Quantum Phase Transitions in Disordered Two-Dimensional Superconductors

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It is argued that with increasing applied magnetic field, a disordered, superconducting thin film will undergo a zero-temperature transition into an insulating state. At this superconductor-insulator transition the field-induced vortices Bose condense. A scaling theory for this field-tuned transition is described. Right at the transition, both the longitudinal and Hall resistivities are predicted to be finite, nonzero, and have universal values.

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An applied magnetic field drastically alters the low-temperature behavior of disordered two-dimensional electron systems. Weak localization[1] is replaced by the integer and fractional quantum Hall effects.[2] Recent attention has focused on analogous bosonic systems—disordered superconducting films—which exhibit a disorder-tuned ($T=0$) superconductor-insulator transition.[3-6] What effect will an applied magnetic field have on this latter system? In this paper we present arguments that a new and fundamentally different superconductor-insulator transition should be accessible by simply tuning the magnetic field. Rather than an unbinding of vortex pairs, this field-tuned transition is driven by the delocalization and Bose condensation of field-induced vortices. A scaling theory is developed for the resistivity near and at the transition. Both resistivities ρ_{xx} and ρ_{xy} are predicted to be universal at the transition with their squares summing to approximately $(h/4e^2)^2$. Experimental results on this new field-tuned transition in amorphous a-InO_x films are reported in a companion paper.[7]

Consider first a disorder-free superconducting film in zero external field. Such a film will undergo a Kosterlitz-Thouless (KT) superconducting transition at some temperature T_c, which, due to enhanced fluctuations in 2D, is typically substantially below the bulk transition temperature T_{c_0}. Below T_{c_0} but above T_c, the Cooper-pair order parameter $\psi(r)$ obtains an appreciable magnitude, but phase fluctuations due to vortex motion prevent (quasi-) long-range order from being established. At T_c, vortices and antivortices bind into pairs, and power-law order is established, $\langle\psi^*(r)\psi(0)\rangle \sim r^{-\eta}$.

As the disorder Δ is increased, both T_c and T_{c_0} will typically be suppressed. Much effort has focused on this initial suppression for weak disorder.[4] In contrast, we focus on "strong" disorder, near Δ_c in Fig. 1, where T_c is driven all the way to zero. The point labeled Δ_c in the figure is a $T=0$ superconductor-insulator transition, which can be accessed by varying the disorder strength[4] (or film thickness[3]). Provided T_{c_0} does *not* also vanish at Δ_c, the long-length-scale physics near Δ_c can be described in terms of vortex unbinding, just as it can at the KT transition: Vortices, paired when superconducting, unbind in the electron-glass insulator. Since one is at $T=0$, though, the vortices must be treated quantum mechanically. As detailed below, vortices are, in fact, themselves bosonic particles. The electron-glass phase, in which the electron pairs are localized, can then be described, near the transition, as a Bose-condensed fluid of unbound vortices.

Consider now the effects of an applied magnetic field. In a pure system the vortices will freeze into an Abrikosov vortex lattice below a melting line B_m. In real systems with disorder, though, (quasi-) long-range crystalline correlations will be destroyed. In 2D, at finite temperature, vortex creep will then destroy phase coherence and lead to a resistance. What happens as $T\rightarrow 0$? A *classical* description of vortex dynamics would predict complete pinning by disorder at $T=0$, and hence zero resistance. This $T=0$ superconducting phase will exhibit

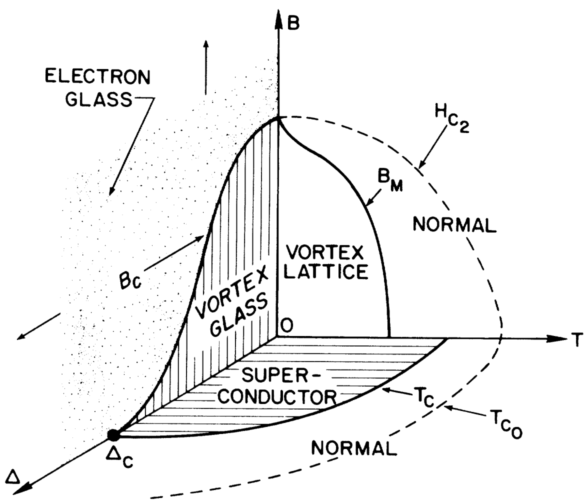

FIG. 1. Schematic phase diagram for disordered superconducting films. *Distinct* $T=0$ superconductor-insulator transitions occur at both critical disorder Δ_c *and* critical magnetic field B_c.

spin-glass-type order[8] in the pair field ψ,

$$\langle|\langle\psi^*(r)\psi(0)\rangle|^2\rangle_{\rm en}\neq 0\,, \tag{1}$$

as $r\to\infty$, where the $\langle\,\rangle_{\rm en}$ denotes an ensemble average, and is thus called a vortex glass. But what about quantum fluctuations of the vortices? At $T=0$ vortices can be *localized* by a combination of disorder and logarithmic repulsive interactions: Once localized they are immobile and do not creep. The 2D vortex glass can thus survive quantum fluctuations.

As the field is increased (at $T=0$) a remarkable possibility arises. With increasing density, the vortices should delocalize and undergo a (Bose) condensation at some critical field B_c. This condensation requires that the electron pairs be localized—in a Cooper-pair glass phase—just as Cooper-pair condensation in the vortex glass requires localized *vortices*. Near the transition there is a direct competition between condensation of Cooper pairs and vortices. This previously unstudied field-tuned superconductor-insulator transition is the focus of this paper.

Almost all of the physics below T_{c_0} and H_{c_2} in the phase diagram should be correctly described by a model of charge-$2e$ bosons, representing the Cooper pairs, moving in a random potential.[5] The continuous transitions, at T_c, Δ_c, B_c, and B_m, will be described correctly, as will the two superconducting phases, the vortex glass and the conventional $B=0$ phase. Although charge-transport properties of the electron glass (e.g., temperature dependence) should be described correctly by the localized Bose-glass phase of a charged-boson model, the spin properties may not be. Keeping this in mind, consider the following boson Hamiltonian on a 2D square lattice: $H=H_0+H_1$, with $(h=2e=1)$,

$$H_0=\tfrac{1}{2}\sum_{ij}V_{ij}(\hat{n}_i-n_0)(\hat{n}_j-n_0)+\sum_i U_i\hat{n}_i\,, \tag{2a}$$

$$H_1=-t\sum_{\langle ij\rangle}\cos(\hat{\phi}_i-\hat{\phi}_j+A_{ij}^{\rm ext})\,. \tag{2b}$$

This is a "phase-only" model wherein the Bose (Cooper pair) field $\psi\to e^{i\phi}$, with a phase ϕ conjugate to the boson number: $[\hat{\phi}_i,\hat{n}_j]=i\delta_{ij}$. Here $V_{ij}\sim(2e)^2/r_{ij}$ represents the Coulomb interaction, U_i is an on-site random potential with mean zero and variance Δ, and $\mathbf{B}=\nabla\times\mathbf{A}^{\rm ext}$ is the external field. The particle density is set by a positive-charge background n_0, taken to be much less than 1. Since the 2D screening length, $\lambda_{\rm 2D}\approx\lambda^2/d$ with d the film thickness, is typically *very* large, it is appropriate to work in the limit $\lambda_{\rm 2D}=\infty$ with no fluctuating gauge field as in (2).

Since the superconductor-insulator transitions at Δ_c and B_c are $(T=0)$ quantum analogs of the vortex-unbinding Kosterlitz-Thouless transition, a formal description in terms of vortices in the quantum regime is clearly desirable (although not essential for the scaling theory below). Fortunately, the vortex degrees of freedom implicit in the quantum Hamiltonian (2) can be made explicit by a mapping to a dual representation with dual Hamiltonian[9] $H'=H_0'+H_1'+H_2'$, with

$$H_0'=\tfrac{1}{2}\sum_{ij}G_{ij}(\hat{N}_i-B)(\hat{N}_j-B)\,, \tag{3a}$$

$$H_1'=-t'\sum_{\langle ij\rangle}\cos(\hat{\theta}_i-\hat{\theta}_j-a_{ij})\,, \tag{3b}$$

$$H_2'=H_0[\hat{n}\to\nabla\times\mathbf{a}]+\sum_i|\mathbf{\Pi}_i|^2\,. \tag{3c}$$

Here $\hat{N}_i$ is a *vortex* number *operator*, conjugate to the phase $\hat{\theta}$ of the vortex field $e^{i\theta}$: The vortices are thus bosons. The votex interaction varies logarithmically, $G_{ij}\sim-\ln(r_{ij})$, and the number of vortices is set by the applied field. In practice the vortex (core) size will be set roughly by T_{c_0} (via $\hbar v_F/k_BT_{c_0}$ with v_F the Fermi velocity) which remains finite at the $T=0$ transitions of interest (see Fig. 1).

When the vortices move, they see a (fictitious) fluctuating gauge field $\mathbf{a}$ in (3b) whose curl is the original boson density. The origin of this gauge field can be understood heuristically as follows: When a Cooper pair hops around a vortex, or similarly when a vortex is taken around a pair, the pair wave function picks up a phase 2π. In the (dual) vortex representation this phase change must be taken up by the *vortex* wave function. The gauge-field coupling in (3b) assures that this happens: Since $\nabla\times\mathbf{a}$ equals the Cooper-pair (boson) density, the Aharonov-Bohm phase factor due to the vortex hopping is precisely 2π for each pair encircled. The sound (or plasmon) mode of the original bosons is described by (3c) in the dual representation, where $\mathbf{\Pi}$ is a momentum conjugate to $\mathbf{a}$ and the Coulomb gauge $\nabla\cdot\mathbf{a}=0$ is assumed. Note the striking similarity between the original and dual representation.

Under the duality transformation, which takes one from particles to vortices, resistivities and conductivities get exchanged. Specifically, one can show formally[6,10] that the dimensionless particle resistivity tensor $\tilde{\rho}_{\alpha\beta}=(4e^2/h)\rho_{\alpha\beta}$ equals the (dimensionless) *vortice conductivity* tensor $\tilde{\sigma}^v_{\alpha\beta}$:

$$\tilde{\rho}_{\alpha\beta}=\tilde{\sigma}^v_{\alpha\beta}\,, \tag{4}$$

with $\alpha,\beta=x,y$. Since a vortex current causes 2π phase slips in ϕ, and hence a voltage $V=(\hbar/2e)\dot{\phi}$, (4) is perhaps not surprising. Equation (4) will come in handy later.

Consider first $B=T=0$. As the disorder Δ is increased, the Hamiltonian (2) should exhibit a transition from a superconducting to Bose-glass insulating phase, which is in the same universality class as the superconductor–electron-glass transition (Δ_c in Fig. 1). Here we briefly recap and extend a scaling theory for this transition, presented previously,[5] before discussing the field-tuned vortex-glass to insulator transition.

Provided the transition at Δ_c is continuous, there will be a diverging length $\xi\sim|\Delta-\Delta_c|^{-\nu}$, which sets the scale of fluctuations about $\langle\psi\rangle\neq 0$ in the superconducting phase and $\langle\psi^*(r)\psi(0)\rangle \sim e^{-r/\xi}$ in the insulating phase, with exponent $\nu \geq 2/d = 1$. A vanishing characteristic frequency $\Omega \sim \xi^{-z}$ is also expected. It was argued in Ref. 5 that the dynamical exponent z is exactly 1, due to the long-ranged Coulomb interaction; i.e., $\hbar\Omega \sim V(r = \xi)$. Near the transition, physical properties should scale with the appropriate powers of ξ and Ω. Consider the current-voltage characteristics. In the presence of a 2D current density J, a vortex picks up *energy* $(h/2e)JL$ from the Lorentz (Magnus) force when it moves a *distance* L. Thus J should enter scaling functions in the dimensionless combination $J\xi/2e\Omega$, or with $\xi \to \xi^{d-1}$ for $d \neq 2$. Likewise, since a Cooper pair picks up an energy $2eEL$ when it moves a distance L in an electric field E, the dimensionless scaling combination is $2eE\xi/h\Omega$. Thus, near the transition the I-V curves should satisfy a $(T=0)$ scaling form

$$E = (h/2e)\xi^{-1}\Omega\tilde{E}_{\pm}(J\xi^{d-1}/2e\Omega). \tag{5}$$

For nonzero currents the electric field must be nonzero. Thus as $\xi \to \infty$, the scaling functions $\tilde{E}_{\pm}(X)$ must have a power-law behavior at large X to cancel the (vanishing) prefactors in (5). This implies that right at the transition, $E \sim J^{(1+z)/(d-1+z)}$. In the 2D case of interest the linear resistivity (or resistance per square) is finite at the $(T=0)$ transition so that the system is "metallic." The Cooper pairs diffuse, something not possible for unpaired electrons[1] in 2D. As argued in Ref. 5, this metallic resistance should be *universal,* independent of all microscopic details.

The form of the scaling functions $\tilde{E}_+(X)$ and $\tilde{E}_-(X)$ for $X \to 0$ are determined by the I-V characteristics in the electron-glass and superconducting phases, respectively. From Mott variable-range hopping of (charged) electron pairs one expects that[11] $J/E \sim \exp[-(T_0/T)^{1/2}] \sim \exp[-(E_0/E)^{1/2}]$, so that $\tilde{E}_+(X) \sim |\ln(X)|^{-2}$ for $X \to 0$. In the $(B=0)$ superconducting phase an applied current generates an electric field by nucleation of vortex-antivortex pairs. At finite temperature this proceeds by thermal activation leading to a power law I-V, but at $T=0$ must proceed by quantum tunneling. At given current density J, tunneling into a state with a vortex and antivortex separated by distance r becomes possible when the Lorentz energy $(h/2e)Jr$ becomes comparable to the vortex-antivortex attractive interaction energy, which varies as $\ln(r)$. Since E will vary with the rate of tunneling, $\sim e^{-r/\xi}$, this implies $\tilde{E}_-(X) \sim e^{-|\ln X|/X}$, for $X \to 0$.

Near the zero-field transition, Δ_c in Fig. 1, temperatures should scale with Ω and magnetic fields with ϕ_0/ξ^2. Thus the KT transition temperature should vanish as $T_c \sim (\Delta_c - \Delta)^{z\nu}$, whereas the vortex- to electron-glass phase boundary should vary as $B_c \sim (\Delta_c - \Delta)^{2\nu}$. Together these imply that near Δ_c,

$$B_c \sim T_c^{2/z}, \tag{6}$$

providing a direct way to measure z, and check the prediction $z=1$.

Consider now the field-tuned superconductor-insulator transition from the vortex- to electron-glass phase. Most of the scaling results described above and in Ref. 5 for the transition at Δ_c apply equally well at this transition. Once again, near the transition one expects a diverging length $\xi_B \sim (B - B_c)^{-\nu_B}$, which sets the scale for correlations of the function in (1), with $\nu_B \geq 2/d = 1$, and a vanishing characteristic frequency $\Omega_B \sim \xi_B^{-z_B}$. Here subscripts denote the field-tuned transition. The scaling argument in Ref. 5 implies $z_B = 1$. The $T=0$ I-V characteristics near the transition should satisfy a scaling form as in (5), but with ξ and Ω replaced by ξ_B and Ω_B, respectively. This implies that right at the transition, $B = B_c$ and $T=0$, there should, once again, be a metallic (i.e., finite) linear resistivity. As at the zero-field transition,[5] this resistivity is expected to be universal, although presumably with different values for the two transitions. The scaling functions $\tilde{E}_{\pm}(X)$ for the vortex-glass to insulator transition should have the same functional forms for small and large X as the $B=0$ transition.

Because of the applied field, a Hall resistivity ρ_{xy} is also expected at the $B \neq 0$ vortex- to electron-glass transition. Like ρ_{xx}, ρ_{xy} should have a universal value at the transition. Close to this transition (i.e., $B \to B_c$ and $T \to 0$) T should be scaled by Ω_B. Thus both resistivities should satisfy scaling forms,

$$\rho_{\alpha\beta}(B,T) = (h/4e^2)\tilde{\rho}_{\alpha\beta}[c_0(B - B_c)/T^{1/z_B\nu_B}], \tag{7}$$

with c_0 a nonuniversal constant. Here $\tilde{\rho}_{\alpha\beta}[Y]$ are (analytic) dimensionless scaling functions, which are finite and nonzero at $Y \equiv c(B - B_c)/T^{1/z_B\nu_B} = 0$. The behavior of $\tilde{\rho}[Y]$ for large positive and negative arguments is determined by the transport properties of the electron- and vortex-glass phases, respectively. In the electron glass, variable-range hopping of (charged) electron pairs implies[11] $\tilde{\rho}_{\alpha\beta} \sim \exp(Y^{z_B\nu_B/2})$ for $Y \to \infty$. In the vortex-glass phase, vortex motion at $T \neq 0$ leads to a nonzero resistivity. At low temperatures motion will proceed by thermally assisted quantum tunneling, leading to a Mott variable-range (vortex) hopping *resistivity.* Because of the logarithmic interaction between vortices which produces a real "Coulomb" gap, though, one expects an Arrhenius form with possible logarithmic corrections,[8] $\rho(T) \sim e^{-T_0|\ln T|/T}$, so that $\tilde{\rho}_{\alpha\beta} \sim \exp(Y^{z_B\nu_B}\ln|Y|)$ for $Y \to -\infty$.

It should be emphasized that at $B = B_c$ the resistivities $\rho^*_{\alpha\beta} \equiv (h/4e^2)\tilde{\rho}_{\alpha\beta}[Y=0]$ are only universal when $T \equiv 0$. The leading finite-temperature corrections should be quadratic and scaled by T_c: $\rho_{\alpha\beta}(B_c,T) = \rho^*_{\alpha\beta} + O(T/T_c)^2$.

Is it possible to estimate the universal resistivities at the vortex- to electron-glass transition? As the magnetic field is increased one expects the model Hamiltonian (2) to undergo a transition from a vortex-glass phase, with long-range order as in (1), to a localized boson phase, called a Bose glass: This transition should be in the same universality class as the vortex- to electron-glass transition in real systems. In the dual representation (3), the localized Bose-glass phase is the *ordered* phase, since the *vortices* (dual particles) have (Bose) condensed. Because the vortices see an effective (fictitious) magnetic field in (3), vortex condensation is only possible at $T=0$ where it will induce spin-glass-type order, as in (1), but with the Copper-pair field ψ replaced by the vortex field $e^{i\theta}$ and made suitably gauge invariant.

It is clear that near the vortex- to electron- (or Bose) glass transition, the Cooper pairs and vortices are playing a dual role. In the vortex-glass phase the Cooper pairs have condensed and the vortices are localized, whereas in the insulating phase the vortices are condensed and the Cooper pairs localized. At the transition, neither vortices nor pairs have condensed: Both are metallic and diffuse with a finite resistance. Indeed, for a model of *logarithmically* interacting bosons (Cooper pairs) the transition is in fact *self-dual*:[12] With $V_{ij} \sim -\ln(r_{ij})$, the first term in (2a) has the same form as (3a) and, moreover, the fluctuations of the fictitious vector potential, $\delta\mathbf{a}$, about its mean value ($\nabla\times\mathbf{a}=n_0$) pick up a mass term $(\delta\mathbf{a})^2$ in (3c). Expressing the partition functions for (2) and (3) as path integrals, the fluctuations in $\delta\mathbf{a}$ (and other noncritical high-k modes) can then be integrated out without changing the long-length-scale properties near the transition. The effective coarse-grained theories thereby obtained will have equivalent forms for the original and dual models: Universality then implies self-duality at the transition. In this case the resistivity scaling functions in (7), $\tilde{\rho}(Y)\equiv\tilde{\rho}_{\alpha\beta}(Y)$, can be equated directly with the *adjoint* of their dual vortex counterparts, the vortex resistivity scaling functions: $\tilde{\rho}_{\alpha\beta}(Y)=\tilde{\rho}^{v}_{\beta\alpha}(-Y)$. [The *adjoints* are related since the fictitious field seen by the vortices in (3b) is of *opposite* sign to the physical field in (2b).] Upon combining this with (4), one deduces that the universal resistivities at the transition satisfy

$$(\rho^*_{xx})^2+(\rho^*_{xy})^2=R_Q^2\,, \tag{8}$$

with $R_Q=h/4e^2$ the quantum resistance. More generally, the scaling functions satisfy $\tilde{\rho}(Y)\tilde{\rho}^{\dagger}(-Y)=1$, which can be solved to give $\tilde{\rho}(-Y)=\tilde{\rho}(Y)/\det\tilde{\rho}(Y)$.

Equation (8) says, in effect, that right at the transition, for every Cooper pair crossing the system there is also precisely *one* vortex crossing the system. In a transport situation, the angle Θ between the particle and vortex currents is the Hall angle, $\rho^*_{xx}=\tan(\Theta)\rho^*_{xy}$. This angle can, in principle, be estimated by exploiting a close similarity between this transition, for logarithmically interacting bosons, and the transition between plateaus in the integer quantum Hall effect (IQHE). Recent work[10] suggests that these two transitions might be in the same universality class and shows that $\Theta=2\arctan(2h\sigma^*_{xx}/e^2)$, where σ^*_{xx} is the conductivity as $T\to 0$ right at the IQHE transition. Provided the universality class is the same, a calculation or measurement of σ^*_{xx} enables an estimate of Θ. Numerical calculations for the IQHE give[13] $\sigma^*_{xx}=(0.45\text{–}0.55)e^2/h$, which implies a small Hall effect at the superconductor-insulator transition ($\rho^*_{xy} \ll \rho^*_{xx}$), whereas IQHE experiments appear to find[14] $\sigma^*_{xx}=(0.2\pm0.1)e^2/h$, which would suggest a more sizable Hall effect at the superconducting transition.

Since Cooper pairs *do not* interact logarithmically, but as $1/r$, Eq. (8) (and estimates for the Hall angle) should not be taken too seriously in comparison with real systems. I suspect, though, that a more appropriate model, with $1/r$ interactions, might well give values not substantially different. In any event the above estimates should serve as a useful guide for experiment.

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[1]For example, P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[2]For example, *The Quantum Hall Effect,* edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).

[3]D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. **62**, 2180 (1989).

[4]A. F. Hebard and M. A. Paalanen, Phys. Rev. B **30**, 4063 (1984).

[5]M. P. A. Fisher, G. Grinstein, and S. M. Girvin, Phys. Rev. Lett. **64**, 587 (1990).

[6]X. G. Wen and A. Zee, Institute for Theoretical Physics report (to be published).

[7]A. F. Hebard and M. A. Paalanen, following Letter, Phys. Rev. Lett. **65**, 927 (1990).

[8]D. S. Fisher, M. P. A. Fisher, and D. A. Huse (to be published).

[9]M. P. A. Fisher and D. H. Lee, Phys. Rev. B **39**, 2756 (1989); D. H. Lee and M. P. A. Fisher, Phys. Rev. Lett. **63**, 903 (1989).

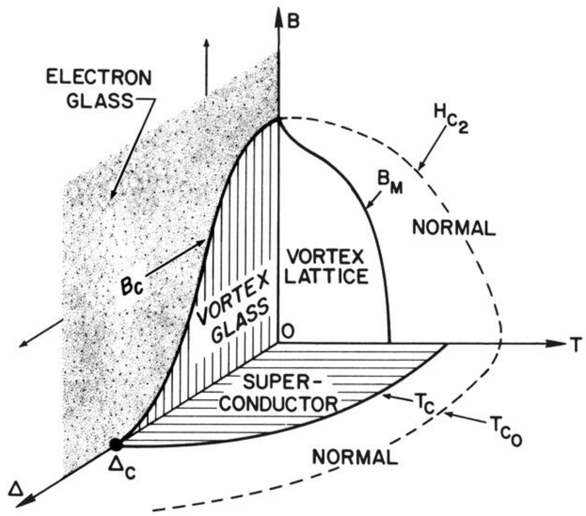
[10]M. P. A. Fisher (unpublished).

[11]A. L. Efros, J. Phys. C **9**, 2021 (1976).

[12]This possibility was suggested to me by S. M. Girvin (private communication).

[13]S. Hikami, Phys. Rev. B **29**, 3726 (1984); R. Singh and S. Chakravarty, Nucl. Phys. **B265 [FS15]**, 265 (1986); J. T. Chalker and G. J. Daniell, Phys. Rev. Lett. **61**, 593 (1988).

[14]D. C. Tsui (private communication).



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