# Form Factors of Excited Baryons at High $Q^2$ and the Transition to Perturbative QCD

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Transition form factors have been extracted from inclusive electron-scattering data in the $Q^2$ range 1 to 10 GeV$^2/c^2$ for the $\Delta(1232)$ and second resonance, which is dominated by the $S_{11}(1535)$. The form factor for the second resonance appears to approach a $Q^{-4}$ dependence expected from leading-order perturbative QCD, while the form factor for the $\Delta(1232)$ decreases faster than $Q^{-4}$, which would be expected when nonleading processes are dominant.

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One of the fundamental problems in physics concerns the structure of hadrons and their excitations in terms of elementary quark and gluon constituents. A central question relates to which models are valid for describing these excitations in different domains of $Q^2$. At low $Q^2$ ($<1$ GeV$^2/c^2$), constituent quark models do a fairly good job of explaining the available data—sparse as they are.[1] In the limit of very high $Q^2$, perturbative QCD (PQCD) descriptions should be valid. However, there is currently strong disagreement as to what domain of $Q^2$ corresponds to the transition from nonperturbative to PQCD descriptions. Some believe[2-6] that the transition may take place for $Q^2$ as low as 4-6 GeV$^2/c^2$, while others maintain[7-9] that the transition should occur in a much higher region of $Q^2$—as high as 100 GeV$^2/c^2$.

During the past several years, proton elastic-scattering form-factor data[10] have provided the primary focus for testing PQCD calculations. However, this body of data in itself has not diminished the controversy. To test theories in a systematic way and further constrain wave functions, measurements of baryon-resonance transition form factors at high $Q^2$ will be important. To separate the contributing electromagnetic multipoles requires measurement of exclusive reactions such as $(e,e'\pi)$ and $(e,e'\eta)$. This will be a very difficult task since the resonances are broad and overlapping, and their form factors become small at high $Q^2$. Furthermore, there is a significant contribution from nonresonant processes.

The current experimental situation for the resonances is that there are no exclusive data for $Q^2$ above 3 GeV$^2/c^2$. Existing single-arm inclusive electron-scattering data have been evaluated by Brasse *et al.*[11] up to about 6 GeV$^2/c^2$. Figure 1(a) shows the virtual photon cross section at $Q^2=1$ GeV$^2/c^2$ as a function of baryon invariant mass $W$ obtained from their parameters. Higher-$Q^2$ inclusive results were obtained at SLAC,[12] and more recently[13] in SLAC experiment E133 spanning the $Q^2$ range 2.5–10.0 GeV$^2/c^2$. These cross sections agree very well with the Brasse evaluation at $Q^2=2.5$ and 4 GeV$^2/c^2$. Figure 1(b) shows the SLAC E133 data at $Q^2=6$ GeV$^2/c^2$.

For $W<2$ GeV, the most significant feature above the elastic-scattering peak is the existence of three maxima: the first-, second-, and third-resonance regions. In this interval there are about twenty known resonances. The first maximum is due to the $\Delta(1232)$ resonance. The second-resonance region is dominated by two strong

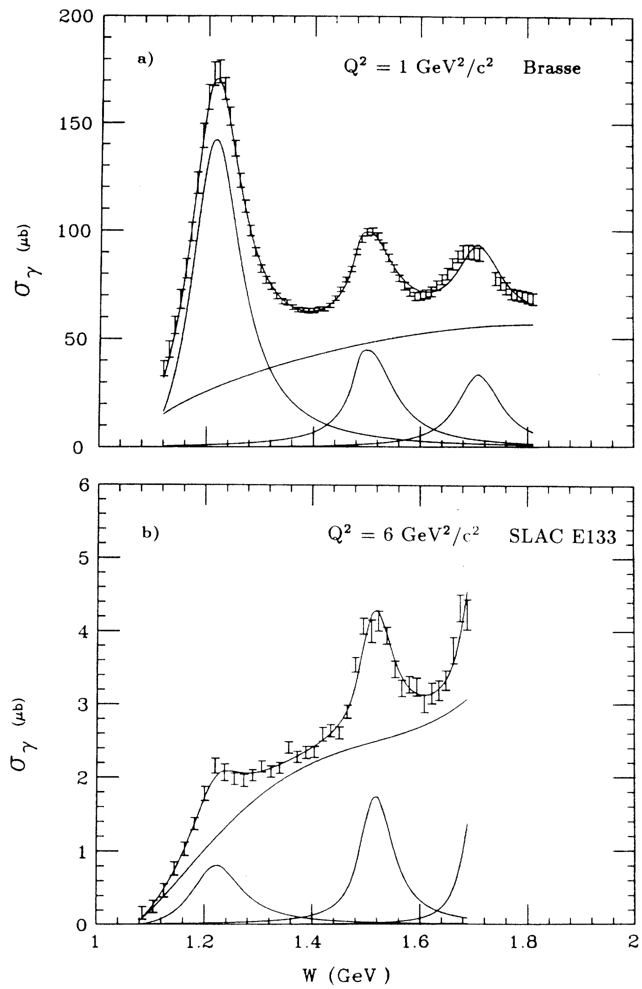


FIG. 1. Examples of least-squares fits to inclusive $(e,e')$ data. (a) Data reconstructed from parameters in Ref. 11. (b) Data from SLAC experiment E133 (Ref. 13). The curves below the data are the individual resonance and nonresonant contributions.

negative-parity states, the $D_{13}(1520)$ and the $S_{11}(1535)$. At low $Q^2$ ($<1$ GeV$^2/c^2$) the $D_{13}(1520)$ dominates, whereas at higher $Q^2$ ($>3$ GeV$^2/c^2$) the $S_{11}(1535)$ is dominant.[14] In the third-resonance region, the strongest excitation at low $Q^2$ is the $F_{15}(1680)$ state. The relative strength of the other states is not well determined, especially at increasing $Q^2$.

This Letter reports form factors in the first- and second-resonance regions, extracted from available inclusive data[11,13] in a $Q^2$ range from 1 to 10 GeV$^2/c^2$, and a reevaluation of exclusive results[14] below 3 GeV$^2/c^2$. The significance of these results is discussed relative to PQCD expectations.

In the first-resonance region the $\Delta(1232)$ is the only contributing state. In the second-resonance region above $Q^2 \sim 3$ GeV$^2/c^2$ the $S_{11}(1535)$ contribution becomes much greater than the $D_{13}(1520)$ state. Thus, for the present analysis only the $\Delta(1232)$, $P_{11}(1440)$, and $S_{11}(1535)$ were included in the least-squares fits to the data. For $Q^2 \leq 4$ GeV$^2/c^2$ corrections were made to subtract the contribution from the $D_{13}(1520)$.

Transverse resonance amplitudes are usually expressed in terms of the electromagnetic helicity amplitudes $A_{1/2}$ and $A_{3/2}$, which are matrix elements between states of total helicity $\frac{1}{2}$ and $\frac{3}{2}$, respectively.[15] For the $\Delta(1232)$, both $A_{1/2}$ and $A_{3/2}$ contribute. At high $Q^2$, helicity conservation requires $A_{1/2} \propto Q^2 A_{3/2}$. In the case of the $S_{11}(1535)$, only $A_{1/2}$ contributes. The helicity amplitudes are related to the virtual-photon transverse cross section by

$$\sigma_T(W_R) = (2M_N/\Gamma_R W_R) A_H^2 ,$$

where $A_H^2 = A_{1/2}^2 + A_{3/2}^2$, and $\Gamma_R$ is the total width at the resonance energy $W_R$. The amplitudes $A_H$ were extracted by fitting standard relativistic Breit-Wigner functions as given in Ref. 16.

For the $\Delta(1232)$ resonance the total width $\Gamma_R(W_R) = 120$ MeV, and the resonance-width damping parameter $X = 180$ MeV gave good fits to the data. For the $S_{11}(1535)$ there is a large $\eta$ branching ratio. The total cross section is then given by $\sigma_T = \sigma_{T\pi} + \sigma_{T\eta}$. The branching ratios in the fit were taken as 50% for each channel.[17] The best-fit width for the $S_{11}(1535)$ was $\Gamma_R(W_R) = 120$ MeV, and the parameter $X$ was taken as 350 MeV as in Ref. 16. Though the parameters $X$ are not well determined the resulting amplitudes were found to be insensitive to variations in $X$.

The nonresonant contribution is always large, and was phenomenologically included in the fit by the form[18] $\sigma_b = (W - W_{\rm th})^{1/2} \sum_{n=1}^{3} C_n (W - W_{\rm th})^n$, where $C_n$ are fitting parameters, and $W_{\rm th} = M_N + m_\pi$.

Figure 1 shows the result of the fits at $Q^2 = 1$ and 6 GeV$^2/c^2$. [At $Q^2 = 1$ GeV$^2/c^2$ there is in fact a significant contribution from the $D_{13}(1520)$ state.]

Several features are noteworthy. (1) The $\Delta(1232)$ becomes relatively weak at higher $Q^2$. (2) The $S_{11}(1535)$ remains quite prominent at all $Q^2$. (3) There is no observable contribution from a resonance in the region corresponding to the $P_{11}(1440)$, contrary to what is observed in the evaluation[11] of older data. (4) The nonresonant and resonant contributions have about the same $Q^2$ dependence, as suggested from the Bloom-Gilman duality.[19]

In order to compare with the $Q^2$ dependence of the proton form factors, following Ref. 3 analogous transition form factors have been defined in terms of the helicity amplitudes,

$$F^2 = \frac{1}{4\pi\alpha} \frac{2M_N}{Q^2} (W_R^2 - M_N^2) A_H^2 .$$

The quantities $Q^4 F$ vs $Q^2$ for the $\Delta(1232)$ and $S_{11}(1535)$ transitions are shown in Fig. 2 up to a maximum $Q^2$ of 10 GeV$^2/c^2$. The errors are statistical. Uncertainties in the radiative corrections, widths $\Gamma_R$, background shape, and the $S_{11}(1535)$ $\eta/\pi$ branching ratios lead to estimated systematic errors in the amplitudes of typically 15%, depending on $Q^2$. For the $\Delta(1232)$ the uncertainty in background shape at high $Q^2$ results in a lower bound compatible with zero at $Q^2 = 10$ GeV$^2/c^2$.

Also shown at lower $Q^2$ are form factors extracted from data obtained from exclusive $(e,e'p)\pi^0$ and $(e,e'p)\eta$ experiments.[14,18] The agreement between the extracted form factors from all data sets is quite good. The elastic-scattering form factor from Ref. 10 is also plotted in Fig. 2.

In the asymptotic PQCD limit, to leading order the cross section for exclusive baryon transitions should involve a minimum of two gluon exchanges. This results in the well-known rule $A_{1/2} \propto Q^{-3}$ or $F \propto Q^{-4}$. In fact, the elastic form factor $F_1$ does appear to approach this behavior above $Q^2 \sim 5$ GeV$^2/c^2$ out to the highest $Q^2$ available[10] ($\sim 35$ GeV$^2/c^2$). In Fig. 2 it is noteworthy that the $S_{11}$ form factor appears to approach the predicted $Q^{-4}$ behavior, as does the proton elastic form factor in the region of their $Q^2$ overlap. On the other hand, the $\Delta(1232)$ form factor appears to fall faster than $Q^{-4}$, which may be due to the suppression of the leading-order PQCD amplitude, and the dominance of higher-order amplitudes, as discussed below.

High-$Q^2$ form-factor calculations are commonly carried out in the light-cone frame and may be factorized in the form $F = \int dx\, dy\, \Phi^* T_H \Phi$, as discussed in Ref. 20 *et seq.* The two main ingredients are the transition operator $T_H$ and the wave functions $\Phi$, which are both functions of the initial and final momentum fractions $x$ $(= x_1, x_2, x_3)$ and $y$ $(= y_1, y_2, y_3)$, respectively, as well as the momentum transfer $Q^2$.

In leading order two gluons are exchanged such that $T_H \propto [\alpha_s(Q^2)]^2/Q^4$, resulting in the $Q^{-4}$ dependence of the form factor.

The wave functions contain the lowest-order three-quark diagrams, as well as the soft contributions due to

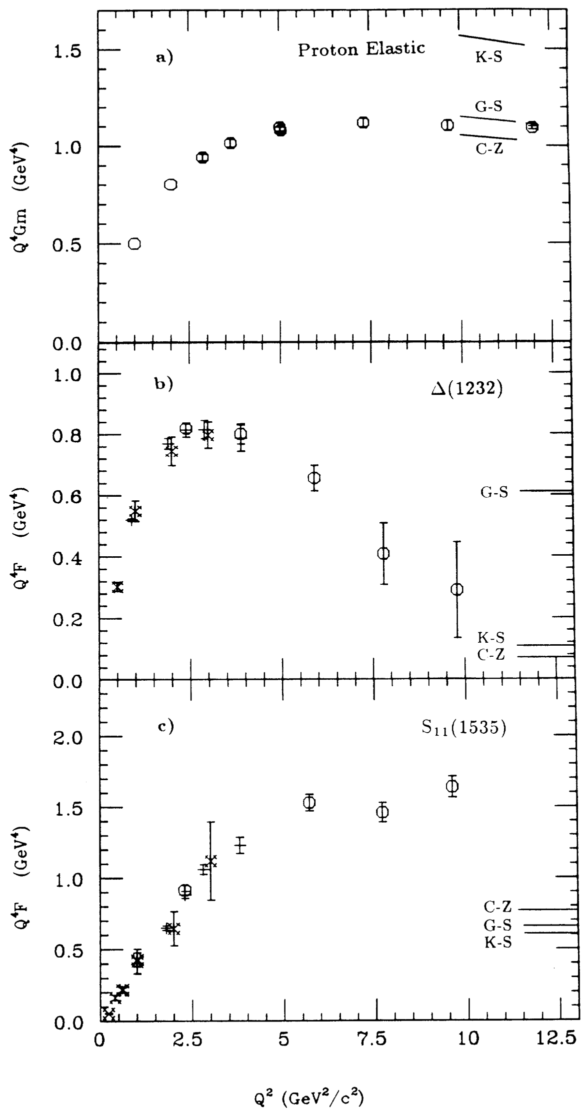


FIG. 2. (a) The quantity $Q^4F_1$ vs $Q^2$ for elastic scattering from the proton. The data are from SLAC (Ref. 10). The solid curves are the results of calculations (Ref. 23) at the values of $Q^2$ shown, using proton distribution functions labeled C-Z (Ref. 21), G-S (Ref. 6), and K-S (Ref. 22). (b),(c) The quantity $Q^4F$ vs $Q^2$ for transitions to $\Delta(1232)$ and $S_{12}(1535)$. For the second resonance, at $Q^2>4$ GeV$^2/c^2$ it was assumed that only the $S_{11}(1535)$ transition contributes. For $Q^2\leq 4$ GeV$^2/c^2$ a contribution due to the $D_{13}(1530)$ using data from Ref. 18 was subtracted. Inclusion of a state at $W=1440$ MeV gave zero amplitude. The form factor $F$ is defined in the text. The fits for $F$ were based on inclusive data reconstructed from the parameters of Ref. 11, denoted by +'s, and from SLAC E133 (Ref. 13), denoted by O's. Also shown at lower $Q^2$, denoted by ×'s, are from factors derived from amplitudes obtained from exclusive $(e,e',p)\pi^0$ and $(e,e',p)\eta$ data (Refs. 14 and 18). The errors shown are statistical. The solid horizontal lines are asymptotic predictions (Refs. 3 and 4). Estimated systematic errors are discussed in the text.

gluon and quark vacuum condensates. Using sum-rule techniques[6,21,22] the nucleon wave function has been expressed as a sum of symmetric and antisymmetric parts with respect to interchange of quarks 1 and 3. $\Phi_N=\phi_{\rm sym}\times\xi_{\rm sym}+\phi_{\rm anti}\xi_{\rm anti}$, where $\phi$ is a longitudinal momentum fraction distribution and $\xi$ is a spin-flavor function.

Good fits were obtained to the proton form-factor data. The curves in Fig. 2(a) are the result of such a calculation,[23] using three model wave functions denoted C-Z (Chernyak and Zhitnitsky[21]), K-S (King and Sachrajda[22]), and G-S (Gari and Stephanis[6]). However, the normalization has been controversial[8] since it depends on the nucleon wave function having a large asymmetry, and most of the elastic form factor comes from a small region near the kinematic limits of $x$ and $y$.

Wave functions using QCD sum-rule techniques have been obtained for the $\Delta(1232)$ (Refs. 3 and 24) and $S_{11}(1535)$ (Ref. 3) states and helicity amplitudes to these states were calculated using the three proton distribution functions C-Z, K-S, and G-S.

For the $p\rightarrow\Delta(1232)$ transition, the $\Delta(1232)$ distribution function has the symmetric form $\Phi_\Delta=\phi_{\rm sym}\xi_{\rm sym}$. The results of the calculation from Ref. 3 for asymptotically large $Q^2$ are shown in Fig. 2(b). As previously noted, the experimental form factor is falling faster than the $Q^{-4}$ dependence predicted by the leading PQCD amplitude. The form factors obtained using the C-Z and K-S proton distributions are small ($Q^4F\sim0.07$ and 0.11 GeV$^4$, respectively). This is due to a cancellation between $\langle\phi_\Delta|T_H|\phi_p^S\rangle$ and $\langle\phi_\Delta|T_H|\phi_p^A\rangle$ in the leading term of $T_H$. On the other hand, the distribution G-S,[6] which has the constraint that the neutron Fermi form factor $F_1\sim0$, does not yield this large cancellation and gives $Q^4F\sim0.61$ GeV$^4$. The large discrepancies suggest significant theoretical uncertainties in the baryon wave functions.

If the leading amplitude of the $\Delta$ transition is indeed small, the shape of the form factor might be explained as follows.[6] At high $Q^2$, the helicity-conserving amplitude (analogous to the Dirac form factor $F_1$ in elastic scattering) dominates the helicity-nonconserving amplitude (analogous to the Pauli form factor $F_2$) in proportion to $Q^2$. That is, $A_{1/2}\propto Q^2A_{3/2}$. In terms of multipoles, this implies the asymptotic equality of $E2$ and $M1$. However, at low $Q^2$ the $\Delta$ is primarily a spin-flip transition in which $E2\ll M1$. The helicity-nonconserving amplitude $A_{3/2}$ is dominant,[25] and the cross section is quite large. If the leading $A_{1/2}$ amplitude is suppressed, then one might expect the $A_{3/2}$ amplitude would remain dominant over a larger range of $Q^2$ than otherwise expected, and $Q^4F$ would decrease as a function of $Q^2$. In fact, there is evidence that the $E2/M1$ ratio is still very small for $Q^2$ up to 3 GeV$^2/c^2$, which is consistent with the dominance of nonleading processes. The data in Fig. 2 provide no clue as to where in $Q^2$ the $Q^4F$ curve levels off. It will be important in the future to find this region, and then to verify whether the $E2$ amplitude becomes comparable to the $M1$.

The $p \rightarrow S_{11}$ form factor, Fig. 2(c), behaves similarly to the elastic form factor because $\Phi_{S_{11}}$ obtained in Ref. 3 is similar to that of a proton, with the antisymmetric part about half as large. The asymptotic results for $Q^4F$ obtained for the three proton distribution functions from Ref. 3 are $\sim$0.60, 0.77, and 0.66 GeV$^4$ for C-Z, K-S, and G-S, respectively. Although all these results are about a factor of 2 lower than the data, a further logarithmic decrease is expected with increasing $Q^2$, and theoretical uncertainties are probably great enough to encompass these discrepancies.

In summary, the form-factor evaluations presented here may provide clues relating to the $Q^2$ regime where PQCD becomes dominant. The $Q^2$ dependence of the $S_{11}(1535)$ transition appears to approach the leading $Q^{-4}$ behavior, as does the proton elastic form factor, possibly indicating an onset of PQCD. However, the $\Delta(1232)$ transition form factor decreases with a greater $Q^2$ dependence, as would be expected if nonleading processes are dominant.

Overall, our understanding of where leading-order PQCD processes become dominant is far from settled. The status of theoretical wave functions is controversial, and discrepancies in the magnitude of theoretically calculated amplitudes are large. On the experimental side, in order to proceed further it will be necessary to perform exclusive experiments at as high $Q^2$ as possible, in which the resonance multipoles may be clearly separated from each other as well as from the nonresonant processes. Such experiments await the construction of new high-energy high-luminosity facilities.

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