Intrinsic Pinning and Lock-In Transition of Flux Lines in Layered Type-II Superconductors

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Reversible flux penetration in uniaxial layered superconductors is studied, taking into account the possible trapping of vortex cores between layers. This is found to induce high intrinsic critical currents parallel to the layers. In low and intermediate fields, the flux lines experience a lock-in transition towards the layer plane, as a function of the field direction or intensity. Applications to anisotropic high-T_c materials is discussed.

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High-T_c cuprate superconductors, made of weakly coupled Cu-O planes or sets of planes, have recently renewed the interest in very anisotropic superconductors.[1] Such systems fall into two classes, depending on the ratio $t_\perp/\Delta$ of the transverse electronic transfer integral coupling the planes to the two-dimensional mean-field order parameter.[2] If $t_\perp/\Delta \gg 1$, one can use the effective-mass model or anisotropic Ginzburg-Landau (GL) theory[3-7] governed by the anisotropy ratio $\varepsilon = (m_\perp/m_\parallel)^{1/2}$. On the other hand, if $t_\perp/\Delta < 1$, coupling occurs by Josephson tunneling between planes and can be treated within the Lawrence-Doniach (LD) theory.[8] High-T_c compounds (especially the more anisotropic Bi-Sr-Ca-Cu-O family) may fall in the second class, at least at low enough temperatures. An indication for this is given by the extremely small values of the coherence length $\xi_\perp$, estimated[9] to be of the order of 1-3 Å, and thus smaller than the distance d between superconducting planes. It is this last situation that we address in this Letter, for a very schematic uniaxial structure made of alternating superconducting (S) and insulating (I) layers. Some of the conclusions should equally apply for an SS'S structure where weak superconductivity (S') is induced by a proximity effect in normal layers (see also Ref. 10).

As far as the screening currents are concerned, Bulaevskii[1] has shown that the LD description essentially reduces to the anisotropic GL one, provided one defines a transverse mass from the Josephson coupling parameter η by $\eta = t_\perp^2/\varepsilon_F \approx h^2/2m_\perp d^2$ (ε_F is the Fermi energy). On the other hand, the core structure is drastically affected by the I layers. The cores of flux lines parallel to the layers fit between the S layers and any motion through the S layers involves an energy barrier, very similar to the Peierls-Nabarro barrier for dislocations in crystals. This offers the possibility to realize high intrinsic critical currents due to pinning at the atomic level and perhaps overcome the limitations occurring for usual flux pinning, due to melting of the flux lattice. More precisely, as suggested by Friedel,[11] for a SIS structure coreless vortex lines appear above some critical field $H_{c1\parallel}$, similarly to Josephson vortices. We show that due to the anisotropy a perfect lock-in of flux lines parallel to the layers occurs for some range of field orientation dependent on field intensity.

Because of the large London penetration lengths $\lambda_i = (m_i c^2/4\pi n_s e^2)^{1/2}$ for current flowing in the direction i, one can in a first analysis separate in the flux-line energy the out-of-core contribution (given by the anisotropic GL theory[4]) and the core contribution which for an element of line ds depends on the coordinate z along the normal to the layers. This analysis should be refined by a calculation of the short-length-scale contributions. It is sufficient for our purpose. Assuming the flux lines are close to the layer direction, for an element (dx,dz), $dx/dz = \tan\theta$ and $\theta' = 90° - \theta$ (see Fig. 1), and provided the line curvature is weak, the line tension is given approximately by

$$\frac{d\tau}{ds} = \left(\frac{\phi_0}{4\pi\lambda}\right)^2 \frac{\varepsilon^{-1/3}\Gamma(\theta)}{1-\gamma(\theta,\varepsilon)} \left[\ln\frac{\kappa_\parallel}{\Gamma(\theta)} + \alpha(z)\right], \qquad (1)$$

where

$$\alpha(z) = \alpha_0 - \alpha_1 \sin^2(\pi z/d),$$

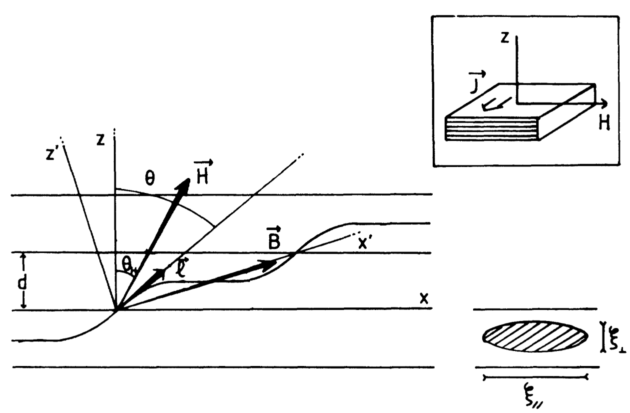

FIG. 1. Soliton lattice shape of the core of a flux line for an oblique field. The direction of the field **H** and the local line direction *l* are, respectively, defined by θ_H and θ. Inset: Configuration for parallel field and intrinsic parallel critical current.

$\lambda=(\lambda_\parallel^2\lambda_\perp)^{1/3}$ is the averaged penetration length, $\kappa_\parallel=(\lambda/d)\varepsilon^{1/3}$, and $\Gamma(\theta)=(\sin^2\theta+\varepsilon^2\cos^2\theta)^{1/2}$. Here $\gamma(\theta,\varepsilon)$ is a small expansion coefficient given for small θ' by $\gamma=\varepsilon^2\theta'^2/(2+\varepsilon^2\theta'^2)$. The usual lower cutoff given by the coherence length ξ for the out-of-core contribution is replaced by the S-layer separation d; $\alpha_1<\alpha_0$ represents the reduction of the core energy when lying between layers. We take here a simple sine modulation, valid if the thicknesses of S and I layers are not too different. More harmonics should be added to thicker I layers. Let us comment on the values of α_0 and α_1. For a line element parallel to the layers and its core in the S layers, the core parameter is α_0 and corresponds to the condensation energy lost in the S layer. It should be of the order of the usual value 0.497 (anisotropic GL theory)[3,12] or perhaps smaller for very thin S layers. For a line with core in the I layers, $\alpha(z)=\alpha_0-\alpha_1$. We take $\alpha_1<\alpha_0$ to account for possible proximity effects leading to a SS'S structure (S' weaker than S). Moreover, α_1 has a temperature dependence due to that of the transverse coherence length (the system crosses over to the three-dimensional behavior when $\xi_\perp>d/\sqrt{2}$ and α_1 vanishes). Finally, for a straight line not parallel to the layers $\alpha(z)$ averages out to $\alpha=\alpha_0-\alpha_1/2$, a value reduced from the three-dimensional value by the presence of coreless portions (in the I layers).

Let us evaluate the critical current for field and current in the layers and orthogonal to each other. The barrier against crossing of the S layers by the vortex cores is [from Eq. (1)] $\delta=(\phi_0/4\pi\lambda)^2\varepsilon^{-1/3}\alpha_1$. The pinning length being of order d, this yields a critical current $J_c\approx c\alpha_1\phi_0\varepsilon^{-1/3}/(4\pi\lambda)^2 d$. Taking, for instance, for Y-Ba-Cu-O, the values $\lambda_\parallel=1300$ Å, $\lambda_\perp=4500$ Å, $d=8$ Å, $\varepsilon=5$ (see Refs. 1 and 13), and $\alpha_1=0.5$ leads to $J_c=6.5\times10^7$ A cm^{-2}, close to the values measured in the best films at low temperatures. This intrinsic critical current is independent of the field intensity, since vortex lines are trapped individually and on their full length. This type of pinning is different from the usual pinning which is a collective effect. J_c is obviously temperature dependent, due to thermal nucleation of kinks (producing creep) but also because of the temperature dependence of the coherence length. The lattice barrier height measured by α_1 must vary as for dislocations, i.e., $\alpha_1(T)=\exp[-c\xi(T)/d]$, where c is some numerical constant.[14]

The problem of first flux penetration has been already studied in the anisotropic London theory.[4-6] The flux lines prefer to be oriented close to the easy directions (here the layer directions); thus the induction **B** is not parallel to the field **H**, making an angle θ_H with the z axis. Let us now include the core trapping and calculate in low fields the Gibbs potential of a single line, given by

$$G=\int ds\left[\frac{d\tau}{ds}-\frac{\phi_0 H}{4\pi}\cos(\theta_H-\theta)\right]. \tag{2}$$

All angles θ (but not θ_H) are supposed to be close to 90°; thus $dz/dx=90°-\theta=\theta'\ll1$. Expanding (1) and (2) for small θ' with $ds=dx[1+(dz/dx)^2]^{1/2}$ and assuming $\varepsilon\theta'\ll1$, G can then be rewritten on a unit length as

$$G=\tau_\parallel-\frac{\phi_0 H}{4\pi}\sin\theta_H-\frac{1}{2}Kq^2+\int dx\left[\frac{1}{2}K\left(\frac{dz}{dx}-q\right)^2+\frac{\delta}{2}\cos\left(\frac{2\pi z}{d}\right)\right], \tag{3}$$

with

$$\tau_\parallel=(\phi_0/4\pi\lambda)^2\varepsilon^{-1/3}(\ln\kappa_\parallel+\alpha-\alpha_1/2)$$

the line energy parallel to the layers, $K=2(\phi_0/4\pi\lambda)^2\times\varepsilon^{5/3}(\ln\kappa_\parallel+\alpha)$, and $\delta=(\phi_0/4\pi\lambda)^2\varepsilon^{-1/3}\alpha_1$. The core term acts like a commensurate potential, the field imposing to the line angle a "misfit" q, given by $q=\phi_0 H\times(\cos\theta_H)/4\pi K$. The minimization of the integral term with respect to $z(x)$ gives rise to the well-known sine-Gordon soliton lattice.[15] It shows a lock-in transition as θ is increased, at a critical value of the misfit $q_c=(4/\pi)\times(\delta/2K)^{1/2}$. For $q<q_c$ the lines enter parallel to the layers. The first critical field is then determined by setting $G=0$ with $\theta=90°$ in (3), yielding for the components of H_{c1} parallel and perpendicular to the lines $H_{c1\parallel}=(4\pi/\phi_0)\tau_\parallel$ and $H_{c1\perp}=(4\pi/\phi_0)\tau_\parallel\cot\theta_H$. The lock-in transition thus implies an extra transverse magnetization. It is important to define H_{c1} here as a function of θ_H and not of θ_B. The critical angle for lock-in of vortices at H_{c1} results from setting $H=H_{c1}$ and $q=q_c$, i.e.,

$$\tan\theta_c=\frac{\pi}{4\varepsilon}\frac{\ln\kappa_\parallel+\alpha-\alpha_1/2}{\sqrt{2\alpha_1(\ln\kappa_\parallel+\alpha)}}\approx\frac{\pi}{4\varepsilon}\left(\frac{\ln\kappa_\parallel+\alpha}{2\alpha_1}\right)^{1/2}. \tag{4}$$

With the previous numerical values this yields $\theta_c=19°$; thus at H_{c1} the region of field orientations for which vortices are trapped between layers is fairly large. In an idealized ellipsoidal sample and close to H_{c1} the components H_i of H are related to those of the external field H_0 by $H_{0i}=H_i(1-n_i)+n_iB_i\approx H_i(1-n_i)$, where n_i are the demagnetizing factors.[16] For a slab of dimensions $a=b\gg c$ one has $\tan\theta_{H_0}/\tan\theta_H\approx(2/\pi)a/c\gg1$.

One expects from the above discussion a sharp variation with θ_H of the critical currents in presence of a field: for $\theta_H<\theta_c$, J_c would be only due to extrinsic pinning of the kinks in the layers. On the other hand, for $\theta_H>\theta_c$, in the absence of metastable kinks, J_c is intrinsic. The activation energy for kink formation in parallel fields can be readily evaluated by setting $q=0$ and minimizing the free energy F for the line core passing from an I layer to another by a kink (here of the sine-Gordon type). A classical solution gives for the kink energy and length, $E_K=(2d/\pi)\sqrt{2K\delta}$ and $L_K=(2d/\pi)\sqrt{K/2\delta}$,

$$E_K=\frac{4d}{\pi}\left(\frac{\phi_0}{4\pi\lambda}\right)^2\varepsilon^{2/3}[\alpha_1(\ln\kappa_\parallel+\alpha)]^{1/2},$$
$$L_K=2\varepsilon\frac{d}{\pi}\left[\frac{1}{\alpha_1}(\ln\kappa_\parallel+\alpha)\right]^{1/2}. \tag{5}$$

With the same numerical values as above one obtains $E_K=0.12$ eV and $L_K=100$ Å. A similar length would be found in the soliton-lattice line shape for well-spaced kinks. This calls for a remark. The kink length is much smaller than the London length λ. The flux tube thus cannot bend on a whole on such a short length. The problem is indeed very similar to short kinks in dislocations in the case of strong Peierls barriers where the elastic curvature is essentially confined in a volume whose dimension is given by the kink length. At larger distances the deformation is much smoother. This simply amounts to choosing the kink length L_K as the upper cutoff in the line-energy calculation.[17] The problem for flux lines is similar since for lengths less than the screening length the phase (and thus the current) deformations are elastic. The modification entering in the logarithm is small: If one would very roughly replace λ by L_K in $\kappa_\parallel$ [Eq. (1)], the kink length would be only reduced to 65 Å. A transition to abrupt kinks is not yet completely excluded for very strong barriers and would need a separate analysis. But the effect would be negligible if α_1 is smaller, or in higher fields (see the following). On the other hand, E_K increases with the interlayer separation d and with the anisotropy factor ε, making creep less likely in more anisotropic materials such as Bi compounds. It has actually been found recently to be unmeasurable in $Bi_2Sr_2CaCu_2O_8$ for field and current parallel to the layers, which corresponds to the geometry studied here.[18] On the other hand, for **H** parallel but **J** perpendicular to the layers, i.e., for flux-line motion parallel to the layers, the quasiabsence of normal cores makes the vortices glide easily, being hardly pinned by defects. This could explain why the critical currents for **H** parallel and **J** perpendicular are so low, even in single crystals.[18] The anisotropy of the critical current for fields parallel to the layers is thus a crucial question. To finish this short discussion of intrinsic pinning effects, let us mention that even in the so-called vortex-liquid phase,[19] the core trapping effect could prevent the wandering of vortices in the z direction and thus preserve the remanence as observed in Bi-Sr-Ca-Cu-O,[20] and also still give rise to large critical currents.

In higher fields $H_{c1} \ll H \ll H_{c2}$, the calculation of the equilibrium lattice energy can be performed as in Ref. 5, but taking into account the periodicity D in the flux-line direction. Each flux line ν is determined by $y=y_\nu$ and $z'=z_\nu+g(x')$, where g is periodic with period D. The equation giving the local field apart from the vortex core in the anisotropic case can be written in the continuum approximation (valid as far as the screening currents are considered) and is

$$h_i=\lambda^2 \sum_{jklst} m_{kl} e_{lsi} e_{ktj} \frac{\partial^2 h_j}{\partial x_s \partial x_t} + \phi_0 \sum_\nu l_{i\nu} \delta(\mathbf{r}-\mathbf{r}_\nu)\,, \quad (6)$$

where e_{ijk} is the Levi-Civita symbol and

$$\mathbf{l}_\nu = [1+g'^2(x')]^{1/2}(1,0,g'(x'))$$

is the local directing vector of vortex line ν (Fig. 1). Equation (6) is a set of differential equations generalizing those of Ref. 5 and where derivatives along the x' direction now appear. Performing the three-dimensional Fourier transform in the $x'yz'$ reference frame yields the linear system $\sum_j B_{ij}(\mathbf{k}) h_j(\mathbf{k}) = \phi_0 \sum_\nu l_{\nu i}(\mathbf{k})$, where the $B_{ij}(\mathbf{k})$ are functions of the k components and the $\mathbf{l}_\nu(\mathbf{k})$ are given by

$$\mathbf{l}_\nu(\mathbf{k}) = \int_0^D \frac{dx'}{D} \mathbf{l}(x',g(x')) e^{i[k_{x'}x'+k_{z'}g(x')]}\,, \quad (7)$$

where $\mathbf{k}'=(k'_{x'},k_y,k'_{z'})$. Solving (6), the magnetic contribution to the free energy is calculated by

$$F=\frac{1}{8\pi}\int d^3r' \left[\mathbf{h}^2+\lambda^2 \sum_{ij} m_{ij}\,\mathrm{curl}_i h\,\mathrm{curl}_j h\right].$$

This expression is made explicit as usual by separating the $k_{x'}=0$ and the $(k_y,k_{z'})=(0,0)$ Fourier components. We take $H_{c1} \ll H \ll H_{c2}$, assume as before that the line direction is close to the layer direction, expand $\mathbf{l}(\mathbf{k}')$ to second order in $g(x')$, and assume that $k_{x'}$ is small (i.e., close to the lock-in transition), which gives in the xyz frame the free energy

$$F=\frac{B^2}{8\pi}\left[1+\left(\frac{dz}{dx}\right)^2\right] + \frac{B\phi_0}{(4\pi\lambda)^2}\varepsilon^{-1/3}\left[1+\left(\frac{dz}{dx}\right)^2\right] \ln\left[\left(\frac{H_{c2}}{B}\right)^{1/2}\right]. \quad (8)$$

The second term neglects further contributions to the line tension, of relative order $1/4\pi$. Moreover, one should, in principle, calculate the direction in which kinks align themselves on different lines, forming a kind of discommensuration. This can be done only to the order $k_{x'}g^2(x')$; thus the periodicity is here simply assumed to be along z'. The core contribution is added phenomenologically as in the low-field case and one gets, similarly to (3) but for a unit volume,

$$G=\frac{B}{8\pi}(B+2H^*) - \frac{HB}{4\pi}\sin\theta_H - \frac{1}{2}K_1 q_1^2 + \int dx \left[\frac{1}{2}K_1\left(\frac{dz}{dx}-q_1\right)^2 + \frac{V}{2}\cos\left(\frac{2\pi z}{d}\right)\right], \quad (9)$$

with the misfit parameter now given by $q_1 = HB(\cos\theta_H)/4\pi K_1$ and H^*, K_1, and V by

$$H^*=\frac{\phi_0}{4\pi\lambda^2}\varepsilon^{-1/3}\left\{\ln\left[\left(\frac{H_{c2}}{B}\right)^{1/2}\right]+\alpha\right\},$$

$$K_1=\frac{B}{4\pi}(B+\varepsilon^2 H^*),\quad V=\frac{B\phi_0}{(4\pi\lambda)^2}\varepsilon^{-1/3}\alpha_1\,. \quad (10)$$

Comparing $(\phi_0/B)K_1$ to K one observes the line stiffening due to the interaction between lines. The lock-in transition where the flux lines become parallel to the layers occurs at $q_1=q_c=(4/\pi)(V/2K_1)^{1/2}$. This leads to the expression of the critical angle for $H_{c1} \ll H$

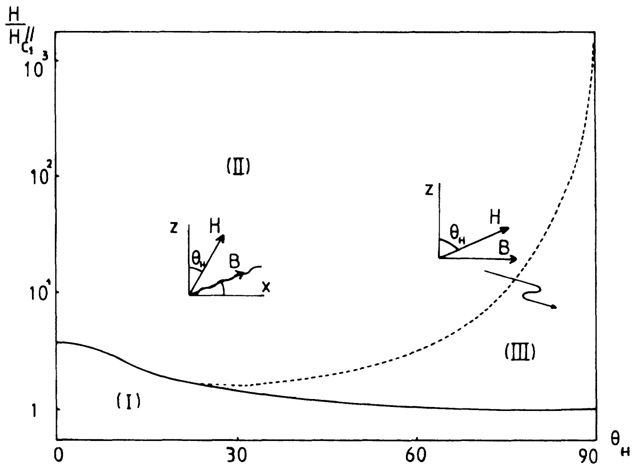

FIG. 2. Phase diagram in the H-θ_H plane. Domains I, II, and III, respectively, refer to the Meissner plane, the anisotropic flux lattice, and the intrinsic flux trapping region. The solid line gives $H_{c1}/H_{c1\parallel}$ and the dotted line the lock-in transition. The parameter values are given in the text ($\varepsilon=5$).

$\ll H_{c2}$ and $B \approx H$,

$$\cos\theta_c = \frac{1}{\pi}\left[2\alpha_1 \frac{H^*}{H}\left(1+\varepsilon^2 \frac{H^*}{H}\right)\right]^{1/2}, \qquad (11)$$

where H^* is of the order of H_{c1}. This result, together with the low-field result, can be represented on a phase diagram (Fig. 2). The region of field orientations for which the flux lines are trapped parallel to the layers increases with the barrier height (measured by α_1) and with the anisotropy factor ε, and decreases with the field intensity. This region offers the possibility of strong intrinsic pinning. Typically, for fields of the order of 100 H_{c1} (≈ 3 T) and $\alpha_1 = 0.5$ the lock-in transition occurs at $\theta_c = 86°$ if $\varepsilon = 5$ (Y-Ba-Cu-O) and $\theta_c = 69°$ if $\varepsilon = 55$ (Bi-Sr-Ca-Cu-O, see Ref. 21). For the kink length one finds $L_K = 150$ Å in the first case and 850 Å in the second. In addition, for a thin slab sample, using $B_\perp = 0$ for $\theta < \theta_c$, one still finds $\tan\theta_{H_0}/\tan\theta_H \approx (2/\pi)a/c$. At the lock-in transition, the field orientation θ must vary very abruptly as a function of the external field orientation. As a consequence, an extremely larger torque should appear. Some recently obtained results[22,23] are consistent with the present picture, but it is difficult to separate the reversible part from the irreversible one which should be present as with any other source of pinning.[24] Further work is needed to study irreversible effects linked to core trapping between layers.

After submitting this paper, we became aware of the recent observation of strong lattice pinning in epitaxial films of Y-Ba-Cu-O.[25]

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