Quantum Nondemolition Measurement of Small Photon Numbers by Rydberg-Atom Phase-Sensitive Detection

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We describe a new quantum nondemolition method to monitor the number N of photons in a microwave cavity. We propose coupling the field to a quasiresonant beam of Rydberg atoms and measuring the resulting phase shift of the atom wave function by the Ramsey separated-oscillatory-fields technique. The detection of a sequence of atoms reduces the field into a Fock state. With realistic Rydberg atom-cavity systems, small-photon-number states down to $N=0$ could be prepared and continuously monitored.

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Quantum harmonic oscillators can be used as ultrasensitive sensors for small classical forces. Hence, procedures to monitor the evolution of these systems with ultimate precision and minimal disturbance have been analyzed in detail.[1] "Squeezing" techniques have been developed, which beat the standard quantum limit in the measurement of an oscillator quadrature amplitude.[2] Quantum nondemolition (QND) methods have also been designed to avoid the "back action" produced by a measurement.[3] For an electromagnetic-field mode F, a version of QND consists in monitoring the photon number N without changing it. This may be achieved by coupling F to a detector via a nonresonant interaction, excluding processes where photons are created or annihilated. QND schemes in which the detector is a probe field quadratically coupled to F in a transparent solid medium have been proposed[4] and demonstrated.[5] The detection of the probe phase yields a QND measurement of F intensity or quadrature amplitude. In this way, one detects relatively intense fields with large-N values, inducing sensible nonlinear effects in solids. Attempts are now being made to increase the sensitivity of the method by replacing the solid medium with an atomic vapor close to resonance.[6]

We discuss here a novel approach to QND, working down to zero photon. The probe is a beam of atoms, laser excited into a Rydberg level f before crossing the cavity sustaining the field F [Fig. 1(a)]. Three Rydberg levels f, e, and i and two allowed transitions $f \rightarrow e$ and $e \rightarrow i$ (with angular frequencies ω_{ef} and ω_{ie}) are relevant in our scheme [Fig. 1(b)]. The detuning δ between the cavity mode (frequency ω) and the $e \rightarrow i$ transition is large enough to preclude photon absorption. Yet, the highly polarizable level e experiences a sensible dynamical Stark shift in a single-photon field (note that the level f is not appreciably shifted since $\omega - \omega_{ef}$ is much larger than δ). The cavity is placed between two field zones R_1 and R_2 driving the $f \rightarrow e$ transition (Ramsey separated-oscillatory-field method[7]). This transition is detected behind R_2 by an atomic ionization counter (IC) which discriminates the states e and f.[8] The presence of N photons in the cavity results in a phase shift, proportional to N, of the e-state amplitude relative to f which alters the probability of detecting the atom in e or f. Monitoring the $f \rightarrow e$ transfer rate thus yields a measurement of N. Since the photon number in the cavity is unchanged, the condition of "back-action evasion" for N is satisfied. On the other hand, the complementary observable (the phase of the field) is scrambled, since each atom modifies the index of refraction in the cavity. Rydberg atoms coupled to a cavity are ideal models for quantum-field measurements[9] and they have already been tested as efficient microwave photon counters.[8,10] These systems operated, however, on resonant N-altering atom-field coupling. Our nonresonant QND method opens new perspectives for the study of weakly excited nonclassical field states.

The dynamical frequency shift $\Delta(\mathbf{r},N)$ induced on an atom in level e, at point $\mathbf{r}$ in the cavity containing N photons, is given by a simple two-level "dressed-atom" calculation:[8]

$$\Delta(\mathbf{r},N) = (\delta/2)\{[1+4E^2(\mathbf{r})d^2N/\hbar^2\delta^2]^{1/2} - 1\}\,. \quad (1)$$

$E(\mathbf{r})$ is the position-dependent rms vacuum field in the

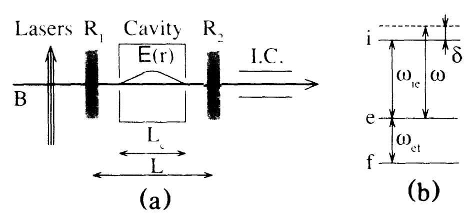

FIG. 1. (a) QND setup for measuring the photon number N in a cavity: The atomic beam B, prepared by lasers in Rydberg level f, crosses successively the field zone R_1, the cavity, and the zone R_2 before detection by the IC counter. The variation of the field intensity along the beam path in the cavity is shown. (b) Diagram of levels e, f, and i: The cavity field, detuned by δ from the $e \rightarrow i$ transition, shifts e by an amount proportional to N. The R_1-R_2 fields induce an $f \rightarrow e$ transition.

cavity, d the electric dipole matrix element on the upward $e \rightarrow i$ transition,[11] and $\delta = \omega - \omega_{ie}$ ($\hbar$ is Planck's constant). As we will see, photon absorption processes can be made negligible with appropriate field geometry provided a "minimal" off-resonance condition $E^2(\mathbf{r}) \times d^2 N/\hbar^2\delta^2 \leq 0.1$ is satisfied. Equation (1) can then be linearized:

$$\Delta(\mathbf{r},N) = [E^2(\mathbf{r})d^2/\hbar^2\delta]N\,. \quad (2)$$

For an order of magnitude, consider that e and i are circular Rydberg states[12] with principal quantum numbers 50 and 51, the cavity sustaining the TE_{121} mode at 51.1 GHz. We then have $d = 10^{-26}$ cm, $E(\mathbf{0}) = 4.35 \times 10^{-3}$ V/m at cavity center ($\mathbf{r} = \mathbf{0}$), and $dE(\mathbf{0})/\hbar = 4.2 \times 10^5$ rad/s. With $\delta = 4.2 \times 10^6\ \mathrm{s}^{-1}$, we satisfy the minimal off-resonance condition for N up to 10 and the shift "per photon" $\Delta(\mathbf{0},1)$ is 4.2×10^4 rad/s. Consider now an atom moving across the length $L_c = 1$ cm of the cavity at velocity v_0. The accumulated phase shift per photon is $\varepsilon = \langle \Delta(\mathbf{r},1) \rangle L_c/v_0$, where the angular brackets denote a spatial average along the atom path [$\langle \Delta(\mathbf{r},1) \rangle = \Delta(\mathbf{0},1)/2$ for the TE_{121} mode exhibiting a half sine-wave variation along the cavity axis]. With $v_0 = 300$ m/s (average velocity of a Rb or Cs thermal beam), ε is 0.7 rad. This phase shift is further increased by slowing down the atomic beam. $v_0 = 35$ m/s, easily achievable by standard laser-cooling techniques, corresponds to $\varepsilon = 2\pi$. Note also that the time of flight across the cavity ($\leq 3 \times 10^{-4}$ s when $v_0 \geq 30$ m/s) is much shorter than the circular Rydberg states radiative decay time (3×10^{-2} s for $n = 50$). Large single-photon shifts are obtained by choosing a relatively small δ detuning, yet without producing sensible photon absorption, due to our choice of adiabatic field-atom coupling (slow-field turn on and off along the atom path in the cavity). Numerical computations with the above parameters show that the probability of photon absorption per atom remains smaller than 10^{-4}. An atom may temporarily absorb a photon, but restores it in the mode before exiting from the cavity. At the same time, the phase shift keeps adding up during the whole cavity crossing time and the atom carries away this phase as a QND "information" on N. The absorption probability is much larger for a mode with a square-shaped $E(\mathbf{r})$ profile, whereas the phase shift is of the same order of magnitude. The slow spatial variation of the field is thus an essential feature of our QND method.

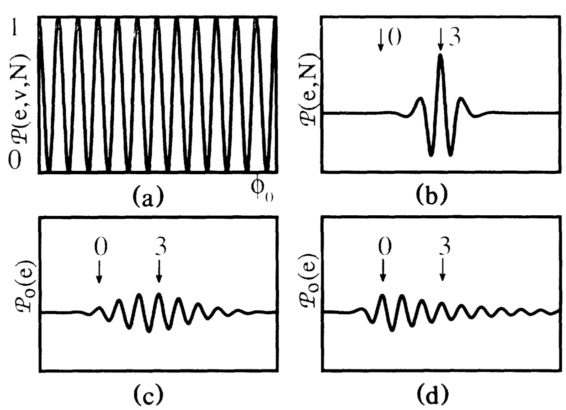

FIG. 2. Transition probability from f to e plotted vs φ_0 for $\varepsilon = 2\pi$: (a) Monokinetic atomic beam (velocity v_0) and field in an N state. (b)-(d) Transition probability averaged over the atom velocity distribution, the cavity sustaining (b) a Fock state, (c) a coherent, or (d) a thermal field. Mean photon number in all cases: $\bar{N} = 3$. Arrows indicate values $\varphi_0 = N\varepsilon$ for $N = 0$ and 3. In each part, the full vertical scale is from 0 to 1 and the full horizontal scale is 24π.

Consider now the effect of the R_1-R_2 oscillatory fields (whose angular frequency ω_r is nearly resonant with ω_{ef}). The atomic beam has a thermal velocity distribution $\mathcal{D}(v)$ centered at $v = v_0$. Call $\varphi_0 = (\omega_r - \omega_{ef})L/v_0$ the dephasing building up between the Ramsey fields and the unperturbed atomic dipole during the time of flight at velocity v_0 over the distance L between R_1 and R_2. The probabilities $\mathcal{P}(e,v,N)$ [$\mathcal{P}(f,v,N)$] of detecting the state e [f] an atom prepared in f and crossing with velocity v the cavity containing N photons is

$$\mathcal{P}(e,v,N) = 1 - \mathcal{P}(f,v,N) = \sin^2(\pi v_0/2v)\cos^2[(\varphi_0 - N\varepsilon)(v_0/2v)]\,. \quad (3)$$

The sine term is the $f \rightarrow e$ transition probability in the absence of Ramsey and Stark detunings, called the "fringe contrast." It is set to 1 for $v = v_0$ by adjusting the Ramsey fields amplitude. This contrast is $\omega_r - \omega_{ef}$ independent, provided the lengths of the $R_1 - R_2$ zones are short enough. The cosine term in Eq. (3) exhibits a well-known fringe pattern when φ_0 is tuned [Fig. 2(a)]. The central (velocity-independent) fringe corresponds to $\varphi_0 = N\varepsilon$. For $\varepsilon = 2\pi$, the pattern is translated by one fringe when N increases by 1. For a field with a photon-number distribution $P_0(N)$, the transition probability $\mathcal{P}_0(e,v)$ is a sum of translated patterns:

$$\mathcal{P}_0(e,v) = \sum_N P_0(N)\mathcal{P}(e,v,N)\,. \quad (4)$$

Note that $\mathcal{P}_0(e,v)$ does not depend upon off-diagonal field density matrix elements and is field-phase insensitive. Next, consider the transition rate averaged over the $\mathcal{D}(v)$ distribution. For each N value, the averaged probability $\mathcal{P}(e,N) = \int \mathcal{D}(v)\mathcal{P}(e,v,N)dv$, considered as a function of φ_0, exhibits a single sharp feature around the velocity-independent fringe position $\varphi_0 = N\varepsilon$, all the other fringes being washed out [Fig. 2(b)]. The averaged probability for a field described by $P_0(N)$ is

$$\mathcal{P}_0(e) = \sum_N P_0(N)\mathcal{P}(e,N) = \sum_N P_0(N)\int \mathcal{D}(v)\mathcal{P}(e,v,N)dv\,. \quad (5)$$

Figures 2(c) and 2(d) show $\mathcal{P}_0(e)$ vs φ_0 for $\varepsilon = 2\pi$, the field being either a coherent or a thermal one [Poisson or exponential $P_0(N)$ distribution with mean photon number $N=3$]. The shape of the fringe pattern allows us to distinguish a coherent from a thermal or a Fock-state field [compare Figs. 2(c), 2(d), and 2(b)]. In order to measure $\mathcal{P}_0(e)$ from which $P_0(N)$ can be deduced, we merely have to detect a sequence of atoms for each value of $\omega_r-\omega_{ef}$ and to determine the mean $f\to e$ transfer rate. The field must be in the same initial state before each atom interacts with it. Practically, this means that the field relaxes back to equilibrium between consecutive atoms, with negligible damping, however, during the time L_c/v_0 ("moderate Q" cavity fed by a stationary microwave source).

We now analyze another experiment where the field is not sensibly relaxing between atoms (very high Q cavity). Repeated measurements are performed on the field initially described by $P_0(N)$. Quantum mechanics prescribes that each atom measured in e or f "reduces" the field density operator, $P_0(N)$ becoming $P_1(N)$ after the first atom, $P_n(N)$ after the nth one. The process being field phase independent, $P_n(N)$ is given by a simple probability argument. Consider an experiment in an ideal (infinite Q) cavity, in which both the atomic state *and* velocity v are measured (the IC counter yielding time-resolved signals allows us to perform also a time-of-flight measurement). Let us call $\{a_k,v_k\}$ a set of measurements on a sequence of n atoms ($1\le k\le n$, a_k stands for e or f). The joint probability $\mathcal{R}(N;\{a_k,v_k\})$ for the field to contain N photons *and* for the $\{a_k,v_k\}$ sequence to be realized is

$$\mathcal{R}(N;\{a_k,v_k\})=P_0(N)\prod_k[\mathcal{D}(v_k)\mathcal{P}(a_k,v_k,N)]\,. \quad (6)$$

Suppose now that the sequence $\{a_k,v_k\}$ has been measured without other information on the field. The *conditional* probability of having N photons then is

$$P_n(N)=\frac{\mathcal{R}(N;\{a_k,v_k\})}{\sum_{N'}[\mathcal{R}(N';\{a_k,v_k\})]}=\frac{P_0(N)\prod_k[\mathcal{P}(a_k,v_k,N)]}{\sum_{N'}P_0(N')\prod_k[\mathcal{P}(a_k,v_k,N')]}\,. \quad (7)$$

Equation (7) shows that measuring the kth atom in level e [or f] with velocity v_k results in multiplying $P_{k-1}(N)$ by $\mathcal{P}(e,v_k,N)$ [or $\mathcal{P}(f,v_k,N)$], with a proper normalization. This leads to a simple recipe to simulate this continuous measurement of the field: First, draw a random velocity v_1 and compute $\mathcal{P}_0(e,v_1)$ from Eq. (4); then, decide the outcome of the first e/f measurement by comparing $\mathcal{P}_0(e,v_1)$ to a random number between 0 and 1. If the outcome is e [f], multiply $P_0(N)$ by $\mathcal{P}(e,v_1,N)$ [$\mathcal{P}(f,v_1,N)$] and normalize by dividing by $\mathcal{P}_0(e,v_1)$ [$\mathcal{P}_0(f,v_1)$]. This gives $P_1(N)$. Draw then a velocity v_2, compute $\mathcal{P}_1(e,v_2)=\sum_N P_1(N)\mathcal{P}(e,v_2,N)$ to be compared to a second random number, and so on. In this way, we get $P_2(N)\cdots P_n(N)$.

We have carried out simulations of such continuous-field measurements for various φ_0 and ε values. The initial field is either coherent or thermal, with $\bar{N}$ ranging from zero to a few tens. Quite generally, $P_n(N)$ is found to converge towards a distribution representing a Fock state somewhere within the width of the initial $P_0(N)$ distribution. Figure 3 shows $P_n(N)$ after 3, 5, and 20 atoms in a typical simulation with $\varphi_0=0.15\pi$, $\varepsilon=\pi$ (initial coherent field with $\bar{N}=10$). The collapse into a Fock state (here $N=13$) requires in this case the detection of ≈ 20 atoms, which we call an "elementary measuring sequence." Another simulation will converge similarly towards another Fock state: The sequence of detected atoms measures the field and hence reduces it to an *a priori* unpredictable energy eigenstate. As expected for a QND method, the histogram of N values obtained by repeating this simulation reproduces the initial $P_0(N)$. Note that each atom does not provide a complete measurement of N, which is "pinned down" to a precise value only by gathering enough information through repeated atom detections: Each one results in multiplying $P_n(N)$ by a function of N presenting peaks and minima, thus decimating efficiently some photon numbers in the distribution, until only one is left. From then on N cannot change any longer. During a measurement, $P_n(N)$ is at each step entirely determined by the $\{a_k,v_k\}$ sequence and is independent of the order in which these values are obtained. It should be possible, in an actual experiment, to compute atom by atom the evolution of the field-density operator from the observed $\{a_k,v_k\}$ and to witness "in real time" the collapse into a Fock state. Note also that an undetected atom does not change $P_n(N)$. This is not true for resonant photon-counting processes[8,10] in which even "unread" atoms emit or absorb photons in the field.

We have also simulated the effect of a repeated QND process on a weakly relaxing field coupled to an external (thermal or coherent) source. The relaxation is neglect-

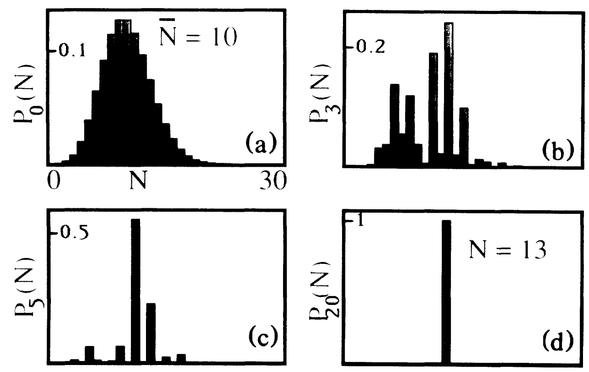

FIG. 3. Evolution of photon-number distribution $P_n(N)$ in a simulation of the $\{a_k,v_k\}$ measuring sequence. (a) Initial distribution ($n=0$, coherent field with $\bar{N}=10$); (b)-(d) $P_n(N)$ after $n=3$, 5, and 20 detected atoms, respectively ($\varepsilon=\pi$, $\varphi_0=0.15\pi$). Note the different vertical axis scale in each part. The full horizontal scale in each part is from $N=0$ to 30. Collapse into the $N=13$ Fock state is clearly observable.

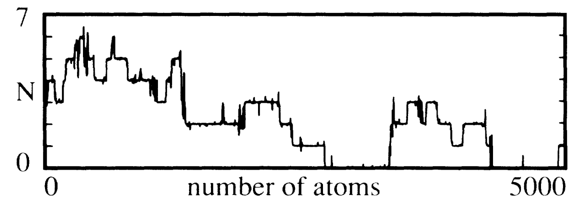

FIG. 4. Photon-number evolution in a simulation of a continuously monitored thermal field coupled into a slowly relaxing cavity ($\bar{N}=3$, $\varphi_0=0.15\pi$, $\varepsilon=\pi$). The "measured" photon number (vertical axis) is plotted against the number of atoms crossing the cavity. The total time scale corresponds to the passage of 5000 atoms and the relaxation time to 2500. Quantum jumps of the field are exhibited.

ed during the time each atom is in the cavity and perturbs the field only when the cavity is empty between atoms.[13] Figure 4 shows a typical evolution of the continuously measured photon number on a thermal field with $\bar{N}=3$. The cavity has a relaxation time corresponding to the passage of 2500 atoms, i.e., about 100 elementary measuring sequences. The field randomly varies among the possible states, spending in each an average time proportional to its stationary probability. Quantum jumps[14] are occurring over the time scale of a measuring sequence. The "noise" corresponds to processes in which field evolution is counteracted by quantum "collapses." Reading the $\{a_k, v_k\}$ sequence provides again a way of telling in which state the field is at a given time. It should be experimentally possible to know when the cavity is empty, achieving in this way, during well determined periods, effective zero-degree field temperature.

We have presented here a QND method to monitor at the microscopic level the evolution of a weakly excited microwave field. Single-photon jumps and the quantum Zeno effect[15] on a field oscillator could be observed in this way. The possibility of applying this method to detect minute forces coupled to the cavity walls is also worth considering.[16]

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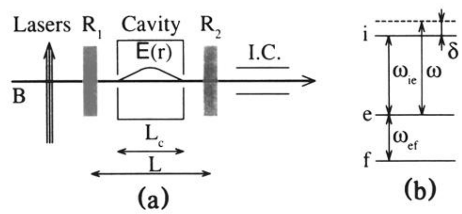

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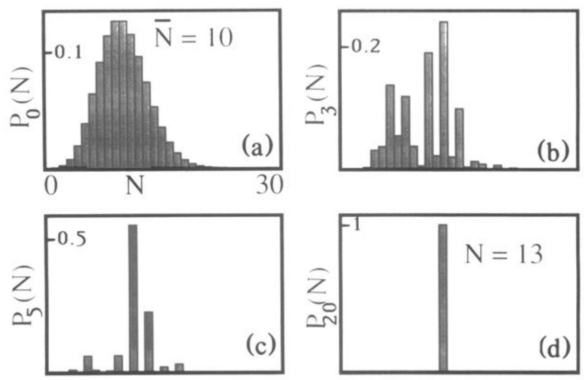

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