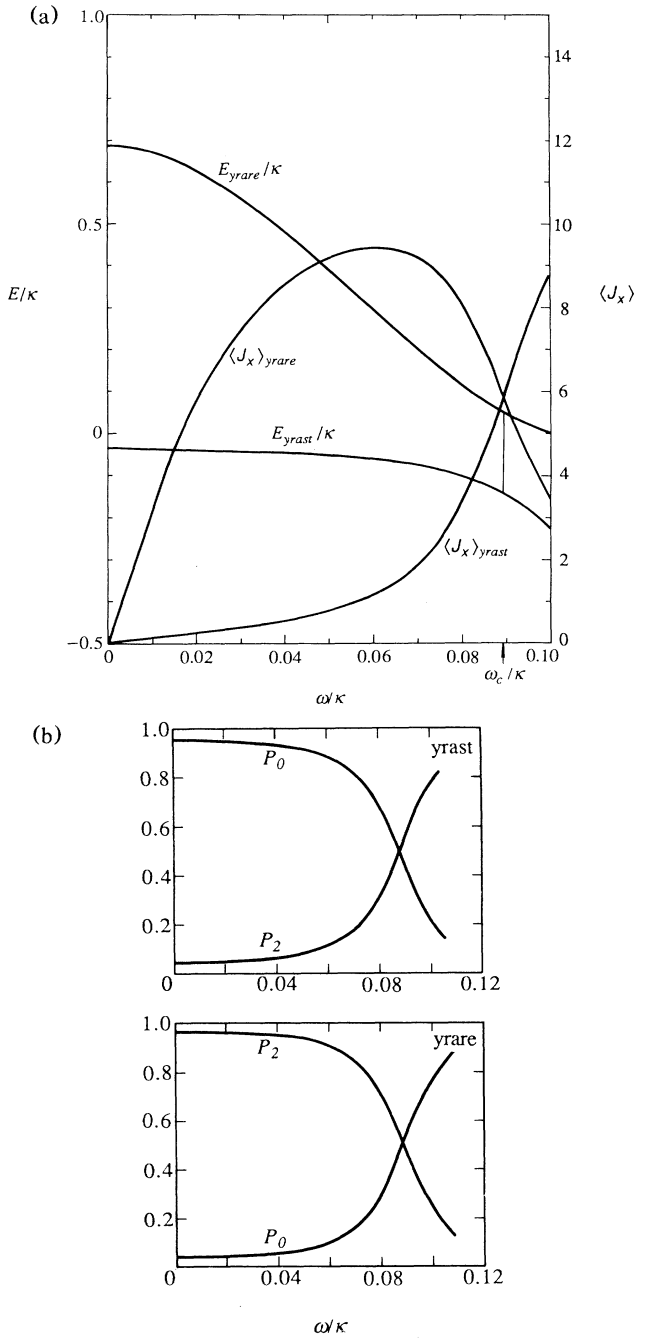


FIG. 3. (a) The eigenenergies and spin alignments of the yrast and yrare bands for a six-particle system in the two-$j$ model with $d/\kappa=-1.5$ ($E_c/\kappa=2.0$, $G/\kappa=0.10$). (b) The same as (a) but for the occupation-number distribution for $i\frac{13}{2}$ orbits.

Coriolis interaction. Therefore, a schematic two-$j$ model is adopted to investigate qualitatively the possibility of sharp backbending. In this model, a set of low-$j$ ($j^\pi=\frac{7}{2}^-$) orbits are inset into the set of high-$j$ ($j^\pi=\frac{13}{2}^+$) orbits and their relative position is characterized by

$$d=\varepsilon(\Omega^\pi=\tfrac{1}{2}^-)-\varepsilon(\Omega^\pi=\tfrac{1}{2}^+). \quad (3)$$

In the PNC treatment the total number of particles is conserved, but the particle numbers in the high-$j$ orbits and in the low-$j$ orbits are not conserved separately. The occupation-number distribution for the high-$j$ shell depends sensitively on the $d$ value, which in turn determines the Fermi energy $\lambda$. Calculations for various $d$ values and different $N$ systems have been carried out and the results show that (a) a weak yrast-yrare interaction indeed emerges for appropriate cases, and (b) a certain type of oscillating character of $V$ indeed occurs. As an illustrative example, the calculated yrast and yrare eigenenergies and spin alignments for $d/\kappa=-1.5$ and $N=6$ are shown in Fig. 3(a), and the occupation-number distribution for the $i\frac{13}{2}$ shell ($P_n$, the probability that the $i\frac{13}{2}$ shell is occupied by $n$ particles) is given in Fig. 3(b). It is seen that the yrast band indeed has a very weak response to the Coriolis interaction; i.e., for not too large $\omega$, $\langle J_x\rangle_{\text{yrast}}$ is very small and $E_{\text{yrast}}$ drops very slowly. On the contrary, the yrare band has a strong Coriolis response, because there are two particles occupying the $i\frac{13}{2}$ shell. With increasing $\omega$, $E_{\text{yrare}}$ drops rapidly and $\langle J_x\rangle_{\text{yrare}}$ rises drastically. When $\omega/\kappa\sim0.09$, the yrast and yrare bands exchange abruptly their features and then a sharp band crossing emerges. Particularly, an abrupt exchange in occupation-number distribution for the high-$j$ shell between the yrast and yrare bands is essential for a sharp band crossing.[15]

The calculation shows that the band-crossing property depends sensitively on the $d$ value and the total particle number $N$, which in turn determine the position of Fermi surface. For a given $N$ and the $d$ values in between two adjacent high-$j$ orbits, the calculated $V$ and $\omega_c$ vary with increasing $d$ value. Some calculated results are listed in Table I. It is seen that a certain type of oscillating character of $V$ is obtained.

Of course, all the calculations for the schematic two-$j$ model merely have qualitative meaning. Because of the low density of low-$j$ orbits in the two-$j$ model, it is extravagant to require such a schematic model calculation to account for the observed results quantitatively. A more conclusive statement can only be drawn from calculations for realistic nuclei. Here we report the preliminary results of calculations for the well-deformed nuclei $^{166\text{-}172}$Yb. Experimentally, it has been well established that sharp backbendings occur in $^{166}$Yb and $^{170}$Yb, while no backbending is found in $^{168}$Yb and $^{172}$Yb. The calculation shows that the observed drastic oscillation of $V$ with increasing neutron number in $^{166\text{-}172}$Yb can be understood from the occupation-number distributions for the $i\frac{13}{2}$ shell, which undergo a great change for the yrast and yrare bands in $^{166}$Yb and $^{170}$Yb when $\hbar\omega>240$ keV, but are almost unchanged within a large range of $\omega$

TABLE I. Variation of $V$ and $\omega_c$.

| | Fermi surface in between $\lvert 642\rvert \frac{5}{2}^+ \leftrightarrow \lvert 633\rvert \frac{7}{2}^+$ | | | $\lvert 633\rvert \frac{7}{2}^+ \leftrightarrow \lvert 624\rvert \frac{9}{2}^+$ | | | $\lvert 624\rvert \frac{9}{2}^+ \leftrightarrow \lvert 615\rvert \frac{11}{2}^+$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $N$ | | 6 | | | 8 | | | 10 | |
| $d/\kappa$ | 0.40 | 0.50 | 0.60 | 0.80 | 0.90 | 1.00 | 1.30 | 1.40 | 1.50 |
| $V/\kappa$ | 0.162 | 0.136 | 0.108 | 0.144 | 0.118 | 0.096 | 0.127 | 0.105 | 0.092 |
| $\omega_c/\kappa$ | 0.056 | 0.082 | 0.122 | 0.067 | 0.104 | 0.151 | 0.073 | 0.114 | 0.162 |

for $^{168}$Yb and $^{172}$Yb. Here the large gap in the Nilsson level scheme ($N=98$) plays an essential role. Details will be published in a forthcoming paper.

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