New Constraint on a Strongly Interacting Higgs Sector

Michael E. Peskin and Tatsu Takeuchi
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
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We show that an integral S over the spectral function of spin-1 states of the Higgs sector is constrained by precision weak-interaction measurements. Current data exclude large technicolor models; asymmetry measurements at the CERN e^+e^- collider LEP and the SLAC Linear Collider will soon provide more stringent limits on Higgs-boson strong interactions.

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The most pressing question in the study of weak interactions is the nature of the Higgs sector. In the standard SU(2)×U(1) theory of weak interactions, some new particle or set of forces is needed to break the gauge symmetry. However, experiment cannot yet distinguish models of these particles ranging from a minimal doublet of scalar fields to elaborate theories with a rich spectrum of resonances.

Among our limited set of constraints on the nature of the Higgs sector, the most important comes from precision measurements of weak-interaction parameters.[1] The famous relation[2] $m_W^2 \cong m_Z^2 \cos^2\theta_W$ implies that the Higgs sector has an approximate global SU(2) "custodial" symmetry.[3] A small deviation from this prediction measures the SU(2)-symmetry violation of the Higgs sector.

In this Letter, we will extend this conclusion to demonstrate that precision weak-interaction experiments also constrain an *isospin-symmetric* observable of the Higgs sector. In essence, we will show that, by comparing weak-interaction parameters, one can constrain not only the isospin asymmetry of this sector but also its total size. A longer and more complete version of this argument will be presented in Ref. 4.

Our analysis will be based on the general formalism for weak-interaction radiative corrections presented in Refs. 5-7. This work begins from the observation that radiative corrections to weak-interaction processes involving light quarks and leptons due to new physics beyond the standard model appears dominantly through vacuum-polarization amplitudes ("oblique corrections"). For example, modifications of the Higgs sector give vacuum-polarization corrections of order α, while vertex and box diagrams are suppressed by an additional factor of $(m/m_W)^2$, where m is the external fermion mass. The oblique corrections organize themselves into compact formulas which permit a general analysis of their effects.

To write these formulas, we use the subscript 1,3 to denote the weak isospin currents $J^\mu_{1,3}$ and the subscript Q to denote the electric-charge current J^μ_Q. We write the Z^0 current as $(e/sc)(J_3 - s^2 J_Q)$, where $s=\sin\theta_W$, $c=\cos\theta_W$. Then the W-boson self-energy is $i(e^2/s^2)\times\Pi_{11}(q^2)$; the Z^0-photon mixing amplitude is $i(e^2/sc)\times[\Pi_{3Q}(q^2) - s^2\Pi_{QQ}(q^2)]$. The Ward identity requires $\Pi_{3Q}(0)=\Pi_{QQ}(0)=0$.

All weak-interaction observables to be measured in the next few years constrain a small number of basic quantities: α, G_F, m_W, m_Z, and the amplitudes $s_*^2(q^2)$, $\rho_*(q^2)$, Z_* of Kennedy and Lynn.[6] These last three amplitudes are defined as follows: We write the Z-f-$\bar f$ vertex as const$\times[I^3 - s_*^2(q^2)Q]$. Thus $s_*^2(q^2)$ is the parameter which determines the weak-interaction forward-backward and polarization asymmetries; e.g., the polarization asymmetry for Z^0 production in e^+e^- annihilation is $A_{LR}\cong 8[\frac{1}{4} - s_*^2(m_Z^2)]$. The quantity ρ_* appears in the effective Lagrangian of low-energy weak interactions:

$$\mathcal{L}_{\text{eff}} = (4G_F/\sqrt{2})\{J_+^\mu J_-^\mu + \rho_*(0)[J_3^\mu - s_*^2(0)J_Q^\mu]^2\}\,.$$

It affects R^ν, the ratio of neutral- to charged-current deep-inelastic ν cross sections. Z_* renormalizes the Z^0 width:

$$\Gamma_Z = Z_*(\alpha_* m_Z/6s_*^2c_*^2)\sum_f (I_f^3 - s_*^2 Q_f)^2 N_f\,;$$

here, $c_*^2 = 1 - s_*^2$, parameters with an asterisk are evaluated at $q^2 = m_Z^2$, and N_f is the effective number of colors for the flavor f, including the QCD correction.

Since the Z^0 mass has now been measured with spectacular accuracy at the CERN e^+e^- collider LEP, it is most convenient to base predictions in weak-interaction theory on the measured values of α, G_F, m_Z. We find it convenient to define a weak mixing angle $\theta_W|_Z$ by

$$\sin 2\theta_W|_Z \equiv [4\pi\alpha_{*,0}(m_Z^2)/\sqrt{2}G_F m_Z^2]^{1/2}\,, \qquad (1)$$

where $\alpha_{*,0}(m_Z^2)$ is the running electric charge, evaluated at the Z^0 mass, with the renormalization computed from known physics only. Current data[1,8] give $\sin^2\theta_W|_Z = 0.23147 \pm 0.00039$. The oblique corrections to the various weak-interaction observables may then be written as

$$m_W^2 - m_Z^2\cos^2\theta_W|_Z = -\left[\frac{e^2c^2}{s^2(c^2-s^2)}\left(\Pi_{33}(m_Z^2) - 2s^2\Pi_{3Q}(m_Z^2) - \frac{s^2}{c^2}\Pi_{11}(0) - \frac{c^2-s^2}{c^2}\Pi_{11}(m_W^2)\right) + \frac{e^2s^2m_W^2}{c^2-s^2}[\Pi'_{QQ}(m_Z^2) - \Pi'_{QQ}(0)]\right],$$

$$s_*^2(q^2) - \sin^2\theta_W|_Z = \left[\frac{e^2}{c^2-s^2}\left(\frac{\Pi_{33}(m_Z^2) - 2s^2\Pi_{3Q}(m_Z^2) - \Pi_{11}(0)}{m_Z^2} - (c^2-s^2)\frac{\Pi_{3Q}(q^2)}{q^2}\right) + \frac{e^2s^2}{c^2-s^2}[s^2\Pi'_{QQ}(m_Z^2) - c^2\Pi'_{QQ}(0) + (c^2-s^2)\Pi'_{QQ}(q^2)]\right], \quad (2)$$

$$\rho_*(0) - 1 = \frac{e^2}{s^2c^2m_Z^2}[\Pi_{33}(0) - \Pi_{11}(0)],$$

$$Z_*(q^2) - 1 = \frac{e^2}{s^2c^2}\frac{d}{dq^2}(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ})|_{q^2=m_Z^2} - e^2\Pi'_{QQ}(0) - \frac{e^2(c^2-s^2)}{s^2c^2}[\Pi'_{3Q}(q^2) - s^2\Pi'_{QQ}(q^2)],$$

where $\Pi'(q^2)$ denotes $\Pi(q^2)/q^2$. We note that these formulas should be used only to compute the nonstandard corrections to these observables; the standard-model radiative corrections are not generally oblique and do not fall into such a simple form.

For the renormalization due to a heavy top quark (another case of a purely oblique correction), the largest term comes from the difference: $\Pi_{11} - \Pi_{33} = (3\alpha/16\pi s^2) \times m_t^2/m_W^2$. This renormalization affects m_W, s_*^2, and ρ_* in a way which can be easily inferred from (2). Any other momentum-independent, isospin-asymmetric contribution to the Π's will have the same phenomenology.

We now propose another simplification of (2) which is appropriate to conventional technicolor models.[9] In these models, the Higgs sector is a copy of the usual strong interactions scaled up to TeV energies. The technicolor interactions conserve conventional isospin and also parity, and we may divide the vacuum-polarization amplitudes of weak isospin currents into contributions from correlators of vector and axial-vector currents: $\Pi_{11} = \Pi_{33} = (\Pi_{VV} + \Pi_{AA})/4$, $\Pi_{3Q} = \Pi_{VV}/2$. If we expand the Π's in powers of q^2 and ignore q^4 and above (making a relative error of m_W^2/M_T^2, where M_T is a technicolor mass scale),

$$\Pi_{VV}(q^2) = q^2\Pi'_{VV}(0),$$
$$\Pi_{AA}(q^2) = \Pi_{AA}(0) + q^2\Pi'_{AA}(0). \quad (3)$$

Inserting (3) into (2), we find that m_W, s_*^2, and Z_* receive corrections proportional to $\Pi'_{VV}(0) - \Pi'_{AA}(0)$. In a large technicolor model, the correction from this source can be as large as that from a heavy top quark.

Let us now formalize these considerations as follows: Define the parameters S and T by[10]

$$\alpha S = -e^2(d/dq^2)[\Pi_{VV}(q^2) - \Pi_{AA}(q^2)]|_{q^2=0}$$
$$= 4e^2(d/dq^2)[\Pi_{33}(q^2) - \Pi_{3Q}(q^2)]|_{q^2=0}, \quad (4)$$
$$\alpha T = (e^2/s^2m_W^2)[\Pi_{11}(0) - \Pi_{33}(0)].$$

Then, if we ignore terms of order q^2 in the isospin-violating pieces of the Π's and terms of order q^4 in the isospin-symmetric terms, we find

$$m_W^2 - m_Z^2\cos^2\theta_W|_Z = m_W^2\frac{\alpha}{c^2-s^2}(c^2T - \tfrac{1}{2}S),$$
$$s_*^2(q^2) - \sin^2\theta_W|_Z = \frac{\alpha}{c^2-s^2}(-s^2c^2T + \tfrac{1}{4}S), \quad (5)$$
$$\rho_*(0) - 1 = \alpha T, \quad Z_* - 1 = \frac{\alpha}{4s^2c^2}S.$$

These equations give a model-independent two-parameter description of oblique weak radiative corrections. By comparing weak-interaction measurements, we can restrict S and T independently. Note, though, that S and T are generally both positive, so that they tend to compensate one another in any single observable.

Before discussing the phenomenological constraints on S and T, let us estimate their values in technicolor models. The combination of vacuum-polarization amplitudes which appears in the definition of S obeys the dispersion relation

$$\Pi_{VV}(q^2) - \Pi_{AA}(q^2) = -\frac{q^2}{12\pi}\int\frac{ds}{\pi}\frac{R_V(s) - R_A(s)}{s-q^2} - f_\pi^2, \quad (6)$$

TABLE I. Measurements used to constrain S and T.

Observable	Measured value	Ref.	Standard model
R_0^ν	0.3095 ± 0.004	15	0.3115
m_W/m_Z	0.8791 ± 0.0036	16	0.8786
Γ_Z	2.540 ± 0.026	8	2.501
$\sin^2\theta_W^0(\nu e)$	0.233 ± 0.014	17	0.231
$A_{FB}^b(Z^0)$	0.133 ± 0.104	18	0.065
$A_{LR}(Z^0)$	?		0.129

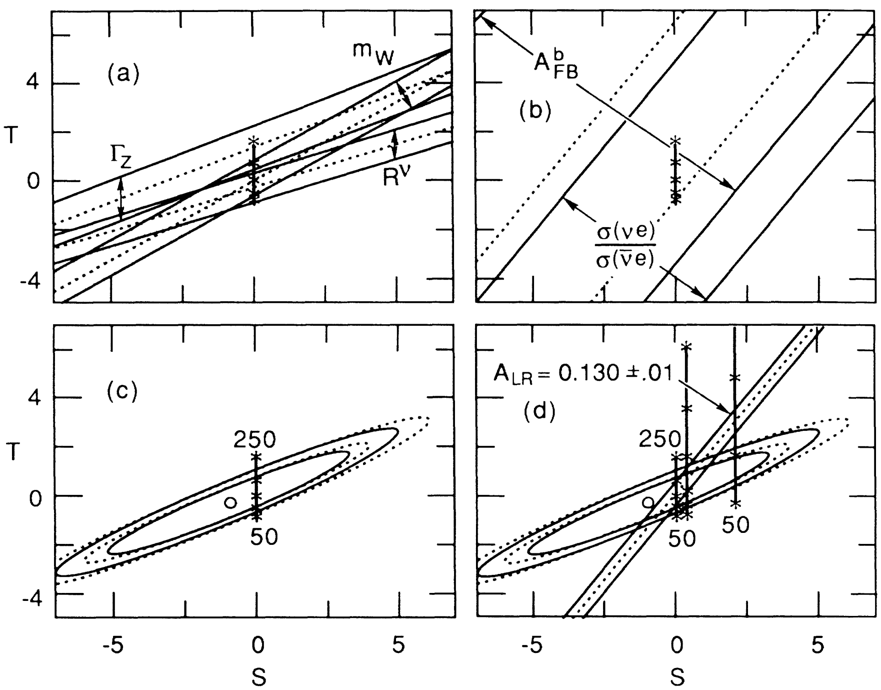

FIG. 1. Allowed region of the S-T plane: (a) Bands in the S-T plane allowed by the measured values of R^ν, m_W/m_Z, and Γ_Z, within 1σ errors; (b) bands allowed by the measurements of νe scattering and A^b_{FB}, within 1σ; (c) likelihood contours based on the five measurements, corresponding to 68% and 90% probability [the dotted curves include only the measurements in (a)]; (d) comparison of these contours with the predictions of two technicolor models, and with the band allowed by an anticipated measurement of A_{LR}. All four parts show the standard-model prediction for T and S as a function of the top-quark mass, with stars at $m_t=50$, 100, 150, 200, and 250 GeV; the last part shows the analogous curves for technicolor models with one doublet and one generation, respectively, and $N_C=4$.

where $R_V(s), R_A(s)$ are the analogs of $R(s)$, the cross section for e^+e^- annihilation to hadrons in units of the point cross section, with the electromagnetic current replaced by the vector and axial-vector isospin currents. We need the $q^2 \to 0$ limit of (6); this gives S as a zeroth Weinberg spectral-function sum rule:[11]

$$S=\frac{1}{3\pi}\int\frac{ds}{s}\left\{[R_V(s)-R_A(s)]-\frac{1}{4}\left[1-\left(1-\frac{m_H^2}{s}\right)^3\theta(s-m_H^2)\right]\right\}. \tag{7}$$

The last term of (7) is the contribution of the standard-model Higgs-boson sector which the technicolor theory replaces. To evaluate (7), we obtained the spectral functions R_V and R_A from the usual strong interactions by a fit by $\sigma(e^+e^-\to \text{hadrons})$ and rescaled these to technicolor in the standard way, using the ratio of f_π values and the relations of the large-N_C limit. The full details of this analysis will be presented in Ref. 4. We found (for $m_H=1$ TeV)

$$S=\begin{cases}0.4+0.08(N_C-4), & 1\text{ doublet},\\ 2.1+0.4(N_C-4), & (U,D,N,E),\end{cases} \tag{8}$$

where the first line of (8) refers to the minimal technicolor model and the second to a model with one generation of technifermions. The value of S is roughly proportional to the total number of weak doublets. For one generation of technifermions, this value of S gives a shift $\Delta m_W=-500$ MeV, in rough agreement with earlier estimates of the technicolor radiative correction.[5,12]

This is a very large correction, and one is tempted to conclude on this basis that such a model is excluded. However, this correction can be canceled by a large value of T. Unfortunately, T cannot be reliably estimated in technicolor models, since it depends in detail on the mechanism for generating the top-quark mass. In Ref. 13, the technicolor correction to T was estimated to be larger than the direct effect of the top quark.

The uncertainties associated with isospin-breaking contributions increase the importance of the parameter S. While T depends on small corrections to the technicolor theory, S is a simple integral over the lowest spin-1 resonances and should thus be reliably estimated. Let us now address the question of whether S can be constrained independently of the value of T.

If we apply (5) to the data, any single measurement selects a line (or, rather, a band) in the S-T plane. By combining measurements, we may determine the region in the S-T plane that they collectively allow. In this analysis, we take the standard-model calculations at the fixed values $m_t = 150$ GeV, $m_H = 1$ TeV based on the measured values of α, G_F, and m_Z as reference values. Our fit to (S,T) is based on the best-measured weak observables, as listed in Table I.[14] The first three quantities in this table—R^ν, m_W/m_Z, and Γ_Z—give the bands in the S-T plane shown in Fig. 1(a). Unfortunately, the three curves are almost parallel. From (5), it is clear that a direct measurement of s_*^2 would cross the plane at a different angle and thus bound S. Unfortunately, the best current measurements of s_*^2, given in the fourth and fifth rows of Table I, produce only the rather wide bands shown in Fig. 1(b). Combining the five measurements, we find the likelihood contours shown in Fig. 1(c). Figure 1(d) compares these measurements to the standard model with a varying top-quark mass, and to two technicolor models. (The dependence of T on m_t is only a rough estimate.[4,13]) The likelihood analysis yields an upper limit

$$S < 3.6 \quad (90\% \text{ C.L.}). \tag{9}$$

This would exclude models with two generations of technifermions—for which we expect $S \approx 4$ for $N_C = 4$—but not yet models with a single generation.

The next two years of data taking at LEP and the SLAC Linear Collider will produce much more accurate direct measurements of $s_*^2(m_Z^2)$, through the measurement of the polarization asymmetry A_{LR} and the forward-backward asymmetry to $b\bar{b}$ at the Z^0. Figure 1(d) shows the band allowed by a polarization asymmetry measurement with 10^5 Z^0's and 40% polarization; such a measurement would either verify or strongly constrain the possibility of a new strong-interaction sector associated with the Higgs bosons.

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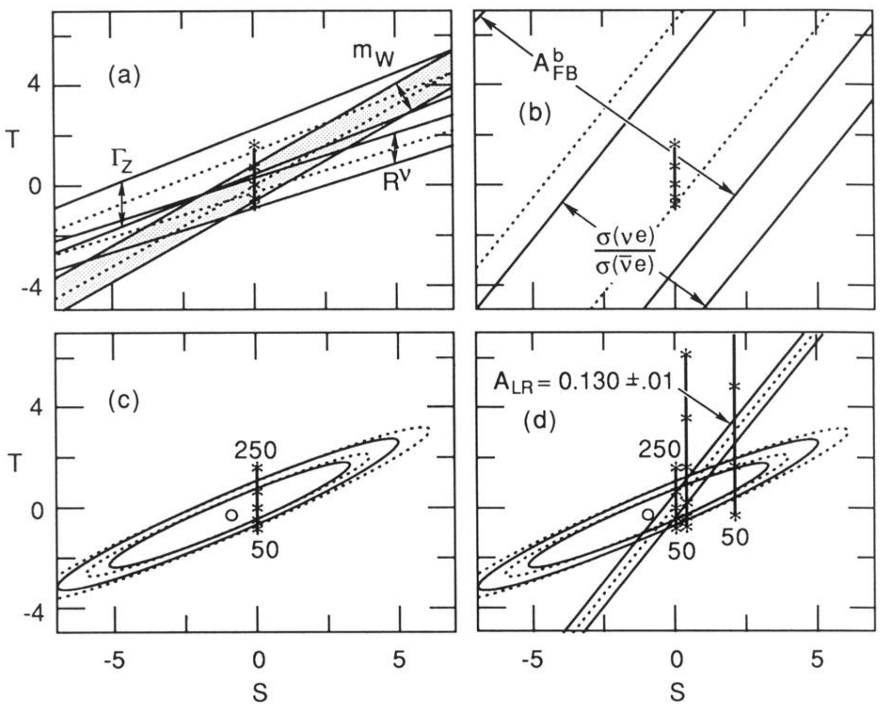

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