Nature of the Extended States in the Fractional Quantum Hall Effect

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We have measured the positions of the longitudinal resistance peaks in the fractional quantum Hall regime and found that they agree well with those predicted by theory in which quantum effects are crucial to the existence of extended states.

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It is believed that in the presence of a transverse magnetic field (B) the localization length diverges at the centers of the Landau levels. The existence of such extended states is a crucial ingredient in the theories of the quantum Hall effect[1] (QHE) and is necessitated by the fact that the transport is dissipationless in the plateau region.[2]

There are two limiting cases that have been considered for the problem of the localization of an electron in this context. The classical percolation model[3,4] assumes a disorder potential that varies slowly on the scale of the magnetic length. In this case, within the semiclassical approximation, it can be shown that the wave functions lie along equipotential contours,[3,4] and the localization length at a given energy is simply the size of the largest contour at this energy. Tunneling of the electrons between different contours at the same energy is neglected in this model. It can be shown that there is a critical energy at which the localization length diverges[3] and, if one assumes a symmetric density of states (which will be assumed in the rest of the paper), this energy corresponds to the center of a Landau level. The other limiting case assumes short-range impurities[5,6] (e.g., δ-function potentials). In this case the localization length is determined by a quantum coherence between the different impurity states. It may not be as intuitively clear as in the classical percolation model that there are extended states in this model, but there is good theoretical evidence that the localization length again diverges at the center of a Landau level.[6]

It has not been possible to determine experimentally which of these two limiting cases is relevant. In the integer-quantum-Hall-effect (IQHE) regime (i.e., for noninteracting electrons) both models predict the same critical filling factors (i.e., the filling factors at which the localization length diverges), given by $\nu_\infty = \frac{3}{2}, \frac{5}{2}, \ldots$. The exponent that governs the divergence of the localization length as one approaches a critical filling factor is presumably different in the two models,[3,7] but the experiments do not measure this exponent directly,[8] and therefore cannot unambiguously determine which model is applicable to experiments.

In contrast, in the fractional-quantum-Hall-effect (FQHE) regime the critical filling factors are predicted to be different for these two models.[9] In this paper we report the measurements of the positions of the FQHE longitudinal resistance peaks in very-low-disorder two-dimensional electron systems (2DES) realized in GaAs/AlGaAs heterojunctions. We find that the results are consistent with the values predicted by the short-range impurity model.

The samples were rectangles several mm on each side cut from wafers described previously.[10] The measurements were done with an Oxford Instruments model TLM-400 (top loading into mixture) dilution refrigerator and a high-field superconducting magnet. The high-resolution data were obtained with very slow magnet current sweep rates (≈ 0.01 T/min) in order to keep the sweep upsweep downshift[11] well within the experimental uncertainty arising from other sources. We have analyzed the data[12] obtained from seven samples cut from five wafers at temperatures from 0.4 K to 13 mK. At higher temperatures the FQHE resistance minima deviate substantially from zero and the positions of the resistance peaks shift somewhat (in excess of our resolution) as the temperature is lowered to 30–20 mK. The magnitudes of the high-temperature shifts appear to be sample dependent. No appreciable shifts in the resistance peak positions were noticed as the temperature was lowered from 30–20 to 12–13 mK.[13] The data for the best-resolution sweeps for the two samples cut from wafers M73 ($n \approx 5.66 \times 10^{10}$ cm^{-2}) and M97 ($n \approx 1.03 \times 10^{11}$ cm^{-2}) are summarized in Table I. Figure 1 gives a part of the longitudinal resistance versus B trace obtained for M97 at 13 mK.

Now we briefly discuss the theories that yield the peak positions in the FQHE regime. The study of scaling in the IQHE regime is greatly simplified by the fact that the IQHE is possible for noninteracting electrons, which allows one to neglect electron-electron interactions, and, consequently, consideration of a single electron with energy equal to the Fermi energy is sufficient. However, incompressibility at fractional filling factors is obtained as a result of interelectron interactions, and it would

TABLE I. Positions of the FQHE longitudinal resistance peaks for the best-resolution sweeps.

Sample	$\nu_1 \to \nu_2$	T (mK)	ν_p (Experiment)	$\nu^{q}_{\infty} = \frac{P_1+P_2}{Q_1+Q_2}$	$\nu^{\rm cl}_{\infty} = \frac{\nu_1+\nu_2}{2}$	Comment
M97 ($n \simeq 1.03 \times 10^{11}$ cm^{-2})	$\frac{5}{7} \to \frac{2}{3}$	13	0.7028 ± 0.0014	...	0.6905	ν^{q}_{∞} not predicted
	$\frac{2}{3} \to \frac{3}{5}$	13	0.6241 ± 0.0012	0.6250	0.6333	
	$\frac{3}{5} \to \frac{4}{7}$	13	0.5824 ± 0.0011	0.5833	0.5857	
	$\frac{4}{7} \to \frac{5}{9}$	13	0.5623 ± 0.0010	0.5625	0.5635	$\frac{5}{9}$ not well developed
	$\frac{4}{9} \to \frac{3}{7}$	13	0.4392 ± 0.0009	0.4375	0.4365	$\frac{4}{9}$ not well developed
	$\frac{3}{7} \to \frac{2}{5}$	13	0.4171 ± 0.0008	0.4167	0.4143	
	$\frac{2}{5} \to \frac{1}{3}$	13	0.3745 ± 0.0012	0.3750	0.3667	
	$\frac{1}{3} \to \frac{2}{7}$	13	0.2957 ± 0.0010	...	0.3095	ν^{q}_{∞} not predicted
M73 ($n \simeq 5.66 \times 10^{10}$ cm^{-2})	$\frac{2}{3} \to \frac{3}{5}$	18	0.6269 ± 0.0025	0.6250	0.6333	
	$\frac{3}{5} \to \frac{4}{7}$	18	0.5828 ± 0.0020	0.5833	0.5857	
	$\frac{3}{7} \to \frac{2}{5}$	18	0.4178 ± 0.0018	0.4167	0.4143	
	$\frac{2}{5} \to \frac{1}{3}$	18	0.3759 ± 0.0015	0.3750	0.3667	
	$\frac{2}{9} \to \frac{1}{5}$	25	0.2138 ± 0.0013	0.2143	0.2111	

clearly be incorrect to neglect interactions when one is discussing a transition between two such incompressible states. Therefore, interactions must be included at least at the level which is sufficient to give the two incompressible states in question.

The classical percolation picture can be straightforwardly extended to describe the transition between two incompressible fractional Hall states. Consider the transition between two incompressible states at ν_1 and ν_2, where $\nu_2 > \nu_1$. In the classical percolation picture, for sufficiently weak disorder, the state at a filling factor ν ($\nu_1 < \nu < \nu_2$) will consist of regions of filling factor ν_2 in a background that is otherwise covered with the incompressible ν_1 fluid. The area covered with the ν_2 fluid is determined by the filling factor ν, and the size of the biggest singly connected region can be identified with the localization length. By analogy with the IQHE, the localization length diverges when half of the area is covered with the ν_1 fluid and the other half with the ν_2 fluid, which happens when

$$\nu^{\rm cl}_{\infty} = \frac{\nu_1 + \nu_2}{2} . \quad (1)$$

The situation is somewhat more complicated in the limit of short-range impurity potential, when quantum tunneling of electrons from one impurity to the other

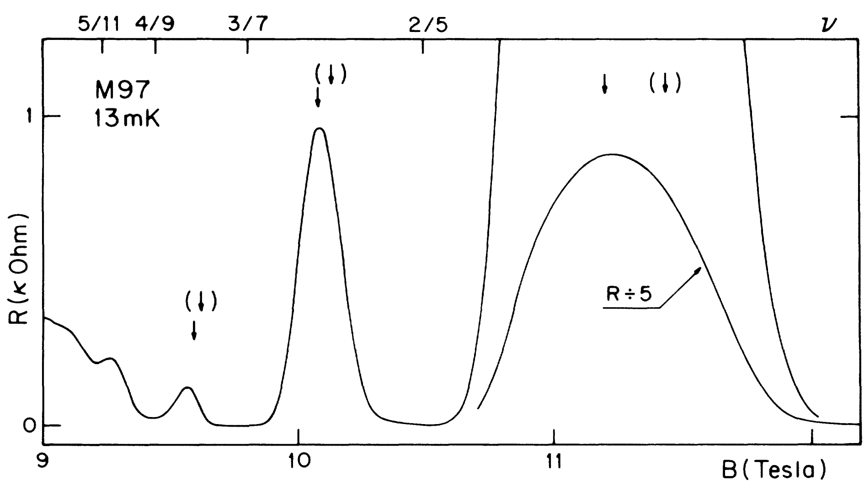

FIG. 1. Longitudinal resistance of the sample cut from the M97 wafer as a function of magnetic field in the range $0.467 < \nu < 0.345$. The arrows outside (inside) the parentheses give the theoretically predicted values ν^{q}_{∞} ($\nu^{\rm cl}_{\infty}$) for the positions of the resistance peaks.

determines the localization length. In this case, another approach has been taken that constructs trial wave functions in the FQHE regime at arbitrary filling factors in the presence of short-range disorder.[9] These trial wave functions are a generalization of the trial wave functions for the incompressible FQHE states proposed in Ref. 14. According to this approach, given the wave function Φ_{ν^*} of the state at filling factor ν^* in the presence of a given impurity configuration, a trial wave function Φ_ν for the state at ν for the same impurity configuration can be written as

$$\Phi_\nu = \prod_{j<k} (z_j - z_k)^{m-1} \Phi_{\nu^*} , \tag{2}$$

where m is an odd integer, $z_j = x_j + iy_j$ denotes the position of the jth electron, and

$$\nu = \frac{\nu^*}{(m-1)\nu^* + 1} . \tag{3}$$

In the simplest cases, Φ_{ν^*} can be taken to be an IQHE state (i.e., the ground state of noninteracting electrons). When ν^* is an integer, the state Φ_{ν^*} corresponds essentially to an integer number of filled Landau levels, and the resulting wave function Φ_ν is precisely that proposed in Ref. 14 to describe the incompressible FQHE states. It has been argued in Ref. 9 that the trial states Φ_ν provide a reasonable description of the physics at arbitrary filling factors. This implies that the transitions between certain FQHE states are analogous to the transitions between certain IQHE states; for example, the transition from $\nu = \frac{1}{3}$ to $\nu = \frac{2}{5}$ is analogous to the transition from $\nu^* = 1$ to $\nu^* = 2$. It has been further argued in Ref. 9 that the critical filling factor ν_∞^q for a transition in the FQHE regime is also related to the critical filling factor of the corresponding transition in the IQHE regime by Eq. (3). For example, given that the localization length in the transition from 1 to 2 diverges at $\frac{3}{2}$, Eq. (3) predicts that the critical filling factor for the $\frac{1}{3} \rightarrow \frac{2}{5}$ transition is $\frac{3}{8}$. Using particle-hole symmetry, the critical filling factor for the $\frac{2}{3} \rightarrow \frac{3}{5}$ transition is predicted to be $\frac{5}{8}$. The state Φ_{ν^*} can also be chosen to be a FQHE state to obtain predictions for some higher-order FQHE states. For example, the transition from $\nu^* = \frac{2}{3}$ to $\nu^* = \frac{3}{5}$ is analogous (with $m = 3$) to the transition from $\nu = \frac{2}{7}$ to $\nu = \frac{3}{11}$, which leads to a critical filling factor $\nu_\infty^q = \frac{5}{18}$ for the transition $\frac{2}{7} \rightarrow \frac{3}{11}$. In general, this theory predicts that the critical filling factor for a transition from $\nu_1 = P_1/Q_1$ to $\nu_2 = P_2/Q_2$ is given by

$$\nu_\infty^q = \frac{P_1 + P_2}{Q_1 + Q_2} . \tag{4}$$

Note that, unlike the classical percolation model, this model does not predict the critical filling factors for all transitions, but only for transitions that are related to transitions between some integer Hall plateaus in the sense described above. For example, there is no prediction for the transition $\frac{1}{3} \rightarrow \frac{2}{7}$ (or the transition $\frac{2}{3} \rightarrow \frac{5}{7}$).

As is clear from Fig. 1 and Table I, the observed critical filling factors are in good agreement with ν_∞^q. This demonstrates the importance of quantum effects (tunneling and interference) in producing extended states in the FQHE regime. The clearest test of the theoretical predictions is provided by the transitions $\frac{2}{3} \rightarrow \frac{3}{5}$ and $\frac{2}{5} \rightarrow \frac{1}{3}$. Note that $\nu_\infty^q - \nu_\infty^{cl}$ has opposite sign for the $\frac{2}{3} \rightarrow \frac{3}{5}$ and the $\frac{2}{5} \rightarrow \frac{1}{3}$ transitions. Therefore, a systematic error in the determination of the ν vs B dependence would reduce one of them and increase the other, which is contrary to our observations.

Interestingly, we find that the peak between $\frac{2}{3}$ and $\frac{3}{5}$

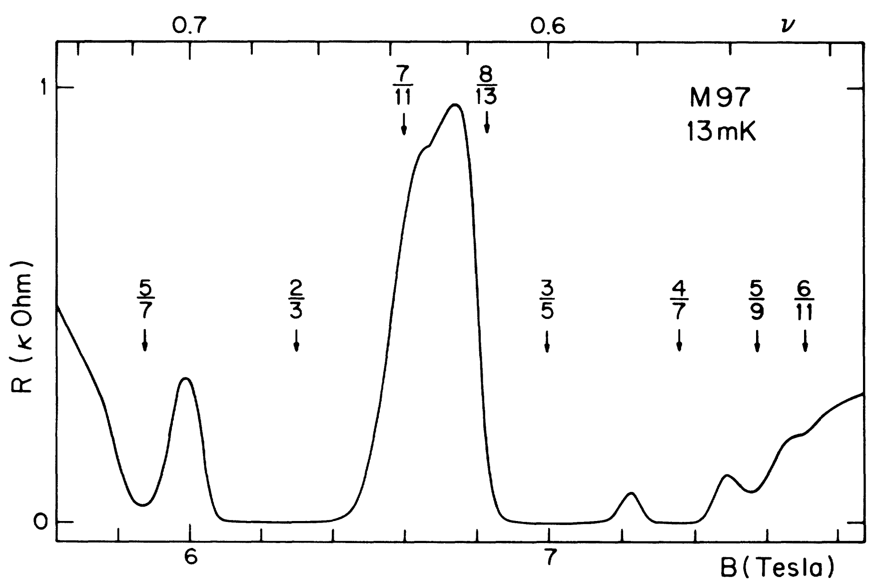

FIG. 2. A continuation of the data in Fig. 1 in the range $0.74 < \nu < 0.53$. The arrows give the *expected* FQHE state positions. The arrows at $\nu = \frac{7}{11}$ and $\frac{8}{13}$ are shown in order to emphasize the absence of these FQHE states at this temperature.

often has a shoulder, suggesting evidence of two peaks; see Fig. 2, for example. The shoulder peak structure is usually weak or sometimes not even seen above the noise level in some samples. It is, however, seen quite often in other samples, often 2 or 3 times stronger than that in Fig. 2. There is also some evidence for such a shoulder peak for the $\frac{1}{3} \rightarrow \frac{2}{5}$ transition. The position of the second peak in Fig. 2 is 0.633 ± 0.002 which is consistent with the prediction of the classical percolation model, $\nu_\infty^{cl} = 0.6333$. This suggests the interesting possibility that between two adjacent FQHE plateaus the localization length diverges not once but twice. This can be understood quite naturally if the actual 2DES is in neither of the limiting cases discussed above, but in an intermediate regime, in which the disorder potential is fairly slowly varying, so that the classical percolation model is a reasonable first approximation, but quantum corrections are also included by allowing the possibility of tunneling between various semiclassical contours. In this case, at ν_∞^{cl} there is a semiclassical contour which is extended, and it is plausible that inclusion of tunneling will not change this. Away from ν_∞^{cl}, all the classical states are finite in size. However, an extended eigenstate is still possible due to tunneling effects. Presumably, one can calculate the filling factor at which there is an extended state by ignoring the finite size of the classical states, which leads to the critical filling factor ν_∞^{q}. Thus the extended states at these two critical filling factors have two different physical origins; in one case the extended state is essentially classical in nature, whereas in the other case it is extended because of the quantum effects.

That the classical peak is not resolved for all the transitions may be a result of the fact that the quantum peak is expected to be much sharper than the classical peak because the localization length diverges with an exponent $\approx \frac{7}{3}$ in the quantum case[7] as opposed to $\frac{4}{3}$ in the classical case.[3] When the two peaks are not resolved, as is the case in the IQHE, the width of the peak may be dominated either by the classical exponent or by the quantum exponent. However, in the fractional regime, in principle, at low enough temperatures one may be able to resolve the two peaks and measure the two exponents separately.

In the end, we would like to emphasize that the predicted values of ν_∞^{q} and ν_∞^{cl} are obtained by assuming a symmetric density of states and sufficiently weak disorder. Corrections to these values are expected for experimental samples; these would depend upon the nature of disorder and the specific transition in question.

In summary, we find that the filling factors at which the localization length diverges are consistent with the predictions of Ref. 9. This implies that tunneling effects are important in the consideration of extended states in the QHE. We also see evidence that during a transition between two adjacent fractional quantum Hall plateaus the localization length diverges at two filling factors. We interpret one divergence to be of classical origin while the other is of quantum origin.

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[11] Part of the current (proportional to the sweep rate) goes through the persistent current switch connected in parallel to the magnet.

[12] We have determined ν from the positions of the minima at $\nu = 1$, $\frac{2}{3}$, $\frac{1}{3}$, and $\frac{1}{5}$ at higher temperatures when the corresponding dips are nearly touching zero yet still sharp. Surprisingly, we have always observed the positions of these dips to vary by $\sim 1\%$ on the two sides of the same sample. The sign of this discrepancy is reversed when the direction of the magnetic field is reversed.

[13] We believe that we can reach electron system temperatures lower than 15 mK since the measured resistance continues to change as the lattice temperature is lowered from 15 to 12 mK. The wiring cutoff frequency in these measurements is $\sim 10^3$ Hz and the residual heat leak to the sample is estimated to be $\leq 2 \times 10^{-14}$ W.

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